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DISPERSION RELATIONS AND GLAUBER MODEL FOR PION-DEUTERON SCATTERING

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A B S T R A C T

Forward dispersion relations are used to calculate the real part of the elastic amplitude for pion-deuteron scattering. The same real part is also calculated from the Glauber multiple scattering model. The results of the two approaches are critically compared in the light of present experiments. A theoretically deduced value for the pion-deuteron scattering lengths is also given. A theoretical prediction for the pion-deuteron total cross-section for energies below 450 MeV is found to agree rather well with experiments.

INTRODUCTION

In a recent paper ¹⁾ we discussed the present status of the Glauber model for the pion-deuteron total cross-section with special emphasis on the region 1-2 GeV. Here the double scattering correction in the theoretical total cross-section contributes only 4 %. A significant test of this term therefore requires a careful treatment of the single scattering term and in addition accurate measurements. Although precision measurements have been done, the situation is rather confusing. Only two measurements cover the whole region ^{2),3)} and they do not agree either with each other or with the Glauber model. But even if none of the present data sets alone can fully satisfy the dreams of the theoreticians when taken together they suggest that future experiments might well do so. From the theoretical point of view the encouraging feature of the data of Carter et al. ²⁾ is that the resonance structure agrees well with theory. The disappointing feature is that the cross-section is 1-2 % higher than the theoretical predictions. The latter normalization is supported by the data of Cool et al. ³⁾ which, however, exaggerates the resonance structure. Independently, the same conclusions can be reached from a study of the real part of the forward scattering amplitude. But before going into details let us recapitulate some well known facts.

Since the deuteron has isotopic spin $T = 0$, the elastic pion-deuteron scattering is independent of the pion charge when electromagnetic effects are neglected. The spin structure is, however, more complicated since the deuteron has total angular momentum $J = 1$. In contrast to the pion nucleon case the elastic amplitude F contains terms bilinear in $\vec{J}$. In the lab system where the incoming pion has momentum $\vec{k}$ and the scattered one $\vec{k}'$, the amplitude can be written

$$F = a + b \cdot (\vec{J} \cdot \vec{n}) + c \cdot (\vec{J} \cdot \vec{n})^2 + \frac{1}{2} d \cdot \{ (\vec{J} \cdot \hat{k}) \cdot (\vec{J} \cdot \hat{k}') + (\vec{J} \cdot \hat{k}') \cdot (\vec{J} \cdot \hat{k}) \} . \quad (1)$$

Here $\hat{n}$ is the unit vector in the direction $\vec{k} \times \vec{k}'$. Due to the bilinear structure in $\bar{J}$, the forward amplitude depends on the target spin. Denoting forward amplitudes by an index 0, we thus find

$$F_0 = a_0 + d_0 \cdot (\bar{J} \hat{n})^2 . \quad (2)$$

In a multiple scattering theory, the scattering is considered as a chain of successive pion-nucleon scatterings. In such a theory the spin dependent term cannot be generated by single scattering terms. It is necessary that the pion is scattered twice. However, it is necessary that both scatterings proceed through the spin-flip part of the elementary pion-nucleon amplitude. For high energies this term is therefore expected to be unimportant.

Hitherto only unpolarized targets have been used in measurements on the deuteron. For this case the differential cross-section is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{3} \text{Tr} [F^* F] . \quad (3)$$

which, for forward scattering, simplifies to

$$\frac{d\sigma}{d\Omega_0} = |a_0 + \frac{2}{3} d_0|^2 + \frac{2}{9} |d_0|^2 . \quad (4)$$

The total cross-section can be obtained from the optical theorem, which for unpolarized targets reads.

$$\sigma_{tot} = \frac{4\pi}{k} \cdot \frac{1}{3} \text{Tr} [\text{Im} F_0] = \frac{4\pi}{k} \cdot \text{Im} (a_0 + \frac{2}{3} d_0) \quad (5)$$

Therefore, measuring the total cross-section gives us the imaginary part of the combination $\mathfrak{F} = a_0 + \frac{2}{3} d_0$. The real part of $\mathfrak{F}$ is the real part that would be measured in a Coulomb interference experiment in much the same way as real parts are measured in pion-nucleon scattering. To make the argument as simple as possible let us neglect the Bethe phase between the real Coulomb amplitude f_c and the nuclear amplitude. For an unpolarized target the differential cross-section near the forward direction is then given by

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{1}{3} \text{Tr} \left[\{f_c + a_0 + d_0(\vec{q}\vec{k})^2\}^* \{f_c + a_0 + d_0(\vec{q}\vec{k})^2\} \right] \\ &= f_c^2 + 2f_c \cdot \text{Re} \{a_0 + \frac{2}{3}d_0\} + |a_0 + \frac{2}{3}d_0|^2 + \frac{2}{9}|d_0|^2. \end{aligned} \quad (6)$$

The interference term is thus proportional to the real part of $\mathfrak{g}$, but the nuclear scattering term contains apart from $|\mathfrak{g}|^2$ the additional term $\frac{2}{9}|d_0|^2$. For high energies this term is completely negligible compared to $|\mathfrak{g}|^2$, as was discussed above. In principle, the real part of the unpolarized amplitude could therefore be measured in exactly the same way as in pion-nucleon scattering.

We can also write down forward dispersion relations for the amplitudes a , b , c and d in much the same way as in the pion-nucleon case. As polarized cross-sections are not known only the dispersion relation using the total unpolarized cross-section will be considered. It relates the real part of $a_0 + \frac{2}{3}d_0$ to its imaginary part. Thus the real part obtained from the dispersion relation is just the real part that would be measured in a Coulomb interference experiment. The actual form of the dispersion relation differs from the pion-nucleon case in two respects. First of all no poles are present, since the pion-deuteron system has isospin unity and no bound two-nucleon state of this isospin exists. Secondly the integration in the dispersion integral extends into the unphysical region.

In the following we will often talk about the real part of the pion deuteron amplitude. With this expression we always mean the real part of $a_0 + \frac{2}{3}d_0$, and we will frequently call this combination the amplitude f for pion-deuteron scattering. The dispersion relation thus takes the form

$$\text{Re } f(\omega) = \text{Re } f(\mu) + \frac{2}{\pi} (\omega^2 - \mu^2) \cdot \mathcal{P} \int_{\omega_0}^{\infty} d\omega' \frac{\omega' \text{Im } f(\omega')}{(\omega'^2 - \mu^2)(\omega'^2 - \omega^2)} \quad (7)$$

Here $\omega_0 < \mu$ is the starting point for the anomalous thresholds and $\text{Re } f(\mu)$ is the threshold value of the real part. The imaginary part $\text{Im } f(\omega)$ in the unphysical region $\omega_0 < \omega < \mu$ cannot be directly measured, but it is a very attractive feature of the dispersion integral above that the contribution from this region to the real part is for high energies a constant independent of the energy ω . Moreover it turns out to be small.

The amplitude can independently be calculated from the Glauber multiple scattering model ⁴⁾. A comparison between the two predictions for the real part is of course of great interest, as the experimental input in the two approaches is quite different. Furthermore the validity of assumptions in one approach may be checked by confrontation with the predictions of the other. Clearly a direct comparison with experimental data would be even more interesting. For the moment, such data are absent but within reach by present techniques.

THE DISPERSIVE APPROACH

The evaluation of the real part from the dispersion relation (7) is not a completely straightforward task. Two main difficulties are encountered. The first one concerns the subtraction constant. In principle it could be determined from a measurement of the pion-deuteron scattering length but for the moment this is not available. A theoretical estimate is therefore necessary. The second difficulty is associated with the unphysical region, where no information on the imaginary part can be obtained from direct measurements. Also here we must resort to theoretical considerations. Let us in turn discuss our treatment of these problems.

For the estimate of the subtraction constant we make use of low energy multiple scattering theory. The details of the calculation are explained in the Appendix. In the multiple scattering expansion, we retain only single and double scattering terms. As the evaluation is done at threshold only S waves contribute to the amplitudes. For this case the double scattering terms can be evaluated in closed form. The result is of course quite different from the high energy result, where one exploits the fact that the amplitudes are strongly peaked in the forward direction. Using the recent scattering lengths of Lovelace et al.⁵⁾ we find

$$\text{Re} f(\mu) \simeq \frac{2}{3} (a_1 + 2a_3) + 2 \cdot \langle \frac{1}{r} \rangle \cdot \left[\left(\frac{a_1 + 2a_3}{3} \right)^2 - 2 \cdot \left(\frac{a_1 - a_3}{3} \right)^2 \right] = 0.21 \text{ GeV}^{-1}. \quad (8)$$

This is in effect a very small value, in qualitative agreement with the predictions of current algebra⁶⁾. The uncertainty comes mainly from uncertainties due to absorption. We think it is good to about 30 %. Even if the estimate is only moderately good, this has only little influence on the value of the real part at high energies as it contributes less than 3 %.

In this connection we would like to point out that the isospin symmetric scattering length $a_1 + 2a_3$ is positive in the analysis of Lovelace et al.⁵⁾ whereas a previous determination⁷⁾ gave a negative sign, a possibility supported by the negative scattering lengths obtained from π mesic atoms for $T = 0$ nuclei. It is therefore of great interest to resolve this ambiguity. We think that a direct experimental determination of the pion-deuteron scattering length would settle this question.

We now come to the second difficulty, the unphysical region. This region is due to the existence of thresholds starting below the elastic threshold. The lowest threshold turns out to be an anomalous threshold. Its position cannot be obtained from first principles. One must instead rely on results from perturbation theory. This problem has previously been investigated⁸⁾. The lowest anomalous threshold is obtained from the triangle diagram in Fig. 1, which gives

$$s_0 = D^2 + \mu^2 - \frac{\mu^2 D^2}{2m^2} - 2\mu D \sqrt{\left(1 - \frac{D^2}{4m^2}\right) \cdot \left(1 - \frac{\mu^2}{4m^2}\right)} . \quad (9)$$

Here D denotes the deuteron mass. The lower integration limit in the dispersion integral (7) obtained from this s_0 is

$$\omega_0 = \frac{1}{2D} (s_0 - D^2 - \mu^2) \simeq -\frac{\mu^2}{2m} - \mu \sqrt{\frac{B}{m}} , \quad (10)$$

where B is the deuteron binding energy. Numerically one finds $\omega_0 = -17.1$ MeV.

The contribution from the unphysical region to the dispersion integral can be calculated in impulse approximation. It then becomes simply the sum of the pion-proton and pion-neutron pole terms, which is

$$\text{Re } f_{\text{pole}} = \frac{2f^2}{m} \cdot \frac{1}{1 - \frac{\mu^2}{4m^2}} \cdot \frac{k^2}{\omega^2 - \left(\frac{\mu^2}{2m}\right)^2} . \quad (11)$$

Although this approximation is rather crude it shows in a simple way that the unphysical region gives a small contribution to high energy scattering. However, at high energies, it is independent of the pion energy. In this limit one finds

$$\text{Re } f_{\text{pole}} = 0.18 \text{ GeV}^{-1} . \quad (12)$$

which is a rather small value. In addition to this pole contribution there is another contribution from the unphysical region coming from a quite different physical process. This is the physical absorption of the pion in the deuteron which extends into the unphysical region and contributes to the dispersion integral. The main contribution from this process comes from the region close to the physical threshold, where the imaginary part in a rather reliable way can be obtained from an extrapolation from the physical region. We therefore turn to an investigation of the inelastic processes near threshold.

Let us take the specific case of π^- deuteron scattering. Apart from the elastic scattering the main contribution to the total cross-section near the physical threshold comes from the following inelastic processes

$$\pi^- + d \rightarrow \pi^- + p + n \quad (13a)$$

$$\rightarrow \pi^0 + n + n \quad (13b)$$

$$\rightarrow n + n + \gamma \quad (13c)$$

$$\rightarrow n + n \quad (13d)$$

Also other processes are present but they are all small corrections to those listed above. A well-known parity argument shows that the exchange process (13b) is very improbable at threshold. This is also confirmed experimentally⁹⁾. Of the remaining processes (13a) gives a finite total cross-section at threshold and consequently does not contribute to the imaginary part at threshold. We are thus left with only two reactions, (13c) and (13d). The electromagnetic process (13c) is of order α . It will not be considered since electromagnetic effects should not be included in our dispersion integral (7). The strong process (13d) is more important than the electromagnetic process (13c) as can be seen from the measured ratio¹⁰⁾

$$S = \frac{\sigma(\pi^- d \rightarrow n n)}{\sigma(\pi^- d \rightarrow n n \gamma)} = 3.02 \pm 0.1 \quad (14)$$

The rather small value of this ratio indicates that the absorption cut should only give a small contribution to the dispersion integral.

The reaction $\pi^+ + d \rightarrow p + p$ has been studied experimentally. From isospin invariance it follows that the rate for this reaction is the same as for $\pi^- + d \rightarrow n + n$. The threshold behaviour is found to be¹¹⁾

$$\sigma(\pi^+ d \rightarrow p p) = \frac{2}{3} \cdot \frac{p^2}{\mu^2} \cdot \left(\frac{\alpha}{\eta} + \beta \eta \right) \quad (15a)$$

where

$$\alpha = 0.24 \pm 0.02 \text{ mb} , \quad (15b)$$

$$\beta = 0.74 \pm 0.05 \text{ mb} . \quad (15c)$$

In (15a), η denotes the cms momentum of the pion in units of μ and p denotes the cms momentum of the proton. From Eqs. (15), we get, neglecting electromagnetic effects,

$$\text{Im} f(\mu) = 0.034 \text{ GeV}^{-1} . \quad (16)$$

This value fixes the scale for the inelastic processes in the unphysical region.

In order to investigate the importance of the unphysical region we have tried two simple parametrizations for the imaginary part. The lower integration limit used in the dispersion integral is the value obtained from the triangle diagram, even if the final result does not depend critically on the exact value used. The parametrizations used are

$$\text{Im} f(\omega) = 0.034 + 0.108 \cdot (\omega^2 - \mu^2) / \mu^2 \text{ GeV}^{-1} , \quad (17a)$$

$$\text{Im} f(\omega) = 0.034 \cdot \omega^2 / \mu^2 \text{ GeV}^{-1} . \quad (17b)$$

Both functions are chosen so as to join the threshold value (16). The two terms in (17a) are obtained from an expansion of (15a) at threshold in the lab momentum of the pion. The two parametrizations give almost the same contribution to the real part. The difference is, at high energies, 0.02 GeV^{-1} . The total contribution from the unphysical region to the real part at high energies using the parametrization (17a) is 0.04 GeV^{-1} . This is indeed a small number. It is furthermore explicitly found that the main part comes from the region close to threshold, where the extrapolation from the physical region is harmless. This is due to the threshold pole in the dispersion integrand.

Before turning to the physical region let us summarize the discussion given above. The contribution from the unphysical region to the dispersion integral is made up of two parts. The first one is the coherent sum of the nucleon pole terms as given by Eq. (11). The second one is the contribution from absorption processes. We will in the following always use the parametrization (17a) for this process and the lower limit in the dispersion integral is taken to be $\omega_0 = -17.1$ MeV as discussed above. For the subtraction constant we use the value given by Eq. (8).

In the physical region the imaginary part is obtained from the total cross-section. Here measurements are available from the low energy region up to 20 GeV ¹²⁾. The experimental situation for energies below 0.5 GeV is shown in Fig. 2. For a discussion of the region 0.5 - 2.0 GeV, we refer to Ref. ¹⁾. For energies above 3 GeV, we have made a least square fit to the data. Our preferred parametrization is

$$\sigma(E) = 40.56 + \frac{39.64}{E^{2/3}} \text{ mb} , \quad (18)$$

where E is measured in GeV. The virtue of this parametrization is that the contribution from the high energy region can be evaluated analytically. This can easily be done for some simple rational powers in the energy, but of these the choice (18) was found to give the best fit. For energies below 1 GeV, we have also made a fit to the data. It is a bit annoying that in the whole region 1 - 2 GeV, there are systematic differences between the different existing measurements. The difference is certainly not big, but for the comparison with theory on the level we discuss here it is crucial. We have therefore chosen to make two different evaluations of the dispersion integral for the two different sets of values. In order to get a smooth transition to the region above 3 GeV, the two sets will differ also in the region 2-3 GeV, where the experimental data are very poor.

Our results for the two inputs are shown in Figs. 4 and 5. Also shown in the same figures are the results of a Glauber model calculation. Before discussing the significance of the results, let us describe how these calculations have been done.

THE MULTIPLE SCATTERING APPROACH

The basis for our calculations is now the Glauber multiple scattering model. It can be used to calculate both the real and imaginary parts of the elastic pion-deuteron forward scattering amplitude from a knowledge of the corresponding amplitudes for pion-nucleon scattering. The imaginary part of this amplitude gives the total cross-section through the optical theorem. The treatment of this term has previously been discussed in detail ¹⁾. It contains two basic contributions of different physical origin ; one single scattering term and one double scattering term. We therefore write the total cross-section as

$$\sigma_{\pi D} = \sigma_s + \sigma_d . \quad (19)$$

The main contribution comes from the single scattering term σ_s . It is independent of any specific multiple scattering theory. For energies below 2 GeV, it was found essential to account for the Fermi motion in the deuteron in an appropriate way. The small double scattering term σ_d on the other hand was considered in the usual high energy approximation where the intermediate scatterings are restricted to the forward direction.

The discussion in Ref. ¹⁾ was confined to the energy region 0.4 - 2.0 GeV, where accurate measurements of the pion-deuteron total cross-section are available. We have here extended this comparison to the low energy region, which in particular includes the (33) resonance region. Our predictions are shown in Fig. 2, together with the experimental data. The entire region is strongly dominated by the single scattering term, which can be accurately treated. The contribution from the double scattering term is negative and in the region 160 - 550 MeV, it reduces the total cross section by 8 - 12 %.

The (33) resonance has an immense effect on the total cross-section. This is clearly seen in our Fig. 2. The resonance is, however, considerably broadened by the Fermi motion, and at the

resonance itself the effective reduction of the cross-section is 18 mb. Also in the double scattering term the (33) resonance produces a sharp peak, which at the resonance, accounts for a reduction of 23 mb.

Our predictions for the total cross-section agree rather well with experiments in the resonance region within the rather large experimental uncertainties ($\sim 10\%$), while in the region 275 - 550 MeV, they are systematically and significantly slightly below the accurate experimental values. This deviation is probably due to a too crude approximation in the double scattering term. Including non-forward intermediate scatterings would decrease the magnitude of the double scattering term and thus increase the theoretical cross-section. There is a clear-cut difference between the region above 1 GeV and the one below as concerns this approximation. In the former region the error produced by this approximation is easily estimated. It is about 0.2 mb or smaller. In the low energy region, however, the magnitude of the error is not so obvious. In addition the double scattering contribution is here more important than for higher energies. In the resonance region one probably also has to worry about absorption effects. It is therefore urgent to study the double scattering term more carefully than has hitherto been done. Such an investigation is now in progress.

We now turn to the real part. Here the method is the same as for the total cross-section. The real part is conveniently considered as a sum of single and double scattering contributions. As usual we use the letter D to denote the real parts and write

$$D_{\pi b} = D_s + D_d . \quad (20)$$

For energies below 2 GeV, the real part is varying more strongly than the total cross-section. It is therefore essential to incorporate the Fermi motion in the single scattering term. The derivation of the appropriate formula is not difficult. The final factors differ somewhat from those in the total cross-section. The reason for this difference

is of course that the total cross-section is not exactly the imaginary part of the elastic forward amplitude ; there is an additional kinematical factor depending on the four-momenta of the participating pion and nucleon. We thus end up with

$$D_s = \int d^3p \{ |\varphi_s(\vec{p})|^2 + |\varphi_D(\vec{p})|^2 \} \cdot \frac{M}{E} \cdot \{ D_+^L(\vec{k}, \vec{p}) + D_-^L(\vec{k}, \vec{p}) \}, \quad (21)$$

where $\varphi_S(\vec{p})$ and $\varphi_D(\vec{p})$ are the S and D wave parts of the deuteron wave function, and $E = \sqrt{\vec{p}^2 + M^2}$ is the nucleon energy. The real parts of the forward π^+ and π^- proton scattering amplitudes in the proton lab system are denoted $D_{\pm}^L(\vec{k}, \vec{p})$. The lab momentum at which the amplitude should be evaluated is the one corresponding to the configuration where the momentum of the pion is $\vec{k}$ and that of the proton $\vec{p}$.

In Fig. 3, we have displayed the real part as calculated from (21), and simultaneously the coherent sum of the pion-nucleon amplitudes, i.e., the single scattering contribution to the real part when the Fermi motion in the deuteron is neglected. The real parts of the pion-proton amplitudes were obtained from forward pion-nucleon dispersion relations ¹³⁾, and the wave function used was the Gartenhaus function as given in ¹⁴⁾, but normalized as in ¹⁾.

The effects of the Fermi motion are qualitatively the same as for the total cross-section as discussed in Ref. ¹⁾. It makes the real part much smoother. At maxima and minima, the Fermi motion correction is far more important than the double scattering correction. Especially noticeable is the situation around 0.6 GeV. Here the coherent sum exhibits a maximum followed by a minimum. When the Fermi motion is included this structure disappears and only a shoulder remains. This behaviour is supported by the dispersion calculation. For energies above 2.5 GeV, the Fermi motion is unimportant due to the absence of structure in the pion-nucleon amplitudes.

Let us now turn to the double scattering term, which we treat in the conventional high energy approximation. Thus we neglect Fermi motion effects. The formula reads ¹⁵⁾

$$D_d = -\frac{1}{k} \cdot \langle \frac{1}{r^2} \rangle \cdot \{ D_+ \cdot A_- + D_- \cdot A_+ - \frac{1}{2} (A_+ - A_-) \cdot (D_+ - D_-) \}, \quad (22)$$

where in the Gartenhaus function

$$\langle \frac{1}{r^2} \rangle = 0.308 \text{ fm}^{-2}. \quad (23)$$

The notations are obvious. $A_{\pm}$ denotes the imaginary parts of the forward elastic lab amplitude for scattering of $\pi^{\pm}$ on protons and k is the pion momentum. As the motion of the nucleons has been neglected, all pion-nucleon amplitudes are lab amplitudes. For energies higher than 0.2 GeV, the double scattering contribution is always positive. When the energy increases from 1.0 GeV to 2.5 GeV, it increases smoothly from 0.1 to 0.6 GeV^{-1} , but there are some fluctuations for lower energies.

The net result for the real part is shown in Figs. 4 and 5, where it is compared with the results obtained by the dispersive method.

In Fig. 4, the data of Carter et al.²⁾ has been used in the evaluation of the dispersion integral. For energies below 0.4 GeV, there are some discrepancies between the dispersive evaluation and the multiple scattering prediction, which are slightly bigger than the double scattering contribution. As the knowledge of the total cross-section is somewhat approximate for these low energies it is difficult to judge how serious this difference really is. In the intermediate energy region 0.4-1.6 GeV the two methods are in beautiful agreement. This is mainly due to our careful treatment of the Fermi motion in the single scattering term. The agreement does not persist in the high energy region. Instead the two curves diverge and the difference becomes greater than the double scattering term. There is a natural guess as to the origin of this discrepancy. As discussed at some length in Ref.¹⁾, there is a systematic difference between the existing measurements of the cross-section over the whole region 1-2 GeV. The measurements of

Carter et al. are here 1-2 % higher than the predictions of the simple Glauber model, which in turn is favoured by the data of Cool et al. Calculations with renormalized Carter data show that it is just around 1.6 GeV that the effect of such a renormalization would set in. And it has just the correct sign. It tends to improve the agreement with the multiple scattering model. The very good agreement in the intermediate region is not destroyed by such a renormalization. This fact lends confidence to our treatment of the subtraction constant and the unphysical region. Specially noticeable is the very good agreement in the region 1.0-1.2 GeV where the double scattering term is very small, only $0.1-0.2 \text{ GeV}^{-1}$. At these energies the unphysical region and the subtraction constant contribute with their full strengths.

In Fig. 5, we have done the same comparison but using the data of Cool et al. In the resonance region the outcome of the dispersion relation is rather different from that of the multiple scattering model. Specially marked is the discrepancy in the region 1.0-1.2 GeV, where it is many times larger than the contribution from the double scattering term. It is difficult to see how such a discrepancy could be blamed on the failure of the theoretical models. A more likely origin of this difference lies in the experimental data which exaggerate the resonance structure as compared to the data of Carter et al. and the predictions of the Glauber model. Such effects certainly turn up in the real part, if deduced from a dispersion integral. There is, however, one encouraging feature of this experimental result. For energies higher than 2.2 GeV, the agreement with the Glauber model is very good. At these energies, the precise variation of the measured cross-section in the resonance region is less important. The total normalization is the crucial feature. This is an alternative way to arrive at the conclusions already deduced from an analysis of the total cross-section : theory supports the energy dependence of the cross-section as found by Carter et al. but supports the total normalization as found by Cool et al.

CONCLUSIONS

There are two independent way to determine the real part of the forward elastic scattering amplitude in pion-deuteron scattering. One of them makes use of conventional dispersion relations. The experimental input is the total charge independent cross-section. For the moment there is no experimental information that can shed light on the subtraction constant or the unphysical region. Here theoretical models are necessary. Fortunately, the unphysical region seems to yield only small contributions in agreement with theoretical estimates. More important, at high energies, it gives a contribution independent of energy.

The other way is to calculate the real part from some multiple scattering model, of course with due attention to the Fermi motion of the target nucleons. The input data are now the pion-nucleon amplitudes. The situation in the deuteron case is very favourable. The model independent single scattering term accounts for more than 90 % of the contribution for energies above 1 GeV. Another attractive feature of the forward direction is that only on-shell amplitudes are needed for this dominant term.

Comparison between the two approaches to the real part shows that the non-trivial part of the dispersion relation can be estimated in a rather accurate way. The unresolved discrepancy between the two precision measurements of the total cross-section reflects itself also in the real part of the amplitude. Direct measurements of the real part of the forward scattering amplitude as well as new precision measurements of the pion-deuteron total cross-section above 1 GeV are highly needed.

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APPENDIX

ESTIMATE OF THE PION-DEUTERON SCATTERING LENGTH

Near threshold the pion-nucleon interaction is adequately described by the S wave terms. The scattering matrix is usually written in the form

$$K = a + b \cdot (\bar{\tau} \cdot \bar{\tau}) \quad , \quad (A.1)$$

where $\bar{\tau}$ and $\bar{\tau}$ are the isotopic spin matrices for the nucleon and the pion. The parameters a and b are related to the scattering lengths through the formulae

$$a = \frac{1}{3} (a_1 + 2a_3) \quad (A.2a)$$

$$b = \frac{1}{3} (a_1 - a_3) \quad . \quad (A.2b)$$

Here a_3 and a_1 denote the scattering lengths in the $T = \frac{3}{2}$ and $T = \frac{1}{2}$ channels. Using these facts we can derive the pion-deuteron threshold amplitude using low energy multiple scattering theory. We neglect the motion of the nucleons inside the deuteron. Furthermore we use the amplitude (A.1) also off the mass shell, which means that the pion-nucleon interaction is assumed to be point-like and of the form $[a + b \cdot (\bar{\tau} \cdot \bar{\tau})] \cdot \delta(\bar{x}_\pi - \bar{x}_i)$. With only single and double scattering terms retained the low energy multiple scattering expansion gives the threshold formula

$$f_{\pi D} = f_{\pi+p} + f_{\pi+n} + \langle \frac{1}{r} \rangle \cdot \{ 2f_{\pi+p} \cdot f_{\pi+n} - f_{c.e.}^2 \} \quad (A.3)$$

The minus sign in front of the charge exchange term comes from the fact that an exchange of neutron and proton in the deuteron changes the sign of the deuteron wave function. The expectation value multiplying the double scattering term is numerically given by

$$\langle \frac{1}{r} \rangle = 0.646 \mu \quad , \quad (A.4)$$

if the Gartenhaus function ¹⁴⁾ is used. We also remark that although the formula (A.3) is written down for π^+ scattering we get the same net result for π^- or π^0 .

The actual evaluation of (A.3) is done in the pion-deuteron c.m. system. We neglect the difference between this system and the pion-nucleon c.m. system. Combining (A.3) and (A.1) we then get

$$f_{\pi D} = 2a + 2 \cdot \left\langle \frac{1}{r} \right\rangle \cdot (a^2 - 2b^2) . \quad (A.5)$$

For the numerical evaluation we use the scattering lengths as given by Lovelace ⁵⁾

$$3a = 0.069/\mu , \quad (A.6a)$$

$$3b = 0.263/\mu . \quad (A.6b)$$

Explicitly displaying single and double scattering contributions they give

$$f_{\pi D} = 0.046/\mu - 0.019/\mu = 0.192 \text{ GeV}^{-1} . \quad (A.7)$$

Due to the exceptional smallness of a , the double scattering term is seen to be comparable in magnitude with the single scattering term. Unfortunately there is still some confusion about the actual value of a . Samaranayake and Woolcock ⁸⁾ find

$$3a = -0.035/\mu , \quad (A.8a)$$

$$3b = 0.292/\mu , \quad (A.8b)$$

which gives

$$f_{\pi D} = -0.023/\mu - 0.024/\mu = -0.341 \text{ GeV}^{-1} . \quad (A.9)$$

a result quite different from (A.7).

In view of the great uncertainty in a , a direct measurement of the pion-deuteron scattering length would be most valuable. The double scattering term is accurately known, and even a moderately good determination of the pion-deuteron scattering length puts a strong restriction on the possible values of the important quantity a .

The main source of uncertainty in one formula (A.5) comes from the neglect of absorption effects. Their contribution to the real part is not exactly known but it is expected to be of the same magnitude as the imaginary part. Dispersion arguments indicate that it is repulsive. Invoking our estimate (16) in the main text we find

$$\operatorname{Re} f_{\pi d} \approx -\frac{1}{2} \operatorname{Im} f_{\pi d} \approx -0.034 \text{ Gev}^{-1} . \quad (\text{A.10})$$

We thus believe our formula (A.5) for the scattering length to be good to this accuracy.

The subtraction constant needed in the dispersion relation is the threshold value of the real part in the lab system. It differs from (A.5) only through a trivial kinematical factor. Using the scattering lengths of Lovelace ⁵⁾ we find

$$\operatorname{Re} f_{\pi d}^L = \left(1 + \frac{\mu}{D}\right) \cdot 0.192 = 0.208 \text{ Gev}^{-1} . \quad (\text{A.11})$$

This is the value used in our dispersion relation.

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FIGURE CAPTIONS

- Figure 1 This triangle diagram gives an anomalous threshold which determines the lower integration limit in the dispersion integral.
- Figure 2 The pion-deuteron total cross-section for energies below 500 MeV. For the experimental points below 300 MeV, there is an additional 10 % uncertainty in the pion energy. The solid line shows our theoretical predictions.
- Figure 3 This figure shows the importance of the Fermi motion. The dashed line is the coherent sum of the real parts of the π proton and π neutron amplitudes and the solid line is the coherent sum folded with Fermi motion.
- Figure 4 The solid line is the real part calculated from dispersion theory using the data of Carter et al. and the dashed line is the real part predicted by Glauber's multiple scattering theory.
- Figure 5 The solid line is the real part calculated from dispersion theory using the data of Cool et al. and the solid line is the real part predicted by Glauber's multiple scattering theory.

FIG.1

FIG.2

FIG.3

FIG.4

KINETIC ENERGY (MeV)

FIG.5