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ADLER'S SELF-CONSISTENCY CONDITION FOR NUCLEI

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A B S T R A C T

It is shown that Adler's self-consistency condition for light nuclei (^4He , ^{12}C , ^9Be , etc.) implies the smallness of the contribution from the unphysical region to charge symmetric π -nucleus forward dispersion relations.

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1. INTRODUCTION

As is well known the PCAC hypothesis provides a relation between the matrix element for any strong interaction process and the matrix element for the same process with the emission of an additional soft pion. To the extent that the pion pole dominates the axial current this relation should remain valid for the corresponding hard pion reaction. Classical examples in particle physics are the Goldberger-Treiman relation and the consistency condition ¹⁾ for the part $A^{(+)}$ of the crossing even forward πN amplitude ^{*)}

$$A_{\pi N}^{(+)}(r=0, r_B=0, k^2=0) = \frac{g_A^2(k^2=0)}{4\pi m} = \frac{4m}{\mu^2} f(k^2=0) \approx 2.14 \mu^{-1}, \quad (1)$$

which is satisfied to 10% or better. In the case of KN elastic scattering an analogous condition deduced from a partially conserved axial kaon current seems to work also, according to a recent discussion ²⁾.

Condition (1) is at unphysical energy $s=m^2$ and unphysical momentum transfer $t=\mu^2$. The physical hard pion amplitude can be continued into these points by means of dispersion relations. The extrapolation in the pion mass into the point $k^2=0$, however, necessitates a model and is therefore more doubtful. The off-mass shell correction deduced in Ref. 1) from the behaviour of the 33 Born partial wave is 2%. We shall consider the smallness of this correction to be justified mainly by the successful application of soft pion methods in this and other places ³⁾. (Cf. footnote on p. 4.)

It is natural to try out these ideas for pion scattering on nuclei. Pion scattering on light nuclei X has been treated by forward dispersion relations elsewhere ⁴⁾. One of the results of that paper was that the unphysical cut, which is due to the compositeness of the target, has little

^{*)} We denote the four vectors of the incoming and outgoing pions by k and q , those of the nucleons by p_1 and p_2 . The conventional variables ν and ν_B are

$$\nu = (p_1 + p_2) \cdot k / (2m) = (t + 2s - 2m^2 - \mu^2 - k^2) / (4m)$$

$$\nu_B = -q \cdot k / (2m) = (t - \mu^2 - k^2) / (4m)$$

where $m(\mu)$ is the nucleon (pion) mass. The normalization in (1) differs from Adler's by $A = A_{\text{Adler}} / (4\pi)$. We use $f^2 = 0.08$.

effect, already in the 33 resonance region. The contribution is smaller than 10%, where the amplitude is large and hence the effect is detectable. It is our purpose to make plausible that Adler's constraint on the πX amplitude is the deeper reason for the smallness of the unphysical contribution to the dispersion integral. In this context it is worth mentioning that in nucleon X scattering, where no such condition exists, the unphysical cut dominates the low energy behaviour ⁵⁾.

2. CONSISTENCY CONDITION FOR NUCLEI; GENERAL REMARKS

The consistency condition for the charge symmetric amplitude $T^{(+)} = \frac{1}{2}(T_{\pi^+X} + T_{\pi^-X})$ for a pion scattered on any nucleus X reads ^{*})

$$T^{(+)}(v=0, v_B=0, k^2=0) = - \sum_i \frac{2 f_i^2(k^2=0)}{\omega_i} \quad (2)$$

In the derivation of the πN condition (1) the pn mass difference has been neglected. In the same spirit, for a nucleus X (of mass M), all nuclear pole terms going over nuclei X_i with mass $M_i = M + \epsilon_i$ appear on the right-hand side of Adler's condition (2), provided $\epsilon_i \ll \mu$ which is true for most nuclear excitations. If the intermediate mass differs appreciably from M (as the hyperon mass in KN scattering does) such a state does not contribute to the right-hand side of (2).

To trace the consequences of (2) we shall first assume, as mentioned in the Introduction, that the dependence of T on the pion mass k^2 is smooth and negligible over a range of order μ^2 , i.e., we neglect the off-mass-shell effect and put $k^2 = \mu^2$. In addition we take momentarily the momentum transfer at the closest physical value $t=0$ (or $v_B = -(\mu^2)/(2M)$). In this case v is equal to ω , the pion laboratory energy. Once subtracted forward dispersion relations ⁴⁾ then define the zero energy amplitude as

^{*}) For practical application X will have isospin 0 or $\frac{1}{2}$ and low spin (assuming spin independence if necessary). The πXX_i coupling constant is defined in analogy to the πN coupling f^2 . The pole positions are $\omega_i = -(\mu^2)/(2M) + \epsilon_i$. Our normalization gives the laboratory cross-section as $d\sigma/d\Omega = |T|^2$.

$$\begin{aligned} & \operatorname{Re} T^{(+)}(\omega=0, t=0, k^2=\mu^2) - \operatorname{Re} T^{(+)}(\omega=\mu) = \\ & = - \sum_i \frac{2 f_i^2(k^2=\mu^2, t=0)}{\omega_i} (1 - \omega_i^2/\mu^2)^{-1} - \frac{2\mu^2}{\pi} \int_{\omega_0 \approx 0}^{\infty} \frac{\operatorname{Im} T^{(+)}(\omega') d\omega'}{k'^2 \omega'} . \end{aligned} \quad (3)$$

It is important to realize that in this formulation the pole terms enter in the same way the condition (2) and the dispersion relation (3). Without needing a precise knowledge of the pole terms the validity of (2) therefore means the smallness of the remaining terms in (3), subtraction constant and dispersion integral (as compared to the pole terms)

$$\operatorname{Re} T^{(+)}(\omega=\mu) - \frac{2\mu^2}{\pi} \int_{\omega_0 \approx 0}^{\infty} \frac{\operatorname{Im} T^{(+)}(\omega') d\omega'}{k'^2 \omega'} \approx 0 \quad (4)$$

The subtraction constant being small (see below) this is a condition on the dispersion integral (containing the large 33 resonance contribution). Therefore, Eq. (4) contains no information on the sign of $\operatorname{Re} T^{(+)}(\mu)$.

We now use the success of Adler's condition in the πN case and the similarity in shape of total πX cross-sections above threshold to the sum of $\pi^+ p$ and $\pi^- p$ total cross-sections (scaling law), to convert (4) into the statement that the unphysical contribution of the integral ($\omega' < \mu$) has to be small in the nuclear case.

In fact, in the πN case (where $\omega_0 = \mu$), Eq. (4) is only 4% of the πN residue $4mf^2/\mu^2$ of condition (1). These 4% come almost exclusively from the integral, the subtraction constant, i.e., the charge symmetric πN scattering length, being very small ⁶⁾, $a_{\pi N}^{(+)} = -0.012 \pm 0.006 \mu^{-1}$, cf. also ⁷⁾. Turning to the nuclear case we remark immediately that due to the scaling property $\sigma_{\pi X} = \frac{A^n}{2} (\sigma_{\pi^+ p} + \sigma_{\pi^- p})$, where $n \approx 0.8$, the integral over physical energies is the same apart from normalization. The nuclear

subtraction constants $T^{(+)}(\mu)$ are also small ^{*}) and behave similarly with atomic number A : $T^{(+)} = 0.035A \mu^{-1}$ ($A \lesssim 30$) as deduced from an optical potential calculation by Krell, quoted in ⁸⁾. Again the experimental values [Eq. (7) below and Ref. 4)] show somewhat weaker A dependence: $\text{Re } T(\mu) = -0.16 \mu^{-1}$ for ${}^4\text{He}$, $-0.26 \mu^{-1}$ for ${}^{12}\text{C}$, etc. We also note that the nuclear pole terms are, as order of magnitude, given by the impulse approximation estimate $A f^2_{4m} / \mu^2 (= A 2.14 \mu^{-1})$, cf. the discussion for ${}^9\text{Be}$ in ⁴⁾.

For all the nuclei having the above-mentioned properties, condition (4), and hence (2), is saturated already by the physical contributions to the integral only, to the same degree as it is in the πN case.

Consequently, the contribution from the unphysical region has to be small itself

$$-\frac{2\mu^2}{\pi} \int_{\omega \approx 0}^{\mu} \frac{\text{Im } T^{(+)}(\omega') d\omega'}{k'^2 \omega'} \approx 0, \quad (5)$$

although it is blown up by the large dispersion denominator near $\omega' \approx 0$, which is due to the evaluation at $\omega=0$. In fact it has to be even smaller than the physical contribution if the condition (2) for nuclei were valid with the same accuracy as the original πN condition (1).

^{*}) In view of this smallness, it is evident that off-mass-shell and rescattering corrections will be large in relative size. These effects have been discussed ⁸⁾ recently for the isospin-antisymmetric pion scattering length on nuclei. Absorption and correlation effects turned out to be small, whereas rescattering reduces the Weinberg ⁹⁾ value $T^{(-)}(\mu) = 0.11 \mu^{-1}$ by typically $0.05 \mu^{-1}$ (for $A \approx 10$). In our case, just counting rescattering graphs, we expect a correction to $\text{Re } T^{(+)}(\mu)$ of order

$$A^2 \left[a_{\pi N}^{(+)^2} - \frac{1}{2} a_{\pi N}^{(-)^2} \right] - A \left[a_{\pi N}^{(+)^2} + a_{\pi N}^{(-)^2} \right] \approx -a_{\pi N}^{(-)^2} \left[\frac{1}{2} A^2 + A \right]$$

which is able to explain negative nuclear scattering lengths (as observed), independent of the controversial sign ^{6), 7)} of $a_{\pi N}^{(+)}$.

This then explains, assuming a reasonably smooth energy variation of $\text{Im } T^{(+)}(\omega)$, why the dispersive real parts for ${}^4\text{He}$ and ${}^{12}\text{C}$ above threshold are also compatible with almost negligible contributions from the unphysical region.

3. NUMERICAL EVALUATION

To get a feeling of what Eq. (5) could mean numerically we use some simple parametrization of $\text{Im } T(\omega)$ below threshold. We just take a scattering length expansion and continue below threshold by the replacement $k \rightarrow i|k|$,

$$\text{Im } T(\omega) = \alpha + \beta |k| + \gamma |k|^2 + \delta |k|^3 + \epsilon |k|^4, \quad k^2 < 0. \quad (6a)$$

For zero effective range the coefficients are related to complex s , p and d wave scattering lengths ^{*)} by

$$\alpha = \text{Im } a_0, \quad \beta = -2k_0 \text{Im } a_0, \quad \gamma \approx -3 \text{Im } a_1, \quad \delta \approx 0, \quad \epsilon \approx 5 \text{Im } a_2 \quad (6b)$$

The largest terms are due to α and γ , leading to the parametrization used in ⁴⁾. Lines I, II and V of the Table show the results of this simplified form, for ${}^4\text{He}$ and ${}^{12}\text{C}$, typical for all light nuclei. The results for the full form of (6) are shown in line III, using the $\pi {}^4\text{He}$ scattering lengths from Eq. (7) below, which are obtained from our fit to the combined phase shifts of Ref. 10). The fitting is compatible with zero effective range,

$$\begin{aligned} \mu a_0 &= -0.16 (\pm 4\%) + i 0.035 (\pm 50\%) \\ \mu^3 a_1 &= 0.24 + i 0.067 \\ \mu^5 a_2 &= 0.034 + i 0.009 \end{aligned} \quad (7)$$

^{*)} The scattering lengths are defined by $\lim_{k \rightarrow 0} k^{2l+1} \text{ctg } \delta_l = a_l^{-1}$. The difference between lab. and c.m. momenta is neglected in (6). The s wave contributions to γ , δ and ϵ are negligible.

Looking at the Table, the mentioned parametrization would indicate an unphysical contribution U not exceeding 60% of the physical one, P . This is small in the sense of Adler's condition (2), but not very small in the sense of Eq. (5). This situation is remedied at once if we impose on (6) the auxiliary condition $\text{Im} T(\omega) = 0$ at $\omega = 0$, as demanded by Adler's condition (2). In line IV the p wave coefficient γ has been adjusted to meet this requirement. U then turns out to be negligible (8%).

The merit of Adler's condition is therefore to suppress the unphysical contribution near $\omega = 0$, where our extrapolation (6) is most doubtful, thereby warranting a small unphysical contribution at physical energies ω also.

Finally we have to justify the use of $t=0$ (or $v_B = -\frac{\mu^2}{2M}$) instead of $t = \mu^2$ (or $v_B = 0$) in the previous discussion of Adler's condition (2). We shall rely very much on the analogy to the πN case. The continuation in t can be made with the help of the Legendre polynomials provided a detailed phase shift analysis exists for a sufficient number of energies. In this way Adler¹⁾ was able to show that

$$A_{\pi N}^{(+)}(v=0, v_B=0, k^2=\mu^2) - A_{\pi N}^{(+)}(v=0, v_B=-\frac{\mu^2}{2m}, k^2=\mu^2) = 0.21 \mu^{-1}$$

using fixed $t = \mu^2$ dispersion relations. This is a 10% correction to the right-hand side of condition (1). We believe that this result gives a correct order of magnitude also for the πX case. The comparison of the full πN amplitude^{*)}

*) The physical $t=0$ amplitude $T_{\pi N}^{(+)}$ reduces trivially to its component $A^{(+)}$ at $\omega=0$. However, in going with the full amplitude to the Adler point one has to define the sequence of limits in the pole term as

$$\lim_{\omega \rightarrow 0} \omega B_{\text{pole}}^{(+)} = \frac{g^2}{8\pi m} \lim_{v_B \rightarrow 0} \lim_{\omega \rightarrow 0} \omega \left(\frac{1}{v_B - \omega} - \frac{1}{v_B - \omega} \right) = 0$$

to get $A^{(+)}$ as limit of the full amplitude. This is in agreement with our procedure. In the reversed order of limits Adler's value $g^2/(4m)$ for $A^{(+)}$ is just cancelled by the limit of a $B_{\text{pole}}^{(+)}$ leading to the conclusion that $T^{(+)}$ should stay approximately zero from $\omega=0$ up to threshold, which is Weinberg's⁹⁾ argument for the smallness of the symmetric s wave scattering length. This is however not relevant for the present discussion of the unphysical region.

$$T_{\pi N}^{(+)}(\omega, \nu_B=0, \mu^2) = A^{(+)} + \omega B^{(+)} \quad \text{with} \quad T_{\pi X}^{(+)}(\omega, \nu_B=0, \mu^2)$$

at unphysical $t = \mu^2$ is possible near threshold and strengthens the belief in this analogy: Legendre continuation gives $T_{\pi^4\text{He}}(\mu, 0, \mu^2) = 1.17 \mu^{-1}$ using Nordberg ¹⁰⁾, or $0.83 \mu^{-1}$ from Eq. (7), whereas ${}^6\text{AT}_{\pi N}^{(+)}(\mu, 0, \mu^2) = 4 \times 0.22 \mu^{-1}$.

In this connection we remark that there is also some support of this analogy for light elements, when looking at the t channel singularity structure of nuclear scattering. For $\pi^4\text{He}$ scattering, e.g., the anomalous threshold due to $N\bar{N}$ exchange starts ¹¹⁾ at $t = 10 \mu^2$. It is therefore not the closest singularity, which instead comes from 2π exchange at $t = 4 \mu^2$. For the deuteron the situation is exceptional, the anomalous cut extending down to $t = 1.7 \mu^2$. Therefore the deuteron is a case apart, also in the present context.

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T A B L E

		α	β	$-\gamma$	ϵ	$U(\mu^{-1})$	U/P	Ref.
$\pi^4\text{He}$	I	0.084	0	0.064	0	0.16	-0.30	10a)
	II	0.035	0	0.21	0	-0.26	0.60	Eq. (7)
	III	0.035	0.011	0.21	0.045	-0.20	0.44	Eq. (7)
	IV	0.035	0.011	0.091	0.045	0.036	-0.08	text
$\pi^{12}\text{C}$	V	0.063	0	0.22	0	-0.25	0.23	4)

Contributions to the dispersion integral in Eq. (4) from the unphysical region U and the ratio U/P to the physical region P. The parameters α , β , γ and ϵ are defined in Eq. (6).

R E F E R E N C E S

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