CM-P00057846

WEAK ELECTRON SCATTERING : $e^- + p \rightarrow \Lambda + \nu$

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A B S T R A C T

It is emphasized that intense, monochromatic electron beams from superconducting accelerators may make it possible to study certain electron induced weak interaction processes. In particular the process $e^- + p \rightarrow \Lambda + \nu$ is analyzed. After a review of a number of familiar points which can be examined via this reaction, i.e., i) time-reversal invariance, ii) the Cabibbo hypothesis, iii) partial conservation of the strangeness-changing axial-vector current (PCAC), we give numerical estimates for the total and differential cross-sections and for possible time-reversal violating terms, based on simple models.

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⁺⁺ Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under AFOSR Grant Nr 68-1365.

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1. INTRODUCTION

It has been realized for some time that electron beams could be used, as an alternative to neutrino beams, to investigate certain weak interaction processes ¹⁾. As a result of the recent development of superconducting electron accelerators, it appears that such experiments may actually become feasible in the not too distant future; thus it is now of interest to examine these processes in some detail and in particular to obtain numerical estimates of the cross-sections, etc., which might be expected. Electron beams offer two advantages over neutrino beams: higher intensity and monochromatic energy; background problems, however, are greatly increased since the electron also interacts electromagnetically with the target. Nevertheless, with intense, monochromatic electron beams, one might be able to study strangeness-changing weak interactions in spite of their small cross-sections by operating at an energy below the associated production threshold, since in this energy region there is no real background of strange particles produced by the strong interactions. Such experiments will be difficult at best, but a great deal can be learned from them.

Of the several such strangeness-changing processes allowed by the $\Delta S = \Delta Q$ rule, i.e.,

$$e^- + p \rightarrow \Lambda + \nu$$

$$e^- + p \rightarrow \Sigma^0 + \nu$$

$$e^- + n \rightarrow \Sigma^- + \nu$$

the first, $e^- + p \rightarrow \Lambda + \nu$, appears to have the largest cross-section and thus would seem to be the most accessible experimentally. For that reason we focus our attention on it. We first review some general results and a number of points, familiar from previous discussions of neutrino interactions, which can be studied via the $e^- + p \rightarrow \Lambda + \nu$ reaction, i.e.,

- i) time-reversal invariance,
- ii) the Cabibbo theory of strangeness-changing weak interactions,
- iii) partial conservation of the axial vector current (PCAC) in the case of the strangeness-changing current.

We then proceed to a detailed analysis of such electron induced processes, and in particular of the process $e^- + p \rightarrow \Lambda + \nu$, and obtain as a result numerical estimates for the total and differential cross-sections as a function of the incident electron energy and estimates of possible time reversal violating terms, based on simple models.

2. GENERAL RESULTS

As a result of the similarity between electron and neutrino induced reactions one can in principle obtain general cross-section formulas for electron induced reactions by applying some simple transformations, which will be discussed below, to the analogous formulas for neutrino reactions obtained previously by a number of authors ²⁾. We however found it nearly as simple to derive the needed results from the beginning. Thus below we give the initial equations which define the notation we have chosen and the final cross-section formula which we used as a basis for subsequent numerical calculations. This final formula agrees with the neutrino scattering results of Adler ²⁾ but disagrees with the results of Cabibbo and Chilton ²⁾ whose Eq. (28) apparently has the wrong sign for the V-A interference term.

According to the current-current theory of weak interactions ³⁾ the matrix element for the strangeness-changing weak process $e^- + p \rightarrow \Lambda + \nu$ can be written as

$$M_{fi} = \frac{G}{\sqrt{2}} (2\pi)^4 \delta^{(4)}(P_e + P_p - P_\Lambda - P_\nu) (V + A)_\alpha l^\alpha \quad (1)$$

where p_e , p_p , p_Λ , and p_ν are the momenta of the electron, proton, lambda, and neutrino, respectively, and where $G^2/2 = 0.6755 \times 10^{-22} \text{ MeV}^{-4}$ is the universal Fermi constant obtained from muon decay ⁴⁾. The matrix element of the lepton current is just ⁵⁾

$$Q^\alpha = \bar{u}(p_\nu) \gamma^\alpha (1 - \gamma_5) u(p_e) \quad (2)$$

while the matrix elements of the vector and axial-vector hadron currents are given by

$$V_\alpha = \bar{u}(p_\Lambda) [F_V \gamma_\alpha + i F_M \sigma_{\alpha\beta} q^\beta] u(p_p) \quad (3)$$

and

$$A_\alpha = \bar{u}(p_\Lambda) [F_A \gamma_\alpha \gamma_5 + i F_T \sigma_{\alpha\beta} q^\beta \gamma_5] u(p_p) \quad (4)$$

where the four-momentum transfer is $q^\beta = (p_e - p_\nu)^\beta = (p_\Lambda - p_p)^\beta$. The form factors F_V , F_M , F_A and F_T defined by Eqs. (3) and (4) are invariant functions of q^2 and are relatively real if time-reversal invariance is valid. The induced scalar and the pseudoscalar form factors have been omitted from the hadron currents since their contribution to the matrix element is proportional to the electron mass, which we will neglect throughout.

The cross-section for producing polarized lambdas, obtained by squaring the matrix element and averaging over initial spins, is

$$\frac{d\sigma_{\text{pol}}}{d(-t)} = \frac{G^2}{8\pi (S - m_p^2)^2} \left\{ A + B \epsilon_{\alpha\beta\gamma\delta} p_e^\alpha p_p^\beta p_\Lambda^\gamma S_\Lambda^\delta + C S_\Lambda \cdot p_e + D S_\Lambda \cdot p_p \right\} \quad (5)$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the completely antisymmetric tensor with $\epsilon_{0123} = +1$.

The four-vector S_Λ covariantly describes the lambda polarization; it satisfies the conditions $S_\Lambda \cdot p_\Lambda = 0$ and $S_\Lambda^2 = -1$ and, in the rest frame of the lambda, it reduces to a unit three-vector along the spin of the lambda⁶⁾. In terms of the usual Mandelstam variables

$$\begin{aligned} s &= (p_e + p_p)^2 \\ t &= (p_e - p_\nu)^2 \\ u &= (p_e - p_\Lambda)^2 \end{aligned} \quad (6)$$

which satisfy the condition $s+t+u = m_p^2 + m_\Lambda^2$, the coefficients A, B, C, D can be written:

$$\begin{aligned} A &= [(s - m_\Lambda^2)(s - m_p^2) + \frac{1}{2}t(s - u)](|F_V|^2 + |F_A|^2) + m_\Lambda m_p t(|F_V|^2 - |F_A|^2) \\ &\quad + t[su + m_\Lambda^2 m_p^2 - \frac{1}{2}(m_\Lambda^2 + m_p^2)(s + u)](|F_M|^2 + |F_T|^2) + m_\Lambda m_p t^2(|F_M|^2 - |F_T|^2) \\ &\quad + t(m_\Lambda + m_p)[t - (m_\Lambda - m_p)^2] \operatorname{Re} F_V F_M^* + t(m_\Lambda - m_p)[t - (m_\Lambda + m_p)^2] \operatorname{Re} F_A F_T^* \\ &\quad + t(s - u) \operatorname{Re}[F_V + (m_\Lambda + m_p)F_M][F_A + (m_\Lambda - m_p)F_T]^* \end{aligned} \quad (7)$$

$$\begin{aligned} B &= 2 \operatorname{Im} [2m_p (F_V F_A^* + t F_T F_M^*) + (s - u) (F_M F_V^* + F_T F_A^*) \\ &\quad + (t - m_\Lambda^2 + m_p^2) (F_M F_A^* + F_T F_V^*)] \end{aligned} \quad (8)$$

$$\begin{aligned} C &= (m_\Lambda + m_p)[t - (m_\Lambda - m_p)^2] (|F_V|^2 + t|F_M|^2) + (m_\Lambda - m_p)[t - (m_\Lambda + m_p)^2] (|F_A|^2 + t|F_T|^2) \\ &\quad + 2m_\Lambda (s - u) \operatorname{Re} (F_V F_A^* + t F_M F_T^*) + (s - u)(t + m_\Lambda^2 - m_p^2) \operatorname{Re} (F_V F_T^* + F_A F_M^*) \\ &\quad + [t - (m_\Lambda - m_p)^2][t + (m_\Lambda + m_p)^2] \operatorname{Re} F_V F_M^* \\ &\quad + [t - (m_\Lambda + m_p)^2][t + (m_\Lambda - m_p)^2] \operatorname{Re} F_A F_T^* \end{aligned} \quad (9)$$

$$\begin{aligned}
D = & m_\Lambda(m_p^2 - s) |F_V - F_A|^2 + m_p(m_\Lambda^2 - u) (|F_V|^2 - |F_A|^2) \\
& + t(m_\Lambda + m_p)(m_\Lambda m_p - u) |F_M|^2 - t(m_\Lambda - m_p)(m_\Lambda m_p + u) |F_T|^2 \\
& + (u - m_\Lambda^2)(m_\Lambda^2 - m_p^2 - t) \operatorname{Re}(F_V F_M^* + F_A F_T^*) + 2m_\Lambda m_p t \operatorname{Re}(F_V F_M^* - F_A F_T^*) \\
& + (u - m_\Lambda^2)(u - s) \operatorname{Re}(F_V F_T^* + F_A F_M^*) + 2m_\Lambda m_p t \operatorname{Re}(F_V F_T^* - F_A F_M^*) \\
& + 2m_\Lambda t (m_p^2 - u) \operatorname{Re} F_M F_T^*
\end{aligned} \tag{10}$$

Equation (5), after a trivial relabelling of the masses, the momenta and the spin vector (and appropriate changes in the numerical values of the form factors), applies to the other two $\Delta S = 1$ reactions, $e^- + p \rightarrow \Sigma^0 + \nu$ and $e^- + n \rightarrow \Sigma^- + \nu$, as well as to the similar $\Delta S = 0$ reaction $e^- + p \rightarrow n + \nu$. For the related $\Delta S = 0$ reaction $e^+ + n \rightarrow p + \bar{\nu}$ which is obtained from the adjoint of the weak current $(V+A)_\alpha^+$ one must modify Eq. (5) by i) replacing F_V, F_M, F_A, F_T by $F_V^*, F_M^*, F_A^*, -F_T^*$ and ii) changing the sign of all terms in the coefficients A and B which are, and in C and D which are not, vector-axial vector interference terms. As indicated previously, in the limit of zero lepton mass, Eq. (5) is also very closely related to the cross-section for the corresponding $\bar{\nu}$ and ν induced reactions, which has been calculated by a number of authors²⁾. Thus, for example, to obtain $d\sigma/dt$ for the reaction $\bar{\nu} + p \rightarrow \Lambda + e^+$ as given, say, by Adler²⁾ one must multiply Eq. (5) by two (to cancel the factor of one half coming from the average over the initial electron spin) and apply ii) above; for $\nu + n \rightarrow p + e^-$ one multiplies Eq. (5) by two and applies i) above.

6.

3. FORWARD SCATTERING

Adler ⁷⁾ observed some time ago that when the momenta of the leptons are parallel and the lepton masses are neglected, the squared matrix element for inelastic weak interactions depends only on the divergence of the hadron current $\partial^\alpha (V+A)_\alpha$. In the case of $e^- + p \rightarrow \Lambda + \nu$ this configuration occurs for forward scattering of the neutrino ($t=0$), where

$$\sum_{\text{spins}} l_\alpha^+ l_\beta \sim \frac{4(s-m_p^2)(s-m_\Lambda^2)}{(m_\Lambda+m_p)^2(m_\Lambda-m_p)^2} q_\alpha q_\beta \quad (11)$$

The cross-section for producing polarized lambdas, Eq. (5) then reduces in the laboratory frame to the simple expression

$$\left. \frac{d\sigma_{\text{pol}}}{d(-t)} \right|_{t=0} = \frac{G^2}{8\pi} \frac{(s-m_\Lambda^2)}{(s-m_p^2)} (|F_V|^2 + |F_A|^2 - 2\lambda \text{Re} F_V F_A^*) \quad (12)$$

where λ , the helicity of the lambda, is $+1$ (-1) when the spin of the lambda is parallel (antiparallel) to its momentum.

It should be noted that in the forward direction, despite the fact that the vector current contributes to the matrix element only through its divergence, the contributions of F_V and F_A to the cross-section are equal, even in the limit of exact $SU(3)$ symmetry ($m_\Lambda \rightarrow m_p$) ⁸⁾. This same result is obtained in the case of "elastic" neutrino interactions, as was noted by Adler ⁷⁾ and originally by Lee and Yang ²⁾. It is a consequence in our case of the fact that the $(m_\Lambda - m_p)$ factor in the denominator of Eq. (11) cancels the similar factor in $\partial^\alpha V_\alpha \sim F_V(m_\Lambda - m_p)$ leaving just F_V .

4. TIME-REVERSAL INVARIANCE

One of the more important aspects of an experimental investigation of the process $e^-p \rightarrow \Lambda + \gamma$ is the possibility of looking for a non-vanishing lambda polarization normal to the scattering plane which, as is well known, would indicate a violation of time-reversal invariance. This transverse polarization is proportional to the coefficient B given in Eq. (8) and therefore vanishes if the form factors are relatively real as required by time-reversal invariance. To be more explicit, the lambda polarization in the direction $\hat{p}_e \times \hat{p}_\Lambda$ is given in the centre of mass (or laboratory) system by

$$P = -\frac{1}{2} [-st^2 - t(s - m_p^2)(s - m_\Lambda^2)]^{1/2} B/A \quad (13)$$

The subsequent parity violating decay $\Lambda \rightarrow p + \pi^-$ leads to an asymmetric angular distribution of the decay products with respect to the lambda polarization. Therefore one can directly detect a transverse lambda polarization by looking for an up-down asymmetry of the decay pion with respect to the scattering plane; the difference between the number of π^- emitted into the upper hemisphere (defined by $\hat{p}_e \times \hat{p}_\Lambda$) and those emitted into the lower hemisphere is just αP , where α is the $\Lambda \rightarrow p + \pi^-$ decay asymmetry parameter $\alpha = 0.647 \pm 0.016$ ⁹⁾.

Electromagnetic corrections, which we have neglected, can contribute to a transverse polarization without a violation of time-reversal invariance. However, such corrections are suppressed by the fine structure constant $\alpha \simeq 1/137$ and are not expected to exceed a few per cent.

8.

5. THE CABIBBO HYPOTHESIS

Cabibbo's extension ¹⁰⁾ of the conserved vector current theory and universality to $SU(3)$ makes definite predictions for the form factors F_V , F_A , F_M and F_T which can be tested in the $e^- + p \rightarrow \Lambda + \nu$ scattering experiment. The vector form factors, F_V and F_M are related to the proton electromagnetic form factors $F_1^p(t)$ and $F_2^p(t)$ by

$$F_V = -\sqrt{3/2} \sin \Theta_V F_1^p(t) \quad (14)$$

and

$$F_M = -\sqrt{3/2} \sin \Theta_V \mu_p F_2^p(t)/(2m_p) \quad (15)$$

where Θ_V is the Cabibbo angle for the vector current, $\mu_p = 1.793$ is the proton anomalous magnetic moment and $F_1^p(0) = F_2^p(0) = 1$. In a similar way the axial-vector form factors F_A and F_T are related to g_A , the neutron beta decay axial-vector form factor, and to g_T , the class II induced tensor form factor often defined for muon capture, by

$$F_A = -\sqrt{3/2} (1 - \frac{2}{3} \xi_A) \sin \Theta_A g_A(t) \quad (16)$$

and

$$F_T = -\sqrt{3/2} (1 - \frac{2}{3} \xi_T) \sin \Theta_A g_T(t)/(2m_p) \quad (17)$$

The parameter $\xi_A = D/(D+F) \simeq 2/3$ characterizes the relative mixture of D and F type axial-vector currents, and ξ_T is similarly defined. In general ξ_A and ξ_T are functions of t ¹¹⁾; however, as nothing is known of their t dependence we will for the purposes of the present calculation put $\xi(t) = \xi(0)$. The quantity θ_A is the Cabibbo angle for the axial-vector current, $g_A(0) = -1.18 \pm 0.02$ ¹²⁾, and $g_T(t)$ is usually neglected ¹³⁾.

The form factors for the similar process $e^- + p \rightarrow \Sigma^0 + \nu$ can be obtained by making the following replacements in Eqs. (14)-(17) respectively: $\sqrt{3} F_1^p \rightarrow F_1^p + 2 F_1^n$, $\sqrt{3} \mu_p F_2^p \rightarrow (\mu_p F_2^p + 2 \mu_n F_2^n)$, $\sqrt{3} (1 - \frac{2}{3} \xi_A) \rightarrow (1 - 2 \xi_A)$, and $\sqrt{3} (1 - \frac{2}{3} \xi_T) \rightarrow (1 - 2 \xi_T)$

where $\mu_n = -1.913$ is the neutron anomalous magnetic moment and F_1^n and F_2^n are the neutron electromagnetic form factors [$F_1^n(0) = 0$, $F_2^n(0) = 1$]. As a consequence of the $\Delta I = \frac{1}{2}$ rule the form factors for the process $e^- + n \rightarrow \Sigma^- + \nu$ will be just $\sqrt{2}$ times those for $e^- + p \rightarrow \Sigma^0 + \nu$, and thus $d\sigma(e^- + n \rightarrow \Sigma^- + \nu) = 2 d\sigma(e^- + p \rightarrow \Sigma^0 + \nu)$.

6. PCAC

Applying the notion of partial conservation of the axial-vector current (PCAC) to the strangeness-changing current yields the Goldberger-Treiman type relation ¹⁴⁾

$$|F_A| = F_K \sin \theta_A g_{K\Lambda N} / (m_\Lambda + m_p) \quad (18)$$

where $F_K \sin \theta_A$ is related to the kaon lifetime, and $g_{K\Lambda N}$ is the $K\Lambda N$ coupling constant ¹⁵⁾. Determinations of $g_{K\Lambda N}$ have varied over a large range, $5 \lesssim g_{K\Lambda N}^2 / 4\pi \lesssim 16$ ¹⁶⁾, while the value of F_A determined from lambda beta decay ($\Lambda \rightarrow p + e^- + \bar{\nu}$) has large errors ¹⁷⁾; hence the relation in Eq. (18) has not yet been satisfactorily tested. A direct determination of F_A in the $e^- + p \rightarrow \Lambda + \nu$ experiment would be valuable either to

give an independent estimate of the value of $g_{K\Lambda N}$ through Eq. (18) or, when an agreement on the value of $g_{K\Lambda N}$ has been reached, to determine the accuracy of PCAC for the strangeness changing current. Of course, one does not expect this relation to be satisfied exactly; the original Goldberger-Treiman relation

$$|g_A| = g_{\pi N} F_\pi / (\sqrt{2} m_p) \quad (19)$$

itself holds only to about 10%.

7. NUMERICAL RESULTS

To obtain numerical estimates for the differential cross-section for this process we have used the Cabibbo theory described above with different values for the vector and axial-vector Cabibbo angles. The parameters have been taken from Brene et al.⁴⁾ who fit the hyperon leptonic decay data finding the values $\theta_V = 0.212 \pm 0.004$, $\theta_A = 0.268 \pm 0.001$, and $\xi_A = 0.665 \pm 0.018$. For the electromagnetic form factors $F_1^{p,n}$ and $F_2^{p,n}$ in Eqs. (14) and (15) we have used the standard relations⁵⁾

$$F_1^{p,n} = \frac{G_E^{p,n} - \frac{t}{4m_p^2} G_M^{p,n}}{(1 - \frac{t}{4m_p^2})} \quad \text{and} \quad F_2^{p,n} = \frac{G_M^{p,n} - G_E^{p,n}}{\mu_{p,n}(1 - \frac{t}{4m_p^2})} \quad (20)$$

together with the empirical results obtained from electron scattering data¹⁸⁾

$$G_E^p = \frac{G_M^p}{(1 + \mu_p)} = \frac{G_M^n}{\mu_n} = \left(1 - \frac{t}{.71 \text{ GeV}^2}\right)^{-2} \quad \text{and} \quad G_E^n = 0. \quad (21)$$

For the momentum dependence of g_A and g_T in Eqs. (16) and (17) we have assumed the form suggested for g_A by neutrino experiments¹⁹⁾

$$g_A(t) = g_A(0) \left(1 - \frac{t}{M^2}\right)^{-2} \quad (22)$$

$$g_T(t) = g_T(0) \left(1 - \frac{t}{M^2}\right)^{-2} \quad (23)$$

with $M = 810$ MeV. It should be emphasized that the possible range of t is sufficiently large that the shape of the form factors significantly affects the numerical results.

Typical results have been computed for the cross-sections of interest, with the form factor $g_T = 0$. The total cross-sections for $e^- + p \rightarrow \Lambda + \nu$ and $e^- + p \rightarrow \Sigma^0 + \nu$ have been numerically integrated and are presented in Fig. 1 as a function of the laboratory energy of the electron, E_e^{lab} . For the purpose of comparison we give also the total cross-sections for the two $\Delta S = 0$ reactions $e^- + p \rightarrow n + \nu$ and $e^+ + n \rightarrow p + \bar{\nu}$ computed in the same model. In Fig. 2 the differential cross-section in the centre of mass system for the reaction $e^- + p \rightarrow \Lambda + \nu$ is presented as a function of $\cos \theta_{\Lambda}^{\text{cm}} = (\hat{p}_e \cdot \hat{p}_{\Lambda})^{\text{cm}}$ for several values of the electron laboratory energy. Note that the threshold energy for the scattering $e^- + p \rightarrow \Lambda + \nu$ lies at $E_e^{\text{lab}} \simeq 194$ MeV, while the lowest threshold for associated production ($e^- + p \rightarrow \Lambda + K^+ + e^-$) lies at $E_e^{\text{lab}} \simeq 912$ MeV.

To estimate the magnitude of the time-reversal violating transverse lambda polarization that might be expected, we have considered two possibilities for introducing complex form factors into the Cabibbo model:

Model I : a purely imaginary class II induced tensor form factor $F_T^{(20)}$;

Model II : a complex phase factor $e^{-i\delta}$ between the strangeness-changing vector and axial-vector currents V_α and $A_\alpha^{(21)}$.

The differential cross-section and the transverse polarization of the lambda have been calculated for various values of the parameters according to Model I by putting $g_T(0) = i|g_T|$ in Eq. (23) and setting $\xi_T = \xi_A$; the results are presented in Table I for various values of $|g_T|$. Similarly for Model II we have replaced $(V+A)_\alpha$ by $(V+Ae^{-i\delta})_\alpha$ in Eq. (1) and put $g_T(0) = 0$; these results are presented in Table II for various values of δ . Perhaps we have been rather optimistic in selecting the range of parameters in these two models, but it should be emphasized that present experiments are not very restrictive.

As mentioned in the Introduction, one of the advantages of $\Delta S = 1$, as opposed to $\Delta S = 0$, reactions is the fact that as long as one operates below the threshold for associated production there is no real background of strange particles. There is an exception to this statement for the process $e^- + p \rightarrow \Lambda + \nu$ however in that one may also obtain lambdas from the reaction $e^- + p \rightarrow \Sigma^0 + \nu$ followed by the decay $\Sigma^0 \rightarrow \Lambda + \gamma$ (22). To detect the lambda however one must measure the decay proton and decay pion in coincidence; also since electron beams, rather than neutrino beams, are being used the initial energy is well defined. Thus, in principle, one can reconstruct the $e^- + p \rightarrow \Lambda + \nu$ scattering completely and separate lambdas obtained from the two processes. For example one could require that the missing centre of mass energy $(p_e + p_p - p_\Lambda)^2$ be zero, corresponding to a missing neutrino, rather than non-zero as would in general be the case when both a gamma and a neutrino were emitted. In practice of course one works with instruments of finite resolution and so for a particular experimental situation one must eventually make a detailed calculation to see what degree of separation could be obtained. One observes however from Fig. 1 that the expected cross-section for Σ^0 production in the energy range of interest is nearly an order of magnitude lower than that for Λ production. Thus even if the experimental

resolution were so poor that no separation could be made, one would expect the lambdas from $\Sigma^0 \rightarrow \Lambda + \gamma$ to make only a 10-15% correction to the rate.

In so far as the measurement of the transverse polarization is concerned, such a correlation still tests time reversal even if the Λ is produced with a γ . Thus, while one certainly will have to correct for lambdas produced in the reaction $\Sigma^0 \rightarrow \Lambda + \gamma$, it does not appear that such a correction will lead to major difficulties or invalidate the test for time-reversal violation.

8. SUMMARY

Much valuable information can be gained from an experimental investigation of the scattering $e^- + p \rightarrow \Lambda + \gamma$. As is evident from the above numerical results, there is some possibility of observing a rather large violation of time reversal invariance. In addition, the Cabibbo theory and PCAC can be investigated.

ACKNOWLEDGEMENTS

We are grateful to Professor A. Schwettman and Professor M. Yearian for several discussions concerning experimental questions and the construction of the superconducting electron accelerator at Stanford University. One of us (R.J.O.) wishes to thank Professor C. Kaysen for his hospitality at the Institute for Advanced Study. Another, (H.W.F.) would like to thank Professor L. Van Hove and Professor J. Prentki for the hospitality of the Theoretical Study Division of CERN.

TABLE CAPTIONSTABLES I AND II

For the process $e^- + p \rightarrow \Lambda + \gamma$ the polarization P of the lambda and the differential cross-section in the laboratory frame are given for various values of $|\varepsilon_T|$ (Model I) and of δ (Model II) at different values of the laboratory electron energy E_e^{lab} . The angles $\theta_{\Lambda}^{\text{cm}}$ and $\theta_{\Lambda}^{\text{lab}}$ are the angles between the electron and lambda momenta in the centre of mass and laboratory frames, respectively.

$E_e^{\text{lab.}}$ (MeV)	$\cos \Theta_{\Lambda}^{\text{c.m.}}$	$\cos \Theta_{\Lambda}^{\text{lab.}}$	t (GeV ²)	$ g_T = 0$		$ g_T = 1$		$ g_T = 5$		$ g_T = 10$	
				$\frac{d\sigma}{d\Omega}^{\text{lab.}}$ $10^{-41} \text{ cm}^2/\text{sr}$	P	$\frac{d\sigma}{d\Omega}^{\text{lab.}}$ $10^{-41} \text{ cm}^2/\text{sr}$	P	$\frac{d\sigma}{d\Omega}^{\text{lab.}}$ $10^{-41} \text{ cm}^2/\text{sr}$	P	$\frac{d\sigma}{d\Omega}^{\text{lab.}}$ $10^{-41} \text{ cm}^2/\text{sr}$	P
500	-1.0	1.000	-0.0000	0.6	0.00	0.6	0.00	0.6	0.00	0.6	0.00
	-0.8	0.882	-0.0296	2.6	0.08	2.6	0.36	2.7	0.61	3.1	0.61
	-0.6	0.858	-0.0592	18.6	0.09	18.7	0.43	20.4	0.69	25.6	0.69
	-0.4	0.861	-0.0889	11.7	0.10	11.7	0.47	13.0	0.71	17.1	0.71
	-0.2	0.874	-0.1185	6.2	0.11	6.2	0.47	7.0	0.71	9.3	0.71
	0.0	0.892	-0.1481	4.8	0.10	4.8	0.47	5.4	0.70	7.2	0.70
	0.2	0.913	-0.1777	4.1	0.10	4.1	0.45	4.6	0.68	6.0	0.68
	0.4	0.934	-0.2073	3.7	0.09	3.7	0.41	4.0	0.64	5.2	0.64
	0.6	0.956	-0.2369	3.3	0.08	3.4	0.36	3.6	0.57	4.5	0.57
	0.8	0.978	-0.2666	3.1	0.06	3.1	0.27	3.3	0.45	3.9	0.45
	1.0	1.000	-0.2692	2.9	0.00	2.9	0.00	3.0	0.00	3.4	0.00
800	-1.0	1.000	-0.0000	0.6	0.00	0.6	0.00	0.6	0.00	0.6	0.00
	-0.8	0.782	-0.0717	7.3	0.10	7.4	0.44	8.2	0.67	10.8	0.67
	-0.6	0.774	-0.1433	19.3	0.12	19.4	0.51	22.8	0.70	33.4	0.70
	-0.4	0.795	-0.2150	7.7	0.13	7.7	0.53	9.3	0.70	14.2	0.70
	-0.2	0.825	-0.2867	5.4	0.13	5.5	0.54	6.7	0.70	10.3	0.70
	0.0	0.856	-0.3584	4.4	0.13	4.4	0.53	5.3	0.70	8.1	0.70
	0.2	0.887	-0.4300	3.7	0.12	3.7	0.51	4.4	0.69	6.5	0.69
	0.4	0.917	-0.5017	3.2	0.11	3.2	0.48	3.7	0.68	5.2	0.68
	0.6	0.946	-0.5734	2.8	0.10	2.8	0.43	3.1	0.64	4.1	0.64
	0.8	0.974	-0.6450	2.4	0.07	2.4	0.33	2.6	0.54	3.2	0.54
	1.0	1.000	-0.7167	2.2	0.00	2.2	0.00	2.2	0.00	2.4	0.00

TABLE I

$E_e^{\text{lab.}}$ (MeV)	$\cos \Theta_{\Lambda}^{\text{c.m.}}$	$\cos \Theta_{\Lambda}^{\text{lab.}}$	t (GeV ²)	$\delta = 0.0$		$\delta = 0.1$		$\delta = 0.3$		$\delta = 0.5$	
				$\frac{d\sigma}{d\Omega}^{\text{lab.}}$ $10^{-41} \text{ cm}^2/\text{sr}$	P	$\frac{d\sigma}{d\Omega}^{\text{lab.}}$ $10^{-41} \text{ cm}^2/\text{sr}$	P	$\frac{d\sigma}{d\Omega}^{\text{lab.}}$ $10^{-41} \text{ cm}^2/\text{sr}$	P	$\frac{d\sigma}{d\Omega}^{\text{lab.}}$ $10^{-41} \text{ cm}^2/\text{sr}$	P
500	-1.0	1.000	-0.0000	0.6	0.00	0.6	0.00	0.6	0.00	0.6	0.00
	-0.8	0.882	-0.0296	2.6	0.02	2.6	0.04	2.5	0.07	2.5	0.07
	-0.6	0.858	-0.0592	18.6	0.02	18.6	0.06	18.5	0.09	18.2	0.09
	-0.4	0.861	-0.0889	11.7	0.02	11.6	0.06	11.5	0.10	11.3	0.10
	-0.2	0.874	-0.1185	6.2	0.02	6.2	0.06	6.1	0.10	6.0	0.10
	0.0	0.892	-0.1481	4.8	0.02	4.8	0.06	4.7	0.09	4.6	0.09
	0.2	0.913	-0.1777	4.1	0.02	4.1	0.05	4.0	0.09	3.9	0.09
	0.4	0.934	-0.2073	3.7	0.02	3.6	0.05	3.6	0.08	3.5	0.08
	0.6	0.956	-0.2369	3.3	0.01	3.3	0.04	3.3	0.06	3.2	0.06
	0.8	0.978	-0.2666	3.1	0.01	3.1	0.03	3.0	0.05	2.9	0.05
	1.0	1.000	-0.2692	2.9	0.00	2.9	0.00	2.8	0.00	2.7	0.00
800	-1.0	1.000	-0.0000	0.6	0.00	0.6	0.00	0.6	0.00	0.6	0.00
	-0.8	0.782	-0.0717	7.3	0.01	7.3	0.04	7.3	0.06	7.2	0.06
	-0.6	0.774	-0.1433	19.3	0.01	19.3	0.04	19.1	0.07	18.8	0.07
	-0.4	0.795	-0.2150	7.7	0.01	7.7	0.04	7.6	0.07	7.4	0.07
	-0.2	0.825	-0.2867	5.4	0.01	5.4	0.04	5.4	0.07	5.2	0.07
	0.0	0.856	-0.3584	4.4	0.01	4.4	0.04	4.3	0.07	4.2	0.07
	0.2	0.887	-0.4300	3.7	0.01	3.7	0.04	3.6	0.06	3.5	0.06
	0.4	0.917	-0.5017	3.2	0.01	3.1	0.03	3.1	0.06	3.0	0.06
	0.6	0.946	-0.5734	2.8	0.01	2.7	0.03	2.7	0.05	2.6	0.05
	0.8	0.974	-0.6450	2.4	0.01	2.4	0.02	2.4	0.03	2.3	0.03
	1.0	1.000	-0.7167	2.2	0.00	2.2	0.00	2.1	0.00	2.0	0.00

TABLE II

R E F E R E N C E S

1. M. Schwartz (private communication) and I.I. Schiff (private communication) both suggested such experiments to the authors some time ago. Such ideas have undoubtedly been considered by many others as well. As electron and neutrino induced weak interactions are very similar much of the discussion of such points as time reversal violation, PCAC, and Cabibbo theory can be found in discussions of neutrino interactions as for example those of Ref. 2). We review these points here for purposes of completeness. To our knowledge however there does not exist in the literature a numerical analysis of electron induced processes or results analogous to those we give in Figs. 1 and 2 or in Tables 1 and 2.
2. For example see the following and references cited therein:
 A. Fujii and Y. Yamaguchi, Nuovo Cimento 43A, 325 (1966);
 N. Cabibbo and F. Chilton, Phys.Rev. 137, B1628 (1965);
 S.M. Berman and M. Veltman, Phys.Letters 12, 275 (1964);
 S. Adler, Nuovo Cimento 30, 1020 (1963), erratum 32, 509 (1964);
 J.S. Bell and S.M. Berman, Nuovo Cimento 25, 404 (1962);
 T.D. Lee and C.N. Yang, Phys.Rev. 126, 2239 (1962);
 N. Cabibbo and R. Gatto, Nuovo Cimento 15, 304 (1960).
3. R.P. Feynman and M. Gell Mann, Phys.Rev. 109, 193 (1958);
 E.C.G. Sudarshan and R.E. Marshak, Proceedings of the Padua-Venice conference on Mesons and Recently Discovered Particles (1957), (Padua-Venice, 1958) and Phys.Rev. 109, 1860 (1958).
4. N. Brene, L. Veje, M. Roos and C. Cronstrom, Phys.Rev. 149, 1288 (1966).
5. The notation for Dirac spinors, etc., is that of J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics (McGraw-Hill, New York, 1964).
6. The components of S_{Λ} are explicitly given by

$$S_{\Lambda}^0 = [1 - (\hat{\underline{s}} \cdot \underline{\beta})^2]^{-1/2} \hat{\underline{s}} \cdot \underline{\beta} \quad \text{and} \quad \underline{S}_{\Lambda} = [1 - (\hat{\underline{s}} \cdot \underline{\beta})^2]^{-1/2} \hat{\underline{s}}$$
 where $\underline{\beta} = \underline{p}_{\Lambda} / E_{\Lambda}$ and $|\hat{\underline{s}}| = 1$.
7. S.L. Adler, Phys.Rev. 135, B963 (1964).

8. It is of interest to observe that this is also true not only in the forward "elastic" scattering of neutrinos on spin $\frac{1}{2}$ baryons but also for the beta decay of spin $\frac{1}{2}$ baryons when the mass of the electron can be neglected and when the leptons are emitted in the same direction.
9. A.H. Rosenfeld et al., Revs.Mod.Phys. 40, 77 (1968).
10. N. Cabibbo, Phys.Rev.Letters 10, 531 (1963).
11. J.S. Bell has emphasized this point (private communication). See also Cabibbo and Chilton, Ref. 2).
12. C.P. Bhalla, Phys.Letters 19, 691 (1966).
13. Compare however: H.P.C. Rood, Nuovo Cimento Supp. 4, 185 (1966); R.J. Blin-Stoyle and M. Rosina, Nuclear Phys. 70, 321 (1965); H. Ohtsubo and A. Fujii, Nuovo Cimento 42, 109 (1966); A. Bietti, Nuovo Cimento 37, 337 (1965) and H.W. Fearing, Phys.Rev. 146, 723 (1966).
14. M.L. Goldberger and S.B. Treiman, Phys.Rev. 111, 354 (1958).
15. These quantities are defined, respectively, by the matrix element $\langle 0 | A_\alpha | K^+(k) \rangle = i F_K \sin \theta_A k_\alpha$ and the interaction Lagrangian $\mathcal{L}_{int} = -i g_K \Lambda_N \bar{\Psi} \gamma_5 K^+ \Psi_N$. In the SU(3) limit $g_K \Lambda_N = -\sqrt{3}(1-2/3\xi)$ $g_{\pi N}$ with the πN coupling constant $g_{\pi N}^2/4\pi \simeq 14.6$ and the F-D mixing parameter $\xi \simeq 2/3$.
16. M. Lusignoli, M. Restignoli, G.A. Snow and G. Violini, Phys.Letters 21, 229 (1966) and Nuovo Cimento 45A, 792 (1966); N. Zovko, Phys.Letters 23, 143 (1966); H.P.C. Rood, Nuovo Cimento 50, 493 (1967); J.K. Kim, Phys.Rev.Letters 19, 1079 (1967); C.H. Chan and F.T. Meiere, Phys.Rev.Letters 20, 568 (1968).
17. J. Barlow et al., Phys.Letters 18, 64 (1965); C. Baglin et al., Nuovo Cimento 35, 977 (1965); G. Conforto, quoted in Ref. 2).
18. E.B. Hughes, T.A. Griffy, M.R. Yearian and R. Hofstadter, Phys.Rev. 139, B458 (1965); G. Weber, Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies, Stanford (1967), p. 59. These expressions, obtained from fits to electron scattering data over a wide range of t , lead to form factors which are essentially identical for $0 \leq -t \leq 1 \text{ GeV}^2$ to the empirical forms obtained in this region by Hughes et al. Note however that even for the relatively low momentum transfers of interest here the approximate forms used in most previous calculations of neutrino processes (Refs. 2 and 19), namely,

$$F_1^p = F_2^p = F_2^n = (1 - t/0.71)^{-2} \quad \text{and} \quad F_1^n = 0,$$

are now in rather poor agreement with experiment. For example, F_1^p and F_2^p differ by nearly a factor of two at $-t=1 \text{ GeV}^2$. At higher energies total cross-section results obtained in this approximation tend to differ by 10-20% from those obtained using the form factors of Eqs. (20) and (21) and differential cross-sections differ at some angles by factors of two.

19. M.M. Block et al., Proceedings of the XII International Conference on High Energy Physics, Dubna (1964), Vol. 2, p. 7;
E.C.M. Young, CERN report 67-12 (1967).
20. N. Cabibbo, Phys.Letters 12, 137 (1964).
21. S.L. Glashow, Phys.Rev.Letters 14, 35 (1965);
B. d'Espagnat and M.K. Gaillard. Second Coral Gables Conference on Symmetry Principles at High Energy (Freeman and Co., San Francisco, 1965), p. 380.
22. We are indebted to J. Prentki for emphasizing this possibility.

FIGURE CAPTIONSFigure 1 :

Total cross-sections for the processes $e^- + p \rightarrow \Lambda + \nu$ and $e^- + p \rightarrow \Sigma^0 + \nu$ and for the related $\Delta S = 0$ processes $e^- + p \rightarrow n + \nu$ and $e^+ + n \rightarrow p + \bar{\nu}$ as a function of the laboratory energy of the electron. The cross-section for $e^- + n \rightarrow \Sigma^- + \nu$ is just twice that for $e^- + p \rightarrow \Sigma^0 + \nu$.

Figure 2 :

The differential cross-section for the process $e^- + p \rightarrow \Lambda + \nu$ in the centre of mass system.

FIG. 1

FIG.2