CM-P00057932

Archives

Ref.TH.1017-CERN

$K \rightarrow 3\pi$ SPECTRUM AND POLE DOMINANCE

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A B S T R A C T

We show that a combination of current algebra and the pole model for $K \rightarrow 3\pi$ decay is incompatible with experiment unless off-mass shell corrections are unexpectedly large. Without such corrections the pole terms give much too large an amplitude and do not give all the zeros required by current algebra.

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The idea that the decays $K \rightarrow 3\pi$ and $\eta \rightarrow 3\pi$ are dominated by the pole diagrams in Fig. 1 has frequently been entertained ¹⁾⁻⁹⁾. This assumption immediately explains the similarity of the spectra observed in these decays. Using Weinberg's amplitude, the Veneziano model, or any model for $\pi\pi$ scattering which incorporates the zeros required by Adler's self consistency condition, the amplitude corresponding to Fig. 1 gives all the zeros required by current algebra in $K \rightarrow 3\pi$ but only one of the zeros required by current algebra in $\eta \rightarrow 3\pi$ ⁹⁾. This may be regarded as a success of the model since these are just the zeros which seem to be present experimentally ^{*)}.

Here we shall discuss K decay in which case it seems reasonable to require the zeros in any model.

The pole model of Fig. 1 might be justified on the basis of the nearby singularity philosophy using an unsubtracted dispersion relation. Note that if all strong amplitudes have the Adler zeros (as is the case with Weinberg amplitudes or the Veneziano amplitude) then all contributions to the $K \rightarrow 3\pi$ amplitude due to intermediate states with the same quantum numbers as the final 3π state have the correct zeros, i.e., the contribution from the cut beginning at $(3M_\pi)^2$ has the correct zeros and the nearest singularity which does not a priori have the zeros is due to the $\pi\pi K$ cut from $(2M_\pi + M_K)^2$. The zeros of the amplitude then result from a nearby singularity approximation which is much more general than a pion pole model.

^{*)} This does not necessarily imply that current algebra is wrong in η decay. Assumptions which have been successfully applied to $K \rightarrow 3\pi$ forbid $\eta \rightarrow 3\pi$ ¹⁰⁾; off-mass shell corrections or higher order terms must be invoked to give a non-zero rate so that it is, perhaps, the large rate and not the absence of zeros which constitutes the theoretical problem. Alternatively, modified electromagnetic Hamiltonians have been suggested to circumvent the difficulty.

The transition amplitude $\langle K(q)|H_W|\pi(q)\rangle$ in Fig. 1 can be calculated in terms of $\langle K(q)|H_W|\pi(q)\pi(0)\rangle$ using current algebra and the strong amplitude can be calculated using the Veneziano model (or Weinberg's model). Neglecting all off-mass shell effects to get an estimate, it then turns out that the model gives an amplitude which is about six times too large. This means that either (1) the pole model is wrong and the continuum also gives a large contribution which partially cancels the pole, or (2) the off-mass shell correction in $\langle K|H_W|\pi\pi\rangle$ is unexpectedly large^{*}) or (3) the strong $\pi\pi$ amplitude varies by a surprising amount off the mass shell (in any case more than can be accounted for by a linear variation).

In the first case there is no reliable way of estimating the continuum contribution. However, from the point of view of graphs, looking for contributions to the amplitude which could partially cancel the pole term, one is naturally led to consider Fig. 2⁵⁾. This modified model gives an amplitude which is a factor of about eight times too large. Furthermore these new terms do not have all the necessary zeros and are in fact substantial at the relevant points. The modified pole model is therefore also untenable unless off-mass shell corrections are very large.

^{*}) Some variation is necessary in models with a spurion when both pions have zero momentum and the kaon stays on its mass shell¹¹⁾ because of the kaon pole attached to the spurion line¹²⁾ (this is also discussed in Ref. ¹³⁾). In the present case there is never a momentum spurion and this pole therefore does not induce variations.

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CALCULATION

We shall use the Veneziano model for $\pi\pi$ and πK scattering since it was the recent successful calculation ⁹⁾ of the spectra in $K \rightarrow 3\pi$ and $\eta \rightarrow 3\pi$ decays using the Veneziano amplitude in Fig. 1, which aroused our interest in this problem. We shall also give some results obtained using Weinberg's model ¹⁴⁾. These models have the same zeros and (approximately) the same values at the thresholds so that they only differ as far as non-linear effects are concerned for small s , t and u .

The $\Delta I = \frac{1}{2}$ rule gives ^{*)}

$$g(q^2) = \langle K^+(q) | H_W | \pi^+(q) \rangle = \sqrt{2} \langle K^0(q) | H_W | \pi^0(q) \rangle \quad (1)$$

Using this result and the Veneziano model, we write the amplitudes corresponding to Figs. 1 and 2 [combined for convenience, the coefficient of $g(M_K^2)$ comes from Fig. 1, and of $g(M_\pi^2)$ from Fig. 2] for the different charged states, thus :

$$A_{+--} = \frac{\beta}{M_K^2 - M_\pi^2} \left[g(M_K^2) V_{pp}(t, u) - \frac{g(M_\pi^2)}{2} (V_{K^*p}(u, t) + V_{K^*p}(t, u)) \right] \quad (2)$$

$$A_{+00} = A_{+-0} = \frac{\beta}{2(M_K^2 - M_\pi^2)} \left[g(M_K^2) (V_{pp}(t, u) - V_{pp}(s, t) - V_{pp}(s, u)) + \frac{g(M_\pi^2)}{2} (V_{K^*p}(t, s) + V_{K^*p}(u, s) + V_{K^*p}(t, u) - V_{K^*p}(s, u) + V_{K^*p}(u, t) - V_{K^*p}(s, t)) \right] \quad (3)$$

*) In the current algebra approach which we shall use to relate $K \rightarrow \pi$ to $K \rightarrow \pi\pi$ this follows automatically ¹²⁾.

$$A_{000} = \frac{\beta}{2(M_K^2 - M_\pi^2)} \left[g(M_K^2) (V_{pp}(t, u) + V_{pp}(t, s) + V_{pp}(s, u)) \right. \\ \left. - \frac{g(M_K^2)}{2} (V_{K^*p}(u, t) + V_{K^*p}(s, t) + V_{K^*p}(s, u)) \right. \\ \left. + V_{K^*p}(t, u) + V_{K^*p}(t, s) + V_{K^*p}(u, s) \right] \quad (4)$$

where s , t and u are the squares of the invariant masses of the various dipion states (symmetry shows which) and V is the Veneziano function^{9), 15)}

$$V_{pp}(s, t) = \frac{\Gamma(1 - \alpha_p(s)) \Gamma(1 - \alpha_p(t))}{\Gamma(1 - \alpha_p(s) - \alpha_p(t))}$$

$$V_{K^*p}(s, t) = \frac{\Gamma(1 - \alpha_{K^*}(s)) \Gamma(1 - \alpha_p(t))}{\Gamma(1 - \alpha_{K^*}(s) - \alpha_p(t))} \quad (5)$$

with the trajectories given by

$$\alpha_p(s) = 0.48 + 0.89s$$

$$\alpha_{K^*}(s) = 0.28 + 0.89s \quad (6)$$

The function β is normalized to the residue at the ρ pole in $\pi - \pi$ scattering giving $\beta \simeq 53$ *). Initially we shall assume that β does not vary off the mass shell.

) This gives $\pi\pi$ and πK scattering lengths within ten per cent of Weinberg's values¹⁵⁾, so that it is inessential which we normalize to. However, it gives $\Gamma_{K^} \sim 32$ MeV (experimentally $\Gamma_{K^*} \sim 50$ MeV). Normalizing to Γ_{K^*} for πK requires that $\beta \cdot g(M_\pi^2)/2$ in Eqs. (2)-(4) is replaced by $\beta_{\pi K} g(M_\pi^2)$ with $\beta_{\pi K} \simeq 42$ and gives a corresponding increase of the πK scattering lengths by a factor $84/53$.

Notice that the part of A_{++-} due to Fig. 2 does not have the required zero as $q_- \rightarrow 0$ ($u = t = M_\pi^2$, $s = M_K^2$), as is also the case if Weinberg's amplitude is used.

We begin by evaluating the contribution of Fig. 1 alone which gives in the centre of the Dalitz plot ($s = t = u = M_K^2/3 + M_\pi^2$) :

$$|A_{+00}| = |A_{+-0}| = \frac{|A_{++-}|}{2} = \frac{|A_{000}|}{3} = \frac{\beta |g(M_K^2) V_{pp}(t, u)|}{2(M_K^2 - M_\pi^2)} \quad (7)$$

(The relation of the A's is given by the $\Delta I = \frac{1}{2}$ rule which is built into our model). Current algebra gives ^{*)}

$$\begin{aligned} g(q^2) &= |\langle K^+(q) | H_W | \pi^+(q) \rangle| \\ &= 2 f_\pi |\langle K_1^0(q) | H_W | \pi^+(q) \pi^-(0) \rangle| \end{aligned} \quad (8)$$

Neglecting q^2 dependence and approximating the right-hand side by its value at the physical decay point gives :

$$\begin{aligned} |A_{+00}| &= |A_{+-0}| = \frac{|A_{++-}|}{2} = \frac{|A_{000}|}{3} = \\ &= \frac{\beta f_\pi |V_{pp}(t, u) \langle K_1^0 | H_W | \pi^+ \pi^- \rangle|}{3} \\ &\approx 2.6 \text{ GeV} \frac{|\langle K_1^0 | H_W | \pi^+ \pi^- \rangle|}{M_K^2 - M_\pi^2} \end{aligned} \quad (9)$$

This result can be compared with the experimentally correct relation :

$$\begin{aligned} |A_{+00}| &= |A_{+-0}| = \frac{|A_{++-}|}{2} = \frac{|A_{000}|}{3} = \\ &\approx 0.44 \text{ GeV} \frac{|\langle K_1^0 | H_W | \pi^+ \pi^- \rangle|}{M_K^2 - M_\pi^2} \end{aligned} \quad (10)$$

*) f_π here is defined by $\partial_\mu J_\mu^i = f_\pi M_\pi^2 \phi^i$, so $f_\pi \simeq 91 \text{ MeV}$.

where $\langle K_1^0 | H_W | \pi^+ \pi^- \rangle$ is the physical amplitude. (This result can be derived from current algebra and an assumption about the variation of the $K \rightarrow 3\pi$ amplitude, which is well satisfied by the amplitudes corresponding to Fig. 1 alone.) The amplitude given by Fig. 1 alone [Eq. (9)] is seen to be 5.9 times larger than the experimental value [Eq. (10)] if all off-mass shell variations are neglected *).

Adding the contribution of the graph of Fig. 2 and still neglecting off-mass shell effects increases the discrepancy to a factor of 7.8 **). In the case of A_{+00} and A_{+-0} the amplitude still has the correct zeros, at $q_+ \rightarrow 0$ and $q_- \rightarrow 0$ respectively, to which it extrapolates linearly from the physical region [the extrapolation on which the theoretical derivation of Eq. (10) relies is also still satisfactory]. The amplitude A_{++-} , however, acquires a slight positive slope with $s = (P_{\pi^+} + P_{\pi^-})^2$ in contrast to the substantial negative slope given by Fig. 1 alone which is observed experimentally. $A_{+--}(q_- \rightarrow 0) = 0.5 \beta \langle K^+ | H_W | \pi^+ \rangle / (M_{K^2} - M_{\pi^2})$ *** compared to the value zero required theoretically and the value $0.72 \beta \langle K^+ | H_W | \pi^+ \rangle / (M_{K^2} - M_{\pi^2})$ at the centre of the Dalitz plot.

Clearly the zero cannot be produced without introducing new terms, or large off-mass shell corrections. The introduction of imaginary parts in the trajectories only changes the modulus of the various terms by a negligible amount and gives then very small relative

- *) In the centre of the Dalitz plot the $\pi^+ \rightarrow \pi^+ \pi^0 \pi^0$ amplitude is $\beta / 2 V_{\pi\pi} \simeq 14.3$. Weinberg's amplitude gives 11.4 at this point (which would reduce the discrepancy to a factor of 4.7).
- **) The Veneziano model gives 4.7 for the $K^+ \rightarrow K^+ \pi^0 \pi^0$ amplitude in the central point. Weinberg's amplitude gives 3.4 (which would reduce the discrepancy to a factor of 6.1).
- ***) Using Weinberg's model $0.5 \beta \simeq 26.5$ is replaced by 19 and $0.72 \beta \simeq 38$ by 30.

phases. Since there is no consistent way to introduce the imaginary part in the Veneziano model, one may try to see which relative phase ϕ between the contributions of Figs. 1 and 2 used as a parameter could provide agreement. No satisfactory answer can be found. With $\phi = 180^\circ$ the discrepancy at the central point is reduced to a factor of 3.8 in the amplitude but the slope in A_{+-} is still wrong. $\phi = 90^\circ$ gives the correct slope in A_{+-} but the discrepancy is a factor of 5.7. In any case, no phase can give the correct zero.

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CONCLUSIONS

We have shown that pole models of $K \rightarrow 3\pi$ decay, using current algebra to estimate the vertex $\langle K | H_W | \pi \rangle$, could only be compatible with experiment if off-mass shell variations were unexpectedly large. These variations must reduce the amplitude due to Fig. 1 to a sixth of the value obtained without off-mass shell corrections and also strongly suppress Fig. 2 relative to Fig. 1, in order to give the correct zeros.

This off-mass shell variation could be present in the $\pi\pi$ and πK scattering amplitudes. We would then require :

$$\beta_{\pi K}(q_1^2 = q_2^2 = q_3^2 = M_\pi^2, q_4^2 = M_K^2) \ll \beta(q_1^2 = q_2^2 = q_3^2 = M_\pi^2, q_4^2 = M_K^2) \\ \simeq \frac{1}{6} \beta(q_1^2 = q_2^2 = q_3^2 = q_4^2 = M_\pi^2)$$

It is easy to imagine functions which satisfy either part of this condition but not both unless the β for the $\pi\pi$ and πK scattering amplitudes differ radically. In any case the variation is unexpectedly large.

Strong variations might also be imagined in $g(q^2) = \langle K(q) | H_W | \pi(q) \pi(0) \rangle$. In order to save the pole model we would require :

$$g(M_\pi^2) \ll g(M_K^2) \approx \frac{1}{6} \langle K | H_W | \pi(q_1) \pi(q_2) \rangle \Big|_{\substack{q_1^2 = q_2^2 = M_\pi^2 \\ (q_1 + q_2)^2 = M_K^2}}$$

To investigate the sort of variation which could occur one may consider a spurion model such as has been extensively investigated by Okubo¹⁶⁾. The weak amplitude is then proportional to a strong amplitude continued to zero kaon four momentum. Writing a Veneziano amplitude for the $K \pi \pi$ spurion vertex it turns out that both $K^+ \rightarrow \pi^+ \pi^0$ and $K_S^0 \rightarrow \pi \pi$ are forbidden if the trajectories are real (the physical amplitude is given by $q_{\text{spurion}} \rightarrow 0$, which corresponds to a soft K zero). Introducing imaginary parts to fit the observed K^+ to K_S^0 branching ratio one obtains

$$g(M_\pi^2) = g(M_K^2) \approx 6.2 \langle K | H_W | \pi(q_1) \pi(q_2) \rangle \Big|_{\substack{q_1^2 = q_2^2 = M_\pi^2 \\ (q_1 + q_2)^2 = M_K^2}}$$

which would be disastrous for the pole model.

The single pole model has the attractive feature that it justifies the presence of the related zeros in the $\eta \rightarrow 3\pi$ and $K \rightarrow 3\pi$ amplitudes which are present experimentally ; this remains true if the next singularities are also taken into account in a dispersion relation in the kaon or eta mass. However, we conclude that the pole model for $K \rightarrow 3\pi$ decay is untenable unless large and (at present) inexplicable off-mass shell variations are present in the strong or the weak vertices or both.

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FIG.1

FIG.2