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ELASTIC SCATTERING AND ISOBAR PRODUCTION AT HIGH ENERGIES

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A B S T R A C T

Unitarity corrections to Regge pole amplitudes, as they may be introduced using the Baker-Blankenbecler formalism, are used in the analysis of the structure of differential cross-sections at small and intermediate transfers.

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1. INTRODUCTION

Many accurate results on pp , $\bar{p}p$, π^+p and K^+p elastic scattering have recently been reported ^{1),2)}. All these reactions show common striking features. When considered as functions of the momentum transfer squared, all differential cross-sections show a noticeable structure after an initial exponential decrease. This is illustrated in Figs. 1, 2 and 3, where some of the presently available results have been displayed. The observed structure is suggestive of multiple scattering effects as introduced in the analysis of reactions in nuclei through Glauber corrections ³⁾. The prominent observed structure at 1 GeV^2 is indeed more pronounced for pp and $\bar{p}p$ scattering where the cross-section is larger than in meson-nucleon collisions. Glauber corrections have been recently applied to different forms of the pp scattering amplitude ^{4),5),6),7)} and models based either on a pure Regge form ⁶⁾ or on a local interaction supplemented by Regge pole exchange ⁷⁾ have all been successful in reproducing the general behaviour of the differential cross-section at small and intermediate transfers in the 10 to 20 GeV range, even though they differ in many details and, of course, in their predictions at much higher energies. A rationale for the included rescattering effects is however difficult to find a priori for particle reactions where the single scattering Regge amplitude could already fulfil all the requirements imposed by unitarity. If a Regge amplitude is built in such a way that it may satisfy the unitarity conditions, it corresponds however to a model where information, which will be eventually gained from the understanding of complicated inelastic reactions, has not been used yet. When this is possible, a more involved singularity structure in the angular momentum plane could be found to be of importance, besides constraints obtained on the pole parameters. In fact, Regge cuts are known to be present in relativistic scattering ⁸⁾, and they have already been introduced on phenomenological grounds to interpret results at variance with the simplest Regge pole predictions ⁹⁾. Regge cuts could be of a great importance in elastic scattering when associated with a leading Regge trajectory having the quantum numbers of the vacuum and it is known that an amplitude where both cut and

pole contributions play an appreciable role may yield many of the features expected with rescattering corrections. The Glauber corrections applied to a Regge pole amplitude generate cuts which have many of the properties known on general grounds ^{8),9),10)}, even though the magnitude of the cut correction thus obtained cannot be taken with confidence.

It is yet unclear however how Glauber corrections are connected with further constraints brought by unitarity which have to be imposed on a form retained for the single scattering term or the eikonal ^{5),6),9)}. It appears that the formalism introduced by Baker and Blankenbelder ¹¹⁾ is to the contrary well suited to introduce unitarity constraints. It has been used to this end by Abarbanel, Drell and Gilman ¹²⁾ who constructed a unitarized form for an amplitude built with a direct interaction term and a diffractive term, in order to discuss the connection between e-p and p-p scattering at large transfers.

In this paper we follow the same approach in order to get an elastic amplitude which would satisfy the unitarity constraints. We aim, however, at reproducing the forward structure and, to this end, we start from an initial simple Regge pole term to which we thus add cut corrections, the effects of which are directly resummed. This is motivated by the fact that, as recalled later, the starting form of the elastic amplitude introduced in the Baker-Blankenbecler formalism has the singularity structure in s which is reproduced by a Regge pole form. We then show how this leads to an understanding of the low and intermediate t value behaviour of the differential cross-section and in particular the structure observed in the neighbourhood of $|t| = 1 \text{ GeV}^2$.

The Baker-Blankenbecler formalism implies simplifying assumptions for the s singularities of the inelastic amplitudes. In order to deal with reactions where the same Regge poles appear in elastic and inelastic amplitudes, which should then be considered on the same level, we first extend the formalism using a matrix form for the

multi-channel amplitudes. This yields in particular expressions suitable for the analysis of diffraction dissociation. We then apply our Regge pole amplitudes, corrected in order to satisfy unitarity, to the analysis of nucleon-nucleon and meson-nucleon scattering and discuss further complications met when comparing proton and anti-proton differential cross-sections. Agreement with present experimental results is displayed on Figs. 1, 2 and 3. The presence of a structure in the neighbourhood of $|t| = 1 \text{ GeV}^2$ does not depend on the slope of the Pommeranchuk trajectory when its details do. It is manifestly associated with the corrections brought by unitarity. Even though this analysis neglects spin this prominent effect should not be much modified in a more refined analysis where spin would be considered.

To the extent that the double scattering corrections thus introduced are identical with those of the Glauber approach, the differential cross-sections calculated in the 10-20 GeV range have some similarity with those previously obtained by Frautschi and Margolis⁶⁾ and Chiu and Finkelstein⁶⁾, even if they do not reproduce them in detail. However other results with no experimental test yet available turn out to be different. The overlap of the results shows how Glauber corrections can be used, in a first approximation, to take into account effects which may be traced to the constraints imposed by unitarity, even though the connection is a priori unclear¹⁰⁾. The observed discrepancies show however that an approach, which is conducted in such a way as to explicitly introduce unitarity constraints, differs in many points from the introduction of Glauber correction terms which are well suited for the analysis of reactions on nuclei but hard to justify for particle scattering at present machine energies.

If the structure observed in the neighbourhood of 1 GeV^2 comes out as the effect of unitarity corrections on a Regge amplitude, the behaviour of the differential cross-section beyond $|t| \approx 2 \text{ GeV}^2$, where it has already decreased by a factor 10^4 from its forward value, cannot be reproduced from a simple Regge form even thus corrected.

2. GENERAL FORMALISM

We start with the brief recollection of the formalism developed by Baker and Blankenbecler ¹¹⁾. The reaction amplitude (for a process where m particles go into n particles) F_{mn} is written in terms of Fourier-Bessel transform as:

$$F_{mn}(s, t, v_{mn}) = \int_0^\infty b db J_0(b\sqrt{-t}) H_{mn}(s, b, v_{mn}) \quad (1)$$

The initial and final states are referred to by the subscripts m and n and specified by a set of variables denoted by v_{mn} . s and t are respectively the centre-of-mass energy and the momentum transfer squared, the reaction being considered as a quasi two-body process with effective masses m_1 . As shown in Ref. ¹¹⁾ the high energy ($s \gg m_1^2$) expression of the unitarity relation can be written in a simple form as follows:

$$\begin{aligned} h_{mn}(s, b, v_{mn}) &= \frac{1}{2i} \left\{ H_{mn}(s+i\varepsilon, b, v_{mn}) - H_{mn}(s-i\varepsilon, b, v_{mn}) \right\} \\ &= \sum_j H_{mj}(s+i\varepsilon, b, v_{mj}) \rho_j H_{jn}(s-i\varepsilon, b, v_{jn}) \end{aligned} \quad (2)$$

where $\rho_j(s, v_j)$ is the phase space factor for a j particle state described by the variables v_j .

The Baker-Blankenbecler set of amplitudes which satisfies the unitarity condition (2) is then written as

$$\begin{aligned} H_{m2}(s, b, v_m) &= B_{m2}(s, b, v_m) [1 + I(s) H_{22}(s, b)] \\ H_{2n}(s, b, v_n) &= [1 + H_{22}(s, b) I(s)] B_{2n}(s, b, v_n) \\ H_{mn}(s, b, v_{mn}) &= B_{m2}(s, b, v_m) I(s) \times \\ &\times [1 + H_{22}(s, b) I(s)] B_{2n}(s, b, v_n) \end{aligned} \quad (3)$$

for inelastic processes and for elastic two-body scattering one has:

$$H_{22} = N_{22} (1 - I(s) N_{22}) \quad (4)$$

where

$$N_{22} = B_{22}(s, \ell) + \sum_{n \neq 2} B_{2n}(s, \ell, v_n) D_{n2}(s, \ell, v_n) \quad (5)$$

I being defined as

$$I(s) = \int_{s_2}^{\infty} \frac{\rho_2(s')}{s' - s - i\epsilon} ds'$$

and where D_{n2} in Eq. (5) is written as

$$D_{n2}(s, \ell, v_n) = - \frac{1}{\pi} \int_{s_n}^{\infty} ds' \frac{\rho_n(s', v_n)}{s' - s - i\epsilon} B_{n2}(s', \ell, v_n)$$

s_n being the threshold for an n particle state described by the variables v_n . The functions B_{mn} which are introduced have not the s singularities attached with unitarity. The s singularities present in the reaction amplitudes are generated by two-body intermediate states. With this restriction, the amplitudes defined by (4) and (3) satisfy identically (2) thus giving a set of unitarized amplitudes from the original set B .

The model may look somewhat restrictive but one should stress that the function N given by (5) has the s singularities which should be reproduced by a Regge pole expression. It appears therefore as very reasonable to take a Regge pole amplitude for N and transform it according to (4) in order to reach an approximation to the amplitude where unitarity is satisfied. This is the procedure which will be followed here when dealing with an elastic amplitude. The imaginary part of H_{22} obtained from (2) and (4) is given by

$$h_{22}(s, \ell) = \rho_2(s) |H_{22}|^2 + \text{Im } N(s, \ell) |1 - I(s) N(s, \ell)|^{-2} \quad (6)$$

The Regge pole approximation includes already some effects of unitarity but not all and should therefore be corrected. The Regge pole expression for N , modified according to (4), appears as a global representation of the effect of all the coupled channels on the elastic amplitude. Expanding (6), the elastic amplitude is then given by a superposition of a Regge pole chosen for N and an infinite set of cut contributions ¹⁰⁾. Their effect is directly resummed in (4).

We next proceed to a generalization of (4) which holds when several channels are considered at the same time as necessary when the same Regge pole affects elastic and inelastic amplitudes. N is then a symmetric matrix as follows from factorization. It connects a finite number of coupled channels which are considered separately. These channels are connected by a submatrix H_0 of the collision amplitude matrix H which is written as

$$H = \left(\begin{array}{c|c} H_0 & H_1 \\ \hline H_2 & H_3 \end{array} \right)$$

The unitarity condition (2) can be rewritten in terms of the submatrices H_i as follows

$$\text{Im } H_0 = H_0(s+i\varepsilon) \rho_1 H_0(s-i\varepsilon) + H_1(s+i\varepsilon) \rho_2 H_2(s-i\varepsilon)$$

$$\text{Im } H_1 = H_0(s+i\varepsilon) \rho_1 H_1(s-i\varepsilon) + H_1(s+i\varepsilon) \rho_2 H_3(s-i\varepsilon)$$

$$\text{Im } H_2 = H_2(s+i\varepsilon) \rho_1 H_0(s-i\varepsilon) + H_3(s+i\varepsilon) \rho_2 H_2(s-i\varepsilon)$$

$$\text{Im } H_3 = H_2(s+i\varepsilon) \rho_1 H_1(s-i\varepsilon) + H_3(s+i\varepsilon) \rho_2 H_3(s-i\varepsilon)$$

(7)

where the diagonal phase space matrix is written as

$$\rho = \left(\begin{array}{c|c} \rho_1 & 0 \\ \hline 0 & \rho_2 \end{array} \right)$$

One may then obtain a simple generalization of the Baker-Blakenbecker formulae with

$$\begin{aligned} H_1 &= (1 + H_0 I) B \\ H_2 &= B^+ (1 + I H_0) \\ H_3 &= B^+ I (1 + H_0 I) B \end{aligned} \quad (8)$$

where B now represents a real matrix and where I is defined as before but with the matrix ρ replacing the phase space factor.

If now one writes

$$H_0 = N (1 - I N)^{-1} \quad (9)$$

where N is symmetric and $\text{Im } N$ calculated to be

$$\text{Im } N = B \rho_2 B^+$$

one checks that (8) and (9) satisfy identically the unitarity relation (7). Relation (6) is generalized as

$$\text{Im } H_0 = ((1 - I N)^T)^{-1} \text{Im } N (1 - I^* N^*)^{-1} + H_0 \rho_1 H_0^* \quad (10)$$

where T and $*$ stand for transposition and complex conjugation. The assumption underlying this decomposition is that all inelastic amplitudes which we do not consider together with the elastic one in H_0 are such that their singularities are sufficiently well described by (8) where B is a real matrix. For simplicity reasons one may be tempted to take as few states as possible in defining H_0 , with the limiting case of considering only elastic scattering. Such

severe criteria may however lead to unstable assumptions for some inelastic amplitudes. This is in particular the case for those corresponding to diffraction dissociation where the Regge pole amplitude is proportional to the one written for elastic scattering.

Under the assumption that the off diagonal elements of N are small when compared to the diagonal ones, one obtains

$$H_{22} = \frac{N_{22}}{1 - I_{22} N_{22}} \quad (11)$$

thus giving back the Baker-Blankenbecler form, for elastic scattering.

For inelastic amplitudes, on the other hand, one may develop the corresponding matrix elements of H_0 to increasing orders in the off diagonal elements of N . This gives terms of the form

$$\begin{aligned} \text{first order} & \quad \frac{N_{ij}}{(1 - I_{ii} N_{ii})^2} \\ \text{second order} & \quad \sum_n \frac{I_{ii} N_{in} N_{nj}}{(1 - I_{ii} N_{ii})^3} \dots \end{aligned} \quad (12)$$

where we have, for simplicity, assumed all two-body phase factors and all elastic amplitudes to be equal, an assumption which we will have to introduce for lack of pertinent information when considering isobar production. The different terms generated, as in (12), can be associated with one step, two step ... reaction processes ¹³⁾.

Neglecting spin effects, the elastic amplitude is described by a single invariant amplitude $F(s,t)$ normalized in such a way that

$$\frac{d\sigma}{dt} = |T|^2 = \frac{|F|^2}{64 \pi k^2 s} \quad (13)$$

where k is the centre-of-mass momentum.

We define $H(b)$ through (1)

$$T = \int_0^\infty b db J_0(b\sqrt{-t}) H(s,b)$$

and

$$H(s, b) = \int_0^\infty q dq J_0(bq) T(s, t) \quad (14)$$

with $t = -|\vec{q}|^2$.

Next, we use (10) with N_{22} obtained from (14) when T is approximated by a Regge pole form

$$T_R(s, t) = \frac{1}{4k\sqrt{4\pi s}} \sum_i R_i(t) i^P \left(\frac{s}{s_0} e^{-i\frac{\pi}{2}} \right)^{\alpha_i(t)} \quad (15)$$

with P taken equal to 0 (1) for positive (negative) signature. The Regge amplitude is of course considered only for negative values of $|t|$ which do not go beyond a few GeV^2 . One may assume that t dependent factors present in a Regge term combine in such a way that R can be approximated by a constant for a proper choice of s_0 ⁵⁾. With s_0 chosen as usual of the order of 1 GeV^2 , this leads to a slope of $1(\text{GeV})^{-2}$ for the leading vacuum trajectory when reproducing the forward exponential peak from (15) and (13). This does not differentiate the Pomeranchuk trajectory from the other ones which are all known to have slopes of the order of $1(\text{GeV})^{-2}$. This leads however to difficulties with the absence of shrinking observed in many reactions¹⁴⁾. This would also go against the present description of the leading vacuum Regge singularities in terms of the finite energy sum rule approach and the duality picture¹⁵⁾. A smaller slope for the Pomeranchuk trajectory implies an exponential drop of $R(t)$ in order to still meet the values obtained for the slope of the diffraction peak in the 10-20 GeV range. Approximating R by $R_0 e^{+B_1 t}$ and taking linear trajectories $\alpha(t) = \alpha(0) + \alpha' t$ one combines (1) and (14) to obtain

$$N_{22} = \frac{1}{8\pi k\sqrt{4\pi s}} \sum_i \left\{ i^P R_0^i \left(\frac{s}{s_0} e^{-i\frac{\pi}{2}} \right)^{\alpha_i(0)} \int \left(\frac{s}{s_0} e^{-i\frac{\pi}{2}} \right)^{-\alpha' q^2} e^{-(B_1 q^2 - i b q \cos \theta)} d^2 q \right\}$$

which gives

$$N_{22} = \frac{1}{8\pi k \sqrt{4\pi s}} \sum_i i^P R_o^i \left(\frac{s}{s_o} e^{-i\frac{\pi}{2}} \right)^{\alpha_i(o)} \frac{1}{\alpha'_i} e^{-\frac{q^2}{4\alpha'_i}} \quad (16)$$

with

$$\gamma = \log\left(\frac{s}{s_o}\right) - i\frac{\pi}{2} + \frac{B}{\alpha'} \quad (17)$$

H_{22} is then obtained from (11) with

$$I_{22}(s) = \frac{1}{\sqrt{4\pi}} \left(i + \frac{1}{\pi} \log\left(\frac{s}{s_1}\right) \right)$$

as it follows from (1) and (3).

s_1 is a cut of energy ($s_1 \gg M_i^2$) which limits the integral to high energy values where the approximate form of unitarity (2) is valid. In the energy range explored at present only the first term is relevant in the calculations ^{11), 12)}.

The final form of the amplitude is obtained from (1). The amplitude then appears as a sum of pole contributions and an infinite set of cut corrections associated with the folding integral of the successive terms obtained when developing the denominator of (10) ¹⁰⁾. The first correction is identical to the one obtained in the expansion of the eikonal. The higher order corrections obtained for H are however different and this gives in particular a different t behaviour for the phase of the elastic amplitude. The Regge branch points have the correct location ⁷⁾ and an imaginary single scattering amplitude yields the expected alternation in sign of the succeeding correction terms.

As it is readily seen from (12) and (16), the effective slope of the differential cross-section will decrease when the order of the process increases. One arrives at a similar though different classification of production mechanisms as Frautschi and Margolis ¹⁵⁾.

The behaviour at large energy can be obtained from (11) by calculating the effective slope at small t given by the superposition of terms summing up the corresponding series. The corrected Regge behaviour leads to a shrinking diffraction peak. The variation with energy of the forward peak is directly linked to the slope of the leading vacuum trajectory. As an illustration Fig. 4 shows the differential cross-sections expected at ISR energy (1700 GeV) for $\alpha' = 1(\text{GeV})^{-2}$ and $0.2(\text{GeV})^{-2}$.

3. ELASTIC SCATTERING

A leading vacuum trajectory with proportional residue function may be used to reach unitarized amplitudes for pp , πp and Kp scattering. The differential cross-sections thus obtained are compared with experimental results¹⁶⁾. This is displayed on Figs. 1 and 3. One finds a structure at $|t| \simeq 1 \text{ GeV}^2$ which results from the unitarity corrections. It is more pronounced in pp scattering where the residue function is larger than in πp and Kp scattering. The Kp and πp data are very similarly reproduced up to $|t| \simeq 2.5 \text{ GeV}^2$ and we actually give only the πp differential cross-section (Fig. 3). The marked structure obtained in pp scattering (Fig. 1) is reached with quite different values for the slope of the vacuum Regge amplitude introduced, B and α' being related in such a way that an almost identical forward peak would result in the 10-20 GeV region. With a value of the slope equal to 1 GeV^{-2} one observes a rapid increase with $|t|$ of the real part which is almost zero in the forward direction. The first vanishing of the imaginary part obtained in the neighbourhood of $|t| = 0.5 \text{ GeV}^2$ does not produce any noticeable effect since it is marked by the behaviour of the real part. The structure obtained is in fact associated with the vanishing of the real part. On the other hand, with a relatively small value for the slope of the leading vacuum trajectory ($\alpha' = 0.2 \text{ GeV}^{-2}$ for example), the real part of the amplitude increases more slowly with $|t|$ and the structure is closer to that expected from a diffraction minimum now associated with the vanishing of the imaginary part. The correction term here introduced does not lead however to any sharp minima¹²⁾. A structure at $|t| \sim 3(\text{GeV})^2$ also becomes apparent when α' decreases.

The important cusp shown by recent data would seem to favour a relatively small slope for the leading trajectory. This cannot be yet considered as evidence.

The variation of the differential cross-section with energy is illustrated on Fig. 4, where we have displayed the values given by the model at ISR energy (1700 GeV) for the two values of α' ($\alpha' = 1$ and 0.2). The structure is less marked when the energy increases and moves toward low $|t|$ values. The total cross-section, obtained through the optical theorem is found to be almost constant with energy.

In order to refine the single scattering amplitude one may add secondary poles and in particular the ω and P' with degenerate trajectories and residue calculated in such a way as to reproduce the difference between the pp and $\bar{p}p$ cross-section. The real part of the pp amplitude has then the correct sign and magnitude¹⁷⁾. Thus, complicating the initial Regge amplitude does not however yield to a better agreement with the experimental results beyond $|t| \simeq 2 \text{ GeV}^2$, a region which does not seem amenable to a description in terms of a corrected simple Regge amplitude, and where a different mechanism may play a prominent role^{4),12)}.

The same P' and ω parameters can be used to calculate the $\bar{p}p$ differential cross-section inverting the sign of the contribution of the ω trajectory in N_{22} . The ω residue function is not supposed to vanish for a value of $|t|$ of the order of 0.2 GeV^2 where the pp and $\bar{p}p$ differential cross-sections cross-over. This is necessary in a simple Regge pole analysis but is known to lead to serious difficulties when the factorization property of the residue functions is used. It predicts that other differential cross-sections go to zero for the same small $|t|$ value when they do not vanish¹⁶⁾. With unitarity corrections associated to the initial Regge amplitude cross-over is rather associated with a cancellation between the pole and the unitarity correction contributions which are odd under charge conjugation. The effect of the P' and ω contributions, which have to be introduced at the same time to enforce a lack of symmetry between the pp and $\bar{p}p$ energy behaviour, do not lead to cross-over even

though the $\bar{p}p$ differential cross-section is found to have a larger slope at small $|t|$ values. In order to obtain a cross-over it is necessary to increase the corrections to the $\bar{p}p$ Regge pole amplitude. This may be done by increasing artificially the phase space factor which appears in (11), writing $\lambda \mathbf{I}$ with $\lambda > 1$ instead of $\mathbf{I}$. This is motivated by the fact that in the 10-20 GeV range there are many more final states available in $\bar{p}p$ than in pp scattering with annihilation channels playing an important role. More states should be taken in the matrix H_0 before a good approximation is obtained. Relation (11) no longer holds and should be replaced by a more complicated expression. For relatively low $|t|$ values this can be safely represented by an increase of the correction term. Figure 2 shows cross-over as it can be then obtained. The value which has to be retained for λ is as high as 1.6. The calculated $\bar{p}p$ differential cross-section shows a marked cusp at $|t| = 0.6 \text{ GeV}^2$ as observed experimentally. λ is expected to decrease to 1 with energy with a corresponding anti-shrinking effect for the $\bar{p}p$ differential cross-section.

As already mentioned factorization should be introduced in the N matrix or for the Regge amplitudes written for N_{22} in different processes. After introducing the correction terms only an approximate factorization property remains. The corrections affecting relatively weakly the imaginary part of the forward amplitude, factorization as effectively checked on the relation between total cross-sections will be maintained. As an example the relation

$$(\beta_{\pi N})^2 = \beta_{\pi\pi} \beta_{NN}$$

written for the residue of the vacuum trajectory leads within 2% to the corresponding relation between cross-sections

$$\sigma_{\pi N} = (\sigma_{\pi\pi} \sigma_{NN})^{\frac{1}{2}}$$

when correction terms are introduced.

4. ISOBAR PRODUCTION

The production mechanism of nucleon isobars with isospin $\frac{1}{2}$ and belonging to the natural parity series is generally referred to as diffractive dissociation¹⁸⁾. In Regge pole theory these reactions are considered on the same footing as elastic scattering. The same Regge trajectory can be exchanged but appears with much smaller residue functions. With our extension of the Baker-Blankenbecler formalism the corresponding differential cross-sections should be obtained from (12), (13) and (14). Here we differ from the corrections introduced with expansion in terms of the eikonal even for the first order term. If it is possible to approximate most of the inelastic amplitudes as in (3), this cannot be done for this particular type of reaction where Pomeron exchange would have to be included through B which is assumed to be real. Diffraction dissociation requires a special treatment and considering it on the same footing as elastic scattering leads to new correction terms as explicitly given in (12). To the contrary the Δ production amplitude could be in a first approximation included in H_2 and calculated from (3). Figure 5 shows the slopes thus expected from a one step and a two step process if a Pomeronchuk trajectory with a standard slope of $1(\text{GeV})^{-2}$ is considered together with constant values for R as it is then approximately the case for elastic scattering with s_0 chosen as 1 GeV^2 . As directly seen from (16) an increase in the number of steps (product of Regge pole terms in the numerator of 12) decreases the resulting effective slope.

The slopes of the $1520(\frac{3}{2}^-)$, $1690(\frac{5}{2}^+)$ and $2190(\frac{7}{2}^-)$ isobars show very similar values, Ref. 19),

1520	3.88 ± 0.45
1690	5.05 ± 0.38
2190	5.14 ± 0.56

whereas the slope observed for the $1470(\frac{1}{2}^+)$ is strikingly larger: 21.1 ± 4.4 .

This difference is most easily understandable in the quark model where the first group of isobars corresponds to "orbital" excitation of the nucleon state ²⁰⁾ while the $\frac{1}{2}^{+}$ would be associated to a more complicated structure. One should then expect two quite different types of t dependence. In fact, the slope observed for the 1470 cannot simply be attributed to a one step reaction but implies a residue function which decreases faster with $|t|$ than with the one introduced for elastic scattering. The slopes observed for the "orbital" excitation states could be interpreted as suggesting the presence of two step processes ¹³⁾. This would however leave unclear the factorization property which seems to hold, at least approximately, in relating isobar production in nucleon and pion induced reactions ²¹⁾, and relies on the introduction of similar residue functions for isobar production and elastic scattering, an assumption which may well not be met by the facts. With a smaller slope for the vacuum trajectory, elastic and isobar productions could, on the other hand, be still described in much the same way, that is with a dominating one step reaction, the residue function for elastic scattering falling faster at small $|t|$ than those for "orbital" excitation. Following this point of view the single scattering amplitude for isobar production would then be written in analogy with the one retained for elastic scattering as proposed by Chou and Yang ²²⁾. The correction terms which we apply to it, following (12), are however different from those which we introduced for elastic scattering.

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FIGURE CAPTIONS

Figure 1 : pp differential cross-section ¹⁾ at 12 and 18 GeV/c.

The solid curves correspond to $\alpha' = 1 \text{ GeV}^{-2}$ and are drawn for 12 and 18 GeV/c.

The differential cross-section obtained at 12 GeV/c with $\alpha' = 0.2 \text{ GeV}^{-2}$ is represented by a dashed curve; only a small variation is then observed between 12 and 18 GeV/c.

Figure 2 : $\bar{p}p$ data ¹⁾ and pp and $\bar{p}p$ differential cross-section drawn for 10 GeV/c.

Figure 3 : Λ^-p scattering.

Experimental results ¹⁾ and calculated cross-section for 10 GeV/c and 18 GeV/c with $\alpha' = 1 \text{ GeV}^{-2}$. The dashed curve is drawn through the experimental points at 6 GeV. The higher curve corresponds to 10 GeV/c.

Figure 4 : pp differential cross-sections at 12 GeV/c and 1700 GeV.

The solid curve corresponds to $\alpha' = 1 \text{ GeV}^{-2}$ and the dashed curve to $\alpha' = 0.2 \text{ GeV}^{-2}$.

Figure 5 : Slopes for isobar production assuming the same t dependence for the single scattering term. The dashed line indicate the observed slope for 1470 and 1690 isobar production.

FIG.1

FIG.2

FIG.3

FIG.4

FIG. 5