CM-P00058246

Ref.TH.1023-CERN

Archives

A FIT TO THE PROTON ELECTROMAGNETIC FORM FACTORS

R. Jengo and E. Remiddi
CERN - Geneva

A B S T R A C T

We propose a compact formula for the electromagnetic form factors of the proton exhibiting an infinite number of poles in the timelike region. We discuss the fit to the existing data in the spacelike region and the prediction for the reaction $e^+e^- \rightarrow \bar{p}p$.

Ref.TH.1023-CERN

7 May 1969

It is known that it is difficult to obtain a fit to the e.m. form factors of the nucleons by means of formulas whose analytic structure can be simply understood in terms of strong interaction dynamics (as required by general consideration based on dispersion relations). Among the existing fits we quote for instance the famous empirical formula ¹⁾

$$G(t) = \frac{1}{\left(1 - \frac{t}{0.71}\right)^2} \quad (1)$$

exhibiting a double pole at $t=0.71$ (instead of single poles at masses of known resonances).

The fits based on the known vector mesons, on the other hand, seem to require additional masses ²⁾, not directly connected with the known strong interaction dynamics. Indeed it is not surprising that the nearby singularities fail to explain the region of large negative t . In this paper we propose a model in which contributions from the whole singularity region of the form factors are taken into account, the cut being approximated by an infinite number of poles. For scattering processes, such an attitude has received considerable support by the Veneziano model ³⁾ in which the cuts of the scattering amplitude are approximated by the infinite set of the equally spaced poles associated with recurrences on a linearly rising Regge trajectory.

Lovelace ⁴⁾ noted first the relevance of the first daughter of the second recurrence of the ρ trajectory in the analysis of the reaction $\bar{p}n \rightarrow 3\pi$, and suggested to take into account its contribution to the e.m. form factors of nucleons.

In the present model we approximate the cut of the form factors by the poles associated with the $J=1$ daughters of all the recurrences of the ρ trajectory.

We note that whereas a finite number of poles allows only for a t^{-n} , n integer, behaviour of the form factor $G(t)$ for $t \rightarrow -\infty$, the sum of an infinite number of poles allows in general for a behaviour t^{-r} , r being any real number.

As a mathematical realization of a model exhibiting such a possibility, we take the form ³⁾

$$G(t) = \sum_{n=n_0}^{\infty} \gamma_n \frac{\Gamma(1-\rho(t))}{\Gamma(n+x+1-\rho(t))} \frac{\Gamma(n+x+1-\rho(0))}{\Gamma(1-\rho(0))} \quad (2)$$

where $\rho(t)$ is the ρ trajectory, and the γ_n are real constants. The parameter x determines the large t behaviours, the n^{th} term of the sum in Eq. (2) behaving at large t as t^{-n-x} .

Each term in Eq. (2), on the other hand, can be represented as a superposition of infinite poles occurring at $\rho(t) = 1, 2, \dots$

$$\frac{\Gamma(1-\rho(t))}{\Gamma(n+x+1-\rho(t))} = \sum_{m=0}^{\infty} \frac{(-1)^m}{m! \Gamma(n+x+1-m) \Gamma(m+1-\rho(t))} \quad (3)$$

As said before, these infinite poles are the $J=1$ daughters of the recurrences of the ρ trajectory. If x is integer, the series in Eq. (3) reduces to the sum of a finite number of poles.

We have investigated the possibility of reproducing the experimental data for the magnetic form factor of the proton, $G_M(t)$, which is the best known one at both small and large t , by means of a limited number of terms in Eq. (2) for the two cases $x=0$ and $x=\frac{1}{2}$.

For $x=0$, consistently with the above discussion, we encountered the usual difficulties connected with formulas based on a finite number of poles.

The $x = \frac{1}{2}$ choice seems to be much more promising. In fact, we obtain a good agreement with the data retaining just three terms in the sum of Eq. (2). We then have two free parameters [we use the normalization $G_M(0) = 1$]. The resulting formula is

$$G_M(t) = \gamma_2 \frac{\Gamma(1-\rho(t))}{\Gamma(\frac{5}{2}+1-\rho(t))} \frac{\Gamma(\frac{5}{2}+1-\rho(0))}{\Gamma(1-\rho(0))} + \gamma_3 \frac{\Gamma(1-\rho(t))}{\Gamma(\frac{3}{2}+1-\rho(t))} \frac{\Gamma(\frac{3}{2}+1-\rho(0))}{\Gamma(1-\rho(0))} + (1-\gamma_2-\gamma_3) \frac{\Gamma(1-\rho(t))}{\Gamma(\frac{9}{2}+1-\rho(t))} \frac{\Gamma(\frac{9}{2}+1-\rho(0))}{\Gamma(1-\rho(0))} \quad (4)$$

where, consistently with previous use ^{4),5)}, $\rho(t)$ is taken as known and given by $\rho(t) = 0.483 + 0.885t$, and by hand adjustment we find

$$\gamma_2 = 1.752$$

$$\gamma_3 = -2.893$$

$$\gamma_4 \equiv 1 - \gamma_2 - \gamma_3 = 2.141$$

When $t \rightarrow -\infty$, the leading term in an expansion of the form of Eq. (2) is the one with minimum n - in the practical case of Eq. (4), it is the first term, with $n=2$, behaving as $t^{-5/2}$, the second and third ones behaving as $t^{-7/2}$, $t^{-9/2}$, respectively.

One may wonder what is the leading term for $t \simeq 0$. In such a case the expansions of Eqs. (2), (4), which give emphasis on the asymptotic behaviour, become inadequate.

Equation (2) can however be rewritten, by formal rearrangement of the terms, as

$$G(t) = \sum_{m=0}^{\infty} \delta_m \frac{\Gamma(m+1-\rho(t))}{\Gamma(n_0+x+1-\rho(t))} \frac{\Gamma(n_0+x+1-\rho(0))}{\Gamma(m+1-\rho(0))} \quad (5)$$

This expansion gives emphasis on the nearby singularities and it is a priori better suited for $t \simeq 0$. In particular, Eq. (4) can be identically rewritten as

$$G_M(t) = \delta_0 \frac{\Gamma(1-\rho(t))}{\Gamma(\frac{5}{2}+1-\rho(t))} \frac{\Gamma(\frac{5}{2}+1-\rho(0))}{\Gamma(1-\rho(0))} + \delta_1 \frac{\Gamma(2-\rho(t))}{\Gamma(\frac{5}{2}+1-\rho(t))} \frac{\Gamma(\frac{5}{2}+1-\rho(0))}{\Gamma(2-\rho(0))} \\ + (1-\delta_0-\delta_1) \frac{\Gamma(3-\rho(t))}{\Gamma(\frac{5}{2}+1-\rho(t))} \frac{\Gamma(\frac{5}{2}+1-\rho(0))}{\Gamma(3-\rho(0))} \quad (6)$$

where

$$\delta_0 = \gamma_2 + \frac{2}{5} \left(\frac{7}{2} - \rho(0) \right) \gamma_3 + \frac{4}{35} \left(\frac{7}{2} - \rho(0) \right) \left(\frac{9}{2} - \rho(0) \right) \gamma_4 = 1.226$$

$$\delta_1 = \left[-\frac{2}{5} \gamma_3 - \frac{8}{35} \left(\frac{9}{2} - \rho(0) \right) \gamma_4 \right] (1 - \rho(0)) = -0.418$$

$$\delta_2 \equiv 1 - \delta_0 - \delta_1 = \frac{4}{35} \gamma_4 (1 - \rho(0)) (2 - \rho(0)) = 0.192$$

We note that the first term in Eq. (6), the only one which has the nearest singularity, the ρ pole, is dominant at $t \simeq 0$. This is not the usual ρ dominance, as the first term contains also all the other poles⁶⁾.

A few remarks on the approximation tacitly assumed.

- 1) For the proton form factors, we should have considered contributions from the ω and φ trajectories. In first approximation, we have assumed that the family of the ρ and its recurrences is degenerate with the family of ω and φ .
- 2) The chosen $\rho(t)$ trajectory is linear and real (the poles are exactly on the real axis).

Due to those approximations the formula should work for large negative t . In fact, however, the agreement is good for the whole range.

The results are shown in Fig. 1. The experimental points are taken from Refs. 7)-10).

The upper part of the curve refers to $0 \leq |t| \leq 3$, the lower part to $|t| \geq 3$. The full line represents Eq. (4) or, which is the same, Eq. (6). For comparison, we have also shown the empirical formula Eq. (1) (dotted line) for $|t| \geq 3$. For $|t| \leq 3$ the two curves coincide graphically, the numerical discrepancy being always less than 1.5%. In the region of large t Eq. (4) seems to be in better agreement with the data than the empirical Eq. (1).

The asymptotic behaviour of Eq. (4), as noted previously, is $t^{-5/2}$, while Eq. (1) gives t^{-2} . In this respect, we recall that it has been noted in Ref. 11) that the data seem to indicate a decrease stronger than t^{-2} .

We want to stress that, contrary to the usual ρ dominance philosophy our formula cannot be approximated by the sum of its nearby singularities, i.e., the first few poles in an expansion of the type of Eq. (3), with the residue fixed from Eq. (4) (this was checked numerically up to the first nine poles). We remind that also in previous fits there was an indication of the importance of short range forces.

A few words about the electric form factor $G_E(t)$. Experimentally ¹²⁾ it is well known up to $t \simeq -1(\text{GeV})^2$, poorly known for $-1 \geq t \geq -4(\text{GeV})^2$. In the whole region $G_E(t)$ is compatible with the empirical formula, Eq. (1).

There is the a priori constraint $G_E(4m^2) = \mu G_M(4m^2)$ where μ , m are the total magnetic moment and the mass of the nucleon. We have obtained a good fit to $G_E(t)$, which satisfies the above constraint, with a formula like Eq. (6), putting in this case

$$\delta_0 = 1.417, \quad \delta_1 = -0.988, \quad \delta_2 \equiv 1 - \delta_0 - \delta_1 = 0.571$$

As each of the three terms of Eq. (6) in the small t region, which is the only relevant one in this case, does not differ very much from the experimental data, the above values for $\delta_0, \delta_1, \delta_2$ are at most indicative. Finally, we note that our form factors are not a priori meaningless in the positive t region. It can be interesting to see which are the predictions of our models for the reaction

$$\bar{e}e^+ \rightarrow \bar{p}p \quad (7)$$

Of course, in order to obtain a detailed prediction, we will need much more information on $G_E(t)$ and a better knowledge of the trajectories actually involved (ρ, ω, φ), in particular of their imaginary parts. Ignoring again the $\rho-\omega-\varphi$ structure, we find that our model involving only the ρ trajectory possesses a noticeable feature - a variation of three - four order of magnitude in the cross-section from a few μb at $t=4(\text{GeV})^2$ to some nanobarns at $t=7(\text{GeV})^2$ - which is rather independent on the detailed choice of $\text{Im } \rho(t)$.

In Fig. 2, we report the predictions for the total cross-section of reaction (7), putting for simplicity $G_E(t)=0$. We have assumed

$$\text{Im } \rho(t) = c \sqrt{t - 4m_\pi^2} \quad (8)$$

and the two curves in Fig. 2 refer to $c=0.1(\text{GeV})^{-1}$ and $c=0.25(\text{GeV})^{-1}$ respectively. With those choices for $\text{Im } \rho(t)$, the widths of the recurrences of the ρ trajectory are of the order of magnitude of the observed widths of known resonances ($100 \text{ MeV} \leq \Gamma < 500 \text{ MeV}$). As the total cross-section depends on $|G_M(t)|^2$ and $|G_E(t)|^2$ separately, the above curves can be regarded as lower limits. If $G_E(t)$ is of the form of Eq. (6), the shape of the curve is essentially unchanged. The non-degeneracy of the ω, ρ, φ families is likely to add other structure without removing the gross effect.

As a last comment, we recall that an experiment ¹³⁾ on the related process

$$\bar{p}p \rightarrow e^-e^+$$

indicates for $\mu_{G_M}(t=6.8(\text{GeV})^2)$ the order of magnitude $0.17 \sim 0.29$. Our model predicts $\mu_{G_M}(6.8) = 0.38 \sim 0.55$ for $c = 0.1 \sim 0.25(\text{GeV})^{-1}$ in Eq. (8), while the empirical formula Eq. (1) gives $\mu_G(6.8) = 0.036$.

R E F E R E N C E S

- 1) R. Hofstadter, F. Bumiller and M.R. Yearian, Revs.Modern Phys. 30, 482 (1958).
- 2) See for instance, K.Y. Ng, Phys.Rev. 170, 1435 (1968).
- 3) G. Veneziano, Nuovo Cimento 57A, 190 (1968).
- 4) C. Lovelace, Phys.Letters 28B, 264 (1968).
- 5) R. Jengo and E. Remiddi, Nuovo Cimento, to be published.
- 6) Independently of the phenomenological use we did of Eqs. (2), (5), one could wonder about the convergence of such series. It might well be that such series are to be regarded as asymptotic ones, Eq. (2) being convenient at large negative t , Eq. (5) at $t \simeq 0$.
- 7) T. Janssens, R. Hofstadter, E.B. Hughes and M.R. Yearian, Phys.Rev. 142, 922 (1966).
- 8) W. Albrecht, H.J. Behrend, F. Wo Brasse, W. Flauger, H. Hultschig and K.G. Steffen, Phys.Rev. Letters 17, 1192 (1967);
W. Albrecht, H.J. Behrend, W. Glauger and H. Hultschig, Phys. Rev. Letters 18, 1014 (1967).
- 9) M. Goitein, R.J. Budnitz, L. Carroll, J. Chen, J.R. Dunning Jr., K. Hanson, D. Invrie, C. Mistretta, J.K. Walker, G.F. Dell, M. Fatino, J.M. Paterson, H. Winick and R. Wilson, Phys.Rev.Letters 18, 1016 (1967).
- 10) D.H. Coward, H. DeStaebler, R.A. Early, J. Litt, A. Minten, L.W. Mo, W.K.A. Panofsky, R.E. Taylor, M. Breidenbach, J.I. Friedman, H.W. Kendall, P.N. Kirk, B.C. Barish, J. Mar and J. Pine, Phys.Rev. Letters 20, 292 (1968).
- 11) T. Massam and A. Zichichi, Nuovo Cimento 1, 387 (1969).
- 12) See for instance, Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies, Stanford (1967).
- 13) M. Conversi, T. Massam, Th. Muller and A. Zichichi, Nuovo Cimento 40, 690 (1965).

FIG. 1

FIG. 2