Stabilized Singlets in Supergravity as a Source of the μ Parameter

Christopher Kolda

School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540

Stefan Pokorski

CERN, Theory Division, CH-1211 Geneva 23, Switzerland

and Institute for Theoretical Physics, Warsaw University, Hoza 69, 00-681 Warsaw, Poland

Nir Polonsky

Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854

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We demonstrate a new and generic mechanism which stabilizes a singlet in supergravity-coupled supersymmetry in the presence of an extended gauge symmetry. Such a symmetry will be broken down to the standard model by the supergravity interactions near the scale of spontaneous supersymmetry breaking in the hidden sector ($\sim 10^{10-11}$ GeV). The resulting singlet expectation value is stabilized and naturally of order the gravitino mass, providing therefore a weak-scale mass for the Higgs fields of the supersymmetric standard model (a μ parameter). The resulting low-energy theory is the minimal supersymmetric standard model, with all new fields decoupling at the intermediate scale. [S0031-9007(98)06339-X]

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One of the primary motivations for low-energy supersymmetry (SUSY) is the stabilization of the weak scale (m_W) against quadratic divergences which appear at the quantum level and can drive m_W up to the Planck scale (M_P). To accomplish such a feat, one typically postulates that there are two separate sectors in the Lagrangian describing all of physics: a visible sector in which exist the standard model (SM) and its SUSY extension, the minimal supersymmetric standard model (MSSM), and a hidden sector in which exist the fields and interactions necessary to spontaneously break SUSY (see, e.g., [1]). Consistency of the model requires that the hidden and visible sectors must not communicate with one another directly via renormalizable, tree-level interactions, and so one must also postulate a mediation mechanism which communicates SUSY breaking from the hidden to the visible sector.

The simplest and most elegant means for communicating SUSY breaking is via supergravity interactions; in fact, this mechanism is inherited directly from the gauging of SUSY and requires no additional sectors or interactions. In this picture, SUSY is spontaneously broken in the hidden sector at a scale M_{SUSY} . This scale is then communicated to the visible sector through Planck-scale suppressed operators so that the visible SUSY-breaking scale is M_{SUSY}^2/M_P . Setting this scale to m_W (in order to have weak-scale SUSY), one finds that the hidden sector scale is then $M_{\text{SUSY}} \simeq \sqrt{m_W M_P} \equiv M_{\text{int}} \simeq 10^{10-11}$ GeV. The resulting picture has three scales (though they are not all independent): M_P , M_{int} , and m_W . The physics in each sector, however, is only sensitive to two of these scales: M_P and M_{int} in the hidden sector, and M_P and m_W in the visible sector. In both cases, one expects that SUSY-

preserving masses should be near M_P , and that SUSY-breaking masses in each sector should be near the scale which controls SUSY breaking in that sector.

However, there is one necessary, and another possible, violation of these expectations in low-energy SUSY. The former is often referred to as the μ problem [2]. The superpotential of the MSSM for phenomenological reasons must contain a term $\mu H\bar{H}$ which mixes the two Higgs doublets responsible for electroweak symmetry breaking, with $\mu \sim m_W$. However, the μ term is both SUSY and $SU(2) \times U(1)$ invariant, and so we expect that its natural scale should be the Planck scale. The other possible violation arises whenever one wishes to extend the gauge structure of the MSSM to include new interactions broken at some scale intermediate to m_W and M_P . In particular, there exist several arguments that there should be new physics in the visible sector at a scale close to $M_{\text{int}} \sim 10^{10-11}$ GeV (for example, a right-handed neutrino). It is remarkable that this new scale for visible sector physics is also roughly the scale of SUSY breaking in the hidden sector, though there is no obvious connection. In fact, we will demonstrate a mechanism which simultaneously solves the μ problem while generating a scale for new visible sector physics close to the scale M_{SUSY} .

Begin by considering the next-to-minimal SUSY standard model (NMSSM), which extends the Higgs sector by a singlet whose vacuum expectation value (vev) in turn provides a dynamical μ term through its superpotential coupling to the usual Higgs doublets [3]. The NMSSM has two well-known problems. At the renormalizable level, it has a Z_3 symmetry. If that symmetry is preserved to all orders, then the vev of S

will break the symmetry at the weak scale and produce cosmologically dangerous domain walls. If, on the other hand, the Z_3 symmetry is not preserved by higher-order terms in the Lagrangian, then S carries no conserved quantum numbers. In this latter case, S will generically develop tadpoles, in the presence of spontaneously broken SUSY, whose quadratic divergences are cut off by the Planck scale [4,5]. The resulting shift in the potential for S causes it to slide to large values far above the weak scale. If it were to couple to the MSSM Higgs fields, they would receive unacceptably large masses, destabilizing the weak scale.

We now present a toy model which solves this destabilization problem of the NMSSM, while at the same time introducing a new visible sector interaction whose scale will naturally fall at M_{int} . Within this model, singlets can couple to MSSM fields, and, in particular, can provide dynamical μ terms at the weak scale. The model itself demonstrates a very general mechanism, though it already contains all of the ingredients necessary to be phenomenologically viable.

The model.—Consider a superpotential

$$W = \lambda_H S H \bar{H} + \lambda_\Sigma S \Sigma \bar{\Sigma}, \quad (1)$$

where $H, \bar{H}$ carry charges ± 1 under a gauge symmetry $U(1)_H$, $\Sigma, \bar{\Sigma}$ are charged ± 1 under another gauge symmetry $U(1)_\Sigma$, and S is a gauge singlet. We require $\Sigma, \bar{\Sigma}$ to be neutral under $U(1)_H$ and, for simplicity, assume that $H, \bar{H}$ are also neutral under $U(1)_\Sigma$, though they need not be. The D term for $U(1)_\Sigma$ is then simply $g_\Sigma(|\Sigma|^2 - |\bar{\Sigma}|^2)$, and similarly for D_H . To apply this toy model to the MSSM, we identify $H, \bar{H}$ as the usual Higgs doublets, and extend $U(1)_H$ to the full standard model gauge group; Σ and $\bar{\Sigma}$ are new fields charged only under $U(1)_\Sigma$.

At the level of the superpotential, there exists the usual Z_3 which forbids explicit mass terms from appearing in W [3]. This symmetry is broken by $S \neq 0$, which could lead to the creation of electroweak scale domain walls via the Kibble mechanism. The appearance of an S^3 term is forbidden by an R symmetry under which $R(W) = 2$ and $R(S) = 0$. However, we will assume that the Z_3 symmetry of the superpotential is only an accidental symmetry. This is a natural expectation since global symmetries are generally not preserved by quantum gravity effects [6]. In particular, we expect that gravity-induced global symmetry breaking will appear as nonrenormalizable, explicit symmetry-breaking terms in the Kähler potential, K . The Z_3 symmetry can thus be a symmetry of the effective superpotential without being a symmetry of the entire action. This is equivalent to the statement that the S field is a true singlet, carrying no conserved quantum numbers.

After SUSY breaking, nonzero tadpoles for S will generically arise with light chiral fields circulating in the loops [4,5,7–9]. These tadpoles appear at $\mathcal{O}(\hbar^n/M_P)$ due to supergravity corrections from Planck-suppressed

operators. We do not know *a priori* at what loop order nonzero contributions are generated. For a flat Kähler metric, nonzero tadpoles do not arise until two-loops [7,8]; however, for a nonflat metric they may arise at one-loop.

We will analyze these tadpole contributions following the formalism laid out in Refs. [5,8]. Putting aside the question of loop order at which the tadpoles arise, it is sufficient to consider these contributions as coming from an effective Kähler potential of the form $K = [1 + (S + S^\dagger)/M_P]\Phi_i\Phi_i^\dagger$. The first term is just the canonical contribution to K of a chiral superfield Φ_i , while the second term explicitly breaks the Z_3 symmetry present in the superpotential. In general, an n -loop induced tadpole generates a term in the effective Lagrangian of the form

$$\begin{aligned} \mathcal{L}_{\text{eff}} &\sim \frac{N}{(16\pi^2)^n} \frac{\Lambda^2}{M_P} \int d^4\theta e^{K/M_P^2} (S + S^\dagger) \\ &\equiv \beta \epsilon m_{3/2} M_P F_S + \gamma m_{3/2}^2 M_P S + \text{h.c.}, \quad (2) \end{aligned}$$

where N counts the number of light chiral superfields that appear in the loops and Λ is the cutoff for the quadratic divergence. Henceforth we assume that supersymmetry breaking is communicated via supergravity interactions and thus that $\Lambda \simeq M_P$, $M_{\text{SUSY}}^2 \simeq m_{3/2}^2 M_P$, and $m_{3/2} \simeq m_W$. The second line of Eq. (2) is derived by expanding the superspace density, e^{K/M_P^2} [8]. The parameter ϵ measures the first derivative of the Kähler potential with respect to the SUSY-breaking fields, or equivalently the scalar vev's of these fields, in Planckian units; hence $|\epsilon|$ ranges from 0 to roughly 1. The variables γ, β are complex coefficients which include the loop suppression factors $(16\pi^2)^{-n}$ and counting factors N , and so whose magnitudes are roughly $\mathcal{O}(10^{-4} - 1)$.

On integrating out all auxiliary fields, one finds that F_S is shifted from its canonical form by the tadpole contribution: $F_S^\dagger = -\frac{\partial W}{\partial S} + \beta \epsilon m_{3/2} M_P$, while all other F and D terms are canonical. The full scalar potential after soft SUSY breaking can then be written as

$$\begin{aligned} V = & \sum_i m_i^2 |\varphi_i|^2 + |\lambda_\Sigma S|^2 (|\Sigma|^2 + |\bar{\Sigma}|^2) \\ & + |\lambda_H S|^2 (|H|^2 + |\bar{H}|^2) + m_{3/2}^2 M_P (\gamma S + \gamma^\dagger S^\dagger) \\ & + |\lambda_\Sigma \Sigma \bar{\Sigma} + \lambda_H H \bar{H} - \beta \epsilon m_{3/2} M_P|^2 \\ & + D \text{ terms} + A, B \text{ terms}, \quad (3) \end{aligned}$$

where the first term is the gravitationally induced soft SUSY-breaking masses, $m_i^2 \sim m_{3/2}^2$, for the fields $\varphi_i = \{S, \Sigma, \bar{\Sigma}, H, \bar{H}\}$. The final terms are gravitationally induced soft-breaking bilinear, B , and trilinear, A , terms which for simplicity we ignore hereafter. Note that the potential as written requires that $m_S^2 \geq 0$ in order to be bounded from below. Indeed, one expects $m_S^2 > 0$ at tree level, and it will only be driven negative if its coupling to either of the two sets of Higgs fields is fairly large.

Henceforth we will take all soft squared masses to be equal to $m_{3/2}^2 > 0$.

Minimization of the potential.—To continue further, we take $\epsilon \approx 1$ which is the generic choice. The potential has three competing minima, all along directions that are D flat up to weak-scale corrections, i.e., $\Sigma \approx \bar{\Sigma}$ and $H \approx \bar{H}$.

At a first minimum, denoted V_1 , the nonvanishing vev's are $S \approx -\gamma^\dagger M_P$ and $|F_S| \approx |\beta m_{3/2} M_P|$ such that

$$V_1 \equiv V_{\min} \approx (|\beta|^2 - |\gamma|^2) m_{3/2}^2 M_P^2. \quad (4)$$

This minimum represents the case usually considered in the literature for singlets with nonzero tadpoles—their vev's are pulled up to the Planck scale, taking with them any matter to which they couple. This is precisely the reason it was argued in Refs. [5,8] that the vev of a true singlet cannot be responsible for the μ term in the MSSM.

At a second minimum, V_2 , the nonvanishing vev's are $\Sigma \bar{\Sigma} = \beta m_{3/2} M_P / \lambda_\Sigma$, $S \approx -\gamma^\dagger m_{3/2} / 2 |\gamma_\Sigma \beta|$, $F_S \sim m_{3/2}^2$, and $F_{\Sigma, \bar{\Sigma}} \approx \lambda_\Sigma m_{3/2} M_P^{1/2}$ such that

$$V_2 \equiv V_{\min} \approx \frac{1}{|\lambda_\Sigma|} \left(|\beta| - \left| \frac{\gamma^2}{2\beta} \right| \right) m_{3/2}^3 M_P. \quad (5)$$

The Σ fields receive vev's of $\sim \sqrt{m_{3/2} M_P}$ to cancel off the F_S contribution to the potential. These large Σ vev's then produce masses for the S field (through the F_Σ terms) which stabilize the S vev against the linear potential. The resulting vev of S is then only $\langle S \rangle \sim m_{3/2} \approx m_W$.

Any gauge symmetry carried by the Σ fields will be broken at the scale of their vev's. Up to the loop factors buried in β , this is the intermediate scale, M_{int} . It is also, not coincidentally, the scale of SUSY breaking in the hidden sector. In fact, one may interpret the physics at this minimum as the tadpoles communicating to the Σ fields the true scale of SUSY breaking, up to the loop factors.

There is also a third minimum, V_3 , which is identical to V_2 except that the would-be MSSM Higgs fields, H and $\bar{H}$, play the role of Σ and $\bar{\Sigma}$ and receive vev's $\sim M_{\text{int}}$, with λ_H replacing λ_Σ in all expressions. This is clearly *not* the desired minimum but is instead another example of how the tadpole can destabilize the weak scale. (Note that points at which $H, \bar{H}, \Sigma, \bar{\Sigma}$ all get vev's simultaneously are not even local minima of the potential.)

One still needs to resolve which of the three minima is the global one. Clearly $|V_{2,3}| \ll |V_1|$, but if $V_1 < 0$ then V_1 will be the global minimum. It is easy to show that $V_1 > 0$ if and only if $|\gamma|^2 < |\beta|^2$, which we will assume hereafter. Given the arbitrariness of β and γ , this is not a restrictive requirement.

But assuming $V_1 > V_{2,3}$, which of either pair of fields $\Sigma, \bar{\Sigma}$ or $H, \bar{H}$ gets a vev? Though the form of the potential for the Σ and H fields is the same, they have differing quadratic pieces (effective masses) coming from their soft masses and their couplings to S . Assuming

equality of soft masses, V at its minimum is positive (given $|\gamma|^2 < |\beta|^2$) and scales as either $|\lambda_\Sigma|^{-1}$ or $|\lambda_H|^{-1}$. Thus the global minimum of the potential occurs where only the pair of fields with largest absolute coupling to S gets vevs $\sim M_{\text{int}}$. That is, if $|\lambda_H| < |\lambda_\Sigma|$, it is energetically unfavorable for $H, \bar{H}$ to receive vevs $\sim M_{\text{int}}$ and so only the $\Sigma, \bar{\Sigma}$ fields do. Note that simple inequality is all that is needed to ensure a gauge hierarchy; no large hierarchy is needed between the two couplings themselves. Thus we conclude that the S -vev can, in fact, provide the μ term of the MSSM as long as $|\lambda_H| < |\lambda_\Sigma|$. The whole question of which gauge group is broken at the scale $m_{3/2}$ and which at M_{int} may rest entirely on the relative size of two couplings (λ_H and λ_Σ) whose ratio is generically $\mathcal{O}(1)$.

Recapping, we have found that if $|\beta|^2 > |\gamma|^2$ and if $|\lambda_H| < |\lambda_\Sigma|$ then S develops a vev at the weak scale and provides a μ parameter of the correct size for the light MSSM Higgs fields. Henceforth we will assume that these conditions hold and examine the solution in minimum V_2 . The R symmetry under which S is neutral and all other superfields are singly charged is explicitly broken by $R(\beta m_{3/2} M_P) = 2$ and the would-be axion is given a mass near the intermediate scale, rendering it harmless. The residual phase symmetry in the MSSM Lagrangian is also explicitly broken by $F_S \sim m_W^2$, which generates a B term in the Higgs sector ($\sim F_S^\dagger H \bar{H}$) and gives mass to the pseudoscalar Higgs/would-be axion. The heavy spectrum contains two massive pseudoscalars (with masses near $\langle \Sigma \rangle$), a massless Goldstone which is eaten by the gauge field, and three massive scalars. In particular, the S scalar itself is not in the light spectrum. It receives a mass via its coupling to Σ and $\bar{\Sigma}$ near M_{int} . Likewise the fermionic partners of the heavy scalars are all heavy.

Finally, it can be verified that our conclusions are unaffected by relaxing the assumption that fields of the MSSM are neutral under $U(1)_\Sigma$, or if the Kähler potential mixes hidden and visible sector fields directly, i.e., $K = Z^\dagger Z S / M_P$. Furthermore, the dominant effective potential contribution $\Delta V \sim (m_{3/2} / M_P) S^4$ is also too small to affect our analysis. On the other hand, a superpotential S^3 must clearly be forbidden [which is the case here for $R(S) = 0$] in order to avoid S receiving a vev to cancel off F_S .

In conclusion, in this Letter, we have demonstrated a new mechanism for obtaining a weak-scale μ term in the MSSM by adding a total singlet in conjunction with a new gauge interaction and its accompanying Higgs sector. We have used the tadpoles endemic to models with singlets to drive the breaking of the new symmetry at the intermediate scale $\sqrt{m_W M_P}$, to remove all vestiges of the singlet from the low-energy theory, and to render all would-be axions harmless. In this way we have pointed out a natural way to tie the introduction of intermediate-scale operators, such as those that might give mass to a

right-handed neutrino, to the solution of the weak-scale μ problem. The model itself is highly generic and can naturally explain the hierarchy between the weak and intermediate scales in terms of the ratio of two couplings which is $\mathcal{O}(1)$. It seems straightforward to generalize our discussion to more complete models of physics at intermediate scales [10].

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