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ON THE GLAUBER SERIES INTERPRETATION OF SCATTERING DATA

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A B S T R A C T

The Glauber¹⁾ theory of high-energy collisions is used in a discussion of the scattering of composite bodies on composite bodies. The scattering amplitude is developed as a multiple scattering series, and it is shown that conclusions drawn on the basis of truncated versions (e.g. inclusion of up to triple scattering only) of such series should be viewed with extreme caution. Because of ease of computation a model of Gaussian density distributions and profile functions is considered in detail and applied to the prediction of the α - α elastic cross-section and to a discussion of "Quark" model interpretation of the p-p elastic cross-section. Nothing in the results can be interpreted as a special support for the ordinary Quark model, mere multiplicity is what counts.

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INTRODUCTION

The observation of diffraction-like scattering patterns in high-energy hadron-hadron collisions²⁾ has led several authors³⁻⁴⁾ to consider the interpretation of the data in terms of multiple scattering series like the Glauber series¹⁾ or integral representations thereof. In the Quark models⁴⁾ this interpretation conveys a direct picture of scattering of composite bodies on composite bodies. In general the Quark model investigations suffer from the fact that the order of multiple scattering included in the computations has been limited to double, triple or quadruple scattering only, whereas the impact of an $A = 3$ on a $B = 3$ composite system would in principle permit up to 9-fold scattering. At the same time the pertinent Glauber series is extremely slowly converging in terms of the multiplicity cut-off n_0 and as we shall see in this note cut-offs for a Quark model of nucleons with $n_0 \leq 6$ are a rather poor approximation for the larger $|t|$ values for which experimental data are available.

The extension of the Glauber formalism to a proper treatment of the impact of a body consisting of A "bits of stuff" on one consisting of B "bits of stuff" involves two practical problems. The first of these is the counting and classification of the pertinent types of multiple scattering. This problem is closely related to the familiar multiple interaction classification in the theory of condensation⁵⁾ and makes the Glauber series look like a partition function. The number of possible essentially different interaction terms is large and grows exponentially with n_0 being 1 for $n_0 = 1$ and 152 for $n_0 = 6$ and the bodies A and B different. The second problem is a natural consequence of the first one. Each interaction term involves a multidimensional complicated form-factor integral and the second practical problem is consequently the evaluation of an enormous number of complicated integrals. This complex of problems will be attacked here under the simplifying assumptions of Gaussian density distributions and profile functions. Although this model is rather crude the general type of results obtained has considerable significance.

METHOD EMPLOYED

We consider the generalization of the elastic scattering amplitude in the Glauber formalism to the collision between e.g. a nucleus composed of A uncorrelated nucleons with one composed of B nucleons. The expression is:

$$F(Q) = \frac{ik}{2\pi} \int e^{iQ \cdot b} \left(\prod_{i=1}^A \tau_A(s_i) d^2 s_i \right) \left(\prod_{j=1}^B \tau_B(v_j) d^2 v_j \right) \left[1 - \prod_{i=1}^A \prod_{j=1}^B (1 - \Gamma(b - s_i + v_j)) \right] d^2 b \quad [1]$$

where τ_A and τ_B are the nucleon density distributions projected onto the plane perpendicular to the incoming momentum $\vec{k}$. The magnitude Q is the perpendicular momentum transfer i.e. $Q^2 = |t|$. The integration variables are the two-dimensional magnitudes s_i , v_j and b which are respectively the coordinates of the i^{th} nucleon of A, the coordinates of the j^{th} nucleon of B (referred to the c.m. of A and B respectively) and b is the overall impact parameter i.e. the distance between the c.m. of A and that of B. All the magnitudes s_i , v_j and b are coordinates in the plane perpendicular to $\vec{k}$ and so are the c.m. coordinates in question. Γ is the profile function for nucleon-nucleon scattering. Eq. [1] involves $2^{A \cdot B} - 1$ terms. In an independent particle model, however, the number of necessary computations is reduced because there are groups of terms which are identical. Still, as mentioned in the introduction, for nontrivial values of A and B hundreds of essentially different terms remain for individual computation even when the products of the Γ 's in eq. [1] are limited to the lowest values ensuring reasonable convergence. Clearly, then, simplification is needed.

The main simplification introduced is the assumption that the density projections, τ , and the profile functions, Γ , are Gaussians. In this case a theorem for the integration and Fourier transformation of multidimensional, arbitrarily rotated and translated Gaussians may be invoked. The theorem says

$$\int e^{-\frac{1}{2} \underline{x}^T \underline{a} \underline{x} - 2 \underline{b}^T \underline{x} - B^2 c + i Q B} d^N \underline{x} dB = \sqrt{\pi^{N+1} / \det |\underline{a}|} \cdot e^{-Q^2 (\det \underline{a} / (4 \cdot \det |\underline{a}|))} \quad [2]$$

where $\underline{x}$ is an N-dimensional vector, $\underline{a}$ an N by N coefficient matrix and

consequently $\underline{x}^T \underline{a} \underline{x}$ a quadratic form in the components of $\underline{x}$. Similarly $\underline{b}$ is an N-dimensional coefficient vector and $\underline{b}^T \underline{x}$ consequently a linear form in the components of $\underline{x}$. Finally B is the one-dimensional Fourier transform variable and c and Q are scalar coefficients. The symbol $\det \underline{a}$ is the determinant of $\underline{a}$ and $\det \underline{A}$ is the determinant of the N+1 by N+1 matrix $\underline{A}$ given by

$$\underline{A} = \left\{ -\frac{c}{b} \begin{vmatrix} \underline{b}^T \\ \underline{a} \end{vmatrix} \right\} \quad [3]$$

The computation of each term in eq. [1] may thus be reduced to the computation of two determinants since in a Gaussian model each term in eq. [1] is factorized into x and y integrals in the plane perpendicular to $\vec{k}$.

The computational scheme of eq.s (2-3) was utilized for the following purposes:

1) The Gartenhaus-Schwartz prescription⁶⁾ for taking the individual c.m. constraints into account in the present case was proved again analytically. The result is contained in eq. [7].

2) In a systematic approach saving computer time the terms in eq. [1] were collected into essentially different groups. The frequency of terms in each group of terms was counted (except for obvious binomial coefficient factors and symmetries) using the identity of determinants as counting criterion. The results were one punched card per group containing information as follows: two binomial coefficient signatures, the frequency of the term inside the smallest configuration where it occurs, the signature of the scattering combination in question and finally the multiple-scattering order to which it belongs. The program in question has so far been used for finding all terms up to 16-fold scattering for $A = B \leq 4$ (316 terms) and all terms up to 6-fold scattering for arbitrary A and B.

3) Using the results obtained under the above headings 1) and 2) and still utilizing eq.s [2] and [3] the predictions for $d\sigma/d\Omega$ were computed as will be discussed below. First, however, a recapitulation of the basis for the computations: the projected densities τ_A and τ_B are assumed to be:

$$\tau_A = N_A \left(\prod_{i=1}^A e^{-as_i^2} \right) \delta^2 \left(\frac{1}{A} \sum_{i=1}^A s_i \right) \quad [4a]$$

$$\tau_B = N_B \left(\prod_{j=1}^B e^{-bv_j^2} \right) \delta^2 \left(\frac{1}{B} \sum_{j=1}^B v_j \right) \quad [4b]$$

where a and b are parameters and where N_A and N_B ensure the normalization of τ_A and τ_B to unity with the c.m. deltafunctions included. The profile functions Γ were assumed to be

$$\Gamma(b-s_i + v_j) = \frac{c \sigma(1-i\rho)}{2\pi} e^{-c(b-s_i+v_j)^2} \quad [5]$$

with c , σ and ρ parameters. The total number of parameters included is consequently A , B , a , b , c , σ and ρ . Also the cut-off multiplicity n_0 could be adjusted as input parameter in order to permit an examination of the convergence of the Glauber series under consideration. The output of the programs were finished drawings of $d\sigma/d|t|$ vs $|t|$. Figs. 1-7 are examples. The programs compute

$$\frac{d\sigma}{d|t|} = \frac{\pi}{k^2} |F|^2 \quad [6]$$

with F corresponding to eq. [1] but including the c.m. constraints of eq. s (4a-b) and now written in the form

$$F = \frac{ik}{2\pi} \sum_{n=1}^{n_0} \sum_{l=1}^{l_n} C_{nl} (-1)^{n+1} \left(\frac{\sigma c(1-i\rho)}{2\pi} \right)^n \frac{\pi a^A b^B}{\det |A_{onl}|} \exp \left[-(Q^2/4) \{ (\det |a_{onl}| / \det |A_{onl}|) - 1/(Aa) - 1/(Bb) \} \right] \quad [7]$$

where C_{nl} is computed from binomial coefficients and term frequency, a_{onl} and A_{onl} are matrices pertaining to the c.m. free case and constructable from a , b , c and term type signature, the two last terms in the exponential of eq. [7] represent the Gartenhaus-Schwartz prescription, and index l runs through the l_n essentially different configurations for each n -fold scattering.

4) Finally eq.s [2-3] permit the deduction of the analytic form for the limiting cases $A=1$, $B \rightarrow \infty$ and $A, B \rightarrow \infty, \infty$. The two cases give formally identical results:

$$F = \frac{ik}{2} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \frac{1}{n!} z^n \left(\frac{1}{\alpha}\right)^{n-1} \exp[-\alpha Q^2/(4n)] \quad [8]$$

with the interpretation of α and z as given in Table 1.

TABLE 1

Interpretation of the symbols α and z in eq. [8] for $A = 1$,
 $B \rightarrow \infty$ and $A, B \rightarrow \infty, \infty$.

	Rez	Imz/Rez	α
$A = 1$ $B \rightarrow \infty$	$\frac{\sigma_B}{2\pi} \rightarrow \frac{\sigma_o}{2\pi}$	ρ	$\frac{1}{c} + \frac{1}{b}$
$A \rightarrow \infty$ $B \rightarrow \infty$	$\frac{\sigma_{AB}}{2\pi} \rightarrow \frac{\sigma_o}{2\pi}$	ρ	$\frac{1}{c} + \frac{1}{a} + \frac{1}{b}$

It is seen that only one combination of a , b and c remains in eq. [8]. Also A and B have of course disappeared as formal parameters. Consequently the case of infinite multiplicity contains σ_o , ρ and α as parameters only.

With these methods and results at hand a series of investigations were undertaken.

APPLICATION TO NUCLEAR SCATTERING

The convergence of the Glauber series in terms of the truncation multiplicity n_o depends of course on the parameters in eq. [7] and on the interval of $|t|$ to be considered. For nuclear scattering the parameter σ is the nucleon-nucleon total cross-section which is rather large. On the other hand, the interval of $|t|$ would usually be fairly small in this case. Still very large n_o values may be necessary, mainly because of the large value of σ . For $A = 1$ i.e. for high energy hadron collisions with nuclei eq. [7] reduces to the standard Glauber series for Gaussian distributions and profile functions, the determinants and the frequency factors are readily evaluated and the usual simple analytical expression results. For $A = 1$ but considering more complicated functions τ_B containing e.g. pair-correlations, triple correlations etc sums of Gaussians may be used to represent τ_B , appropriate counting and

classification schemes may be constructed and fairly simple expressions derived. For $A = 2$ (deuterons on nuclei) the counting scheme is again fairly simple but introduction of correlations in the deuteron and/or in the nucleus would soon lead to lengthy expressions were it not for the fact that the c.m. delta function for the deuteron can be used to advantage in eliminating one integration variable. For $A \geq 3$ and $B \geq 3$ complications are abundant for general models and the Gaussian picture provides a rather unique situation because of the analytic although complicated form of eq. [7]. As an example Fig. 1 illustrates the outcome of eq. [7] for elastic α - α scattering at high energy. In this case $A = B = 4$. The nucleon-nucleon total cross-section is assumed to be $\sigma = 39$ mb, the ratio of real to imaginary nucleon-nucleon scattering amplitude is put equal to its forward value $\rho = -0.25$ and c is determined from the $d^2\sigma / d|t|^2$ value for nucleon-nucleon forward scattering using the value $c = 18$ (GeV/c)². Furthermore a spherically symmetric Gaussian density distribution for the α particle leading to eq. (4a) would imply

$$\langle r_A^2 \rangle = \frac{3}{2} \frac{1}{a} \frac{A-1}{A} \quad [9]$$

Thus $a = b$ is fixed by the radius of the α particle and the value $a = b = 1/84$ (GeV/c)² has been used. The resulting prediction for the α - α elastic scattering cross-section (neglecting Coulomb effects and based on the Gaussian model) is the curve marked 16 in Fig. 1.

The remaining curves in Fig. 1 represent the results when eq. [7] is truncated after $n_0 = 2, 5, 8$ and 11 as marked on the curves. Cases for $n_0 > 10$ are in practice indistinguishable for the interval $0 \leq |t| \leq 1.5$ (GeV/c)² considered. The case $n_0 = 2$ is, of course, much too primitive. It is perhaps more surprising that including 5-fold scattering gives a misfit from the first interference dip and upwards in $|t|$ and that, similarly, 8-fold scattering is not enough to give correct results beyond the second dip.

APPLICATION TO "QUARK" MODELS

A diagram similar to that of Fig. 1 is shown in Fig. 2 but this time for parameters pertinent to p-p scattering as interpreted through "Quark" models.

"Quark" models are defined here as models where A "bits of stuff" constitute a nucleon. The Quark model is the case of $A = 3$. Now σ and ρ refer to scattering of one "bit of stuff" on one "bit of stuff", in this terminology. Calculations were carried out in such a fashion that consistency with the total cross-section and the initial ($|t| = 0$) value of $d^2\sigma/d|t|^2$ for p-p scattering as interpreted by the Krisch plot ⁷⁾ was maintained. For practical purposes this may be taken to mean the following constraints on the parameters

$$\alpha = \frac{1}{c} + \frac{2}{a} \frac{A-1}{A} \simeq 17.6 \text{ (GeV/c)}^2 \quad [10]$$

and

$$\sigma \simeq 126/A^2 \text{ (GeV/c)}^2 \quad [11]$$

which together with $A = B$ limits the free parameters to A, c and ρ . For the sake of the argument we shall furthermore assume $\rho = 0$ in the present section.

First we examine the effects of truncation. Fig. 2 shows the $A = B = 4$ case for $c = 1/6 \text{ (GeV/c)}^2$ and the effects of truncation are indicated in the same way as in Fig. 1. The resulting warning against truncation at low values of n_0 is obvious. Although this point has been illustrated here by means of the simple Gaussian model it is quite general and model independent. A similar investigation for $A = 3$ showed that $n_0 = 7$ is to be recommended corresponding to $n_0 = 8$ for the $A = 4$ case illustrated in Fig. 2 and for the same $0 \leq |t| \leq 15 \text{ (GeV/c)}^2$ interval.

If one wants to use the Gaussian model for more detailed examinations then the conclusions are as follows: For $A = B \geq 2$ it is possible to obtain $d\sigma/d|t|$ curves which fill out enough of the $(|t|, d\sigma/d|t|)$ plane so that both the experimental curves and the Krisch plot are well covered. For small values of A and B this can be done by varying c as indicated in Figs 3-5 for $A = B = 2, 3$ and 4 respectively. As A becomes larger the possible range of curves shrinks as can be noted by following the results through Figs 3-5 for a fixed value of c until for $A \rightarrow \infty$ only one curve remains i.e. the curve marked $\rho = 0$ in Fig. 7, and corresponding to eq. [8] with the $A, B \rightarrow \infty, \infty$ interpretation of Table 1. However for all possible A, B and c combinations the $(|t|, d\sigma/d|t|)$ plane can be filled again by adopting values of $\rho \neq 0$.

VARIATIONS OF THE REAL PART OF THE AMPLITUDE

As long as one has $\rho = 0$, F is, of course, pure imaginary for all $|t|$ values discussed here. F changes sign at each deep dip in $d\sigma/d|t|$. Note that all deep dips in Fig.s 1-7 ought to go to zero, only lack of computational resolution yields the present pictures. The $\rho = 0$ case should therefore be considered the special case where $\varphi = \tan^{-1}(\text{Re}F/\text{Im}F)$ changes abruptly by π at each deep dip in $d\sigma/d|t|$. If $\rho \neq 0$, φ changes smoothly as a function of $|t|$. At any rate a Glauber type interpretation of $d\sigma/d|t|$ means considerable variation in φ vs. $|t|$. The effects of $\rho \neq 0$ are illustrated in Fig. 6 for the $A = B = 3$ case with $c = 1/7$ (GeV/c)² and for the ρ values used as parameters for the curves. The conclusion is therefore that all experimental p-p data including the Krisch representation may be fitted in a multitude of ways: for each $A = B \geq 2$, for each fixed c value (inside certain limits) by letting ρ vary.

Combining eq.s [9] and [10] one finds that the present "Quark" model radius for the proton cannot exceed $\langle r_A^2 \rangle \simeq \frac{3}{4} \alpha \simeq 13.2$ (c/GeV)² which is rather small. If one wants as large a $\langle r_A^2 \rangle$ value as possible then $1/c$ must be small. In order for $1/c$ to be small and still have the $\rho = 0$ $d\sigma/d|t|$ vs. $|t|$ curve below the Krisch plot $A = B$ must be large as can be seen from Fig.s 3-5 and 7. This may even be used as an argument against the Quark model ($A = 3$).

In most cases the necessary ρ variations would be rather full of structure, but very smooth expressions can also be found to "do the trick". Thus the curve for $A = B \rightarrow \infty$ in Fig. 7 and marked $\rho = -.3-.5\sqrt{|t|}$ and compared to the Krisch curve (marked Krisch):

$$\frac{d\sigma}{d|t|} = 90.e^{-10|t|} + .74 e^{-3.45|t|} + .0029 e^{-1.45|t|} \quad [12]$$

(in mb/(GeV/c)²) follows this simple ρ variation and is perhaps an even better fit to the data than the Krisch curve when the uncertainties in the experimental data are taken into account. Roughly speaking such a ρ variation would correspond to a q' variation for the scattering amplitude for "bits of stuff" on "bits of stuff" which had a real part $\sim \gamma_1 + \gamma_2 q'^2$ with q' the momentum transfer for this artificial scattering process and γ_1, γ_2 constants. This

would result from a rough relation $n \simeq \sqrt{|t|}$ between the order n of the dominant Glauber term in eq. [8] and $|t|$. In detail this rough estimate will, however, not be sufficient. Thus it would not be advisable to consider the special fit in Fig. 7 to be of particular significance.

CONCLUSIONS

The convergence of the Glauber series is rather slow in terms of the number of multiple scatterings which must be included before truncating the series. Conclusions concerning Quark models based on too early a truncation must be considered with caution. An examination of the $A = 2, 3, 4$ and ∞ "Quark" models leads to the conclusion that inside the limitation of the Gaussian form factors utilized there is no support for the ordinary $A = 3$ Quark model and that if anything the high A or $A \rightarrow \infty$ models have particular advantages.

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FIGURE CAPTIONS

Fig. 1. The figure illustrates two points. Firstly the curve marked 11, 16 yields the prediction resulting from the present model for $d\sigma/d|t|$ vs. $|t| = Q^2$ for high-energy collisions between α particles. All numerical data are fixed by the nucleon-nucleon cross-section and the α particle radius. The data used are $A = B = 4$, $a = b = 1/84.0 \text{ (GeV/c)}^2$, $c = 1/18.0 \text{ (GeV/c)}^2$, $\sigma = 38.8 \text{ mb}$ and $\rho = -0.25$. Secondly the curves illustrate the effect of truncating the Glauber series at n_0 -fold scattering i.e. omitting n_0+1 and higher terms where n_0 is used as numbering of the curves. The remaining parameters are unchanged.

Fig. 2. The figure illustrates the convergence of the Glauber series. Just as in Fig. 1 the numbering n_0 of the curves corresponds to the truncation of the Glauber series beyond n_0 -fold scattering and $A = B = 4$. This time, however, the data correspond to a "Quark" model with four "bits of stuff" colliding with four "bits of stuff". The input parameters are adjusted so as to fit the total p-p cross-section and the slope for small $|t|$ of $d\sigma/d|t|$ for p-p elastic scattering. The data used are $a = b = 1/7.73 \text{ (GeV/c)}^2$, $c = 1/6.0 \text{ (GeV/c)}^2$, $\sigma = 3.06 \text{ mb}$ and $\rho = 0.0$. When comparing Figures 1 and 2 note the difference in the $|t|$ scale.

Fig. 3. The figure gives computed $d\sigma/d|t|$ values vs. $|t|$ for an $A = B = 2$ "Quark" model. The input parameters are adjusted to the total cross-section and the initial $d\sigma/d|t|$ slope for p-p scattering. For the accuracy here desired this means $2 < r_A^2 > + 1/c = 17.6 \text{ (c/GeV)}^2$ and $\sigma = 126/(A \cdot B)$. The curves correspond to the values of c given in the figure. In all cases $\rho = 0$.

Fig. 4. As Fig. 3 but for an $A = B = 3$ "Quark" model.

Fig. 5. As Fig. 3 but for an $A = B = 4$ "Quark" model.

Fig. 6. As Fig. 4 for $c = 1/7 \text{ (GeV/c)}^2$ and for the ρ values indicated at the curves.

Fig. 7. Reproduction of the Krisch representation of the p-p data together with a fit obtained with an $A, B \rightarrow \infty, \infty$ "Quark model" and with ρ as a function of t as indicated in the figure. The curve marked $\rho = 0$ is the $A, B \rightarrow \infty, \infty$ case for $\rho = 0$ and using the parameters of eq.s [10] and [11] as interpreted with the aid of eq. [8] and Table 1.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG.6

FIG. 7