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CHARGE EXCHANGE REACTIONS AND
CHARGE AND MASS DISTRIBUTIONS IN NUCLEI

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A B S T R A C T

It is shown that the measurements of high energy charge-exchange reactions on nuclei with incident pions or kaons will furnish a sensitive method for obtaining information on a possible difference between charge and mass distributions in nuclei. The interpretation of experimental results when they become available is discussed on the basis of the Glauber or eikonal approximation applied to Saxon-Woods distributions.

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INTRODUCTION

The possibility of a difference between the charge and mass distributions or between the charge and neutral distributions in nuclear matter is an important question for nuclear structure studies. Clearly charge exchange reactions such as

are specifically sensitive to the neutron density distribution [1a) and (2a) from left to right, (1b) and (2b) from right to left] and to the proton density distribution [1a) and (2a) from right to left, (1b) and (2b) from left to right] respectively. Furthermore for reasonably high energy pions or kaons the total cross-sections of the three pion charge states on nucleons are similar in value and the same is true for kaons. For this reason, the corrections to be envisaged in comparing yields of reactions a) to those of b) are small and easy to compute. The total cross-sections involved are of such magnitude that considerable attenuation occurs over distances in nuclear matter comparable to nuclear radii. Should the neutron density extend beyond the proton distribution then the proton sensitive reactions will be suppressed relative to the neutron sensitive ones.

In the following, we examine the yields of the two types of charge exchange reactions in such intervals of momentum transfer and energy loss by the meson that we can be confident that coherent effects

and Coulomb distortion are unimportant and that sufficiently highly excited nuclear states are involved to permit application of the nuclear closure approximation without elementary particle excitation occurring. We must have incident mesons in the GeV energy range so that the Glauber theory ¹⁾ can be expected to be reliable, but not so high that the meson-nucleon charge exchange cross-section has become too small. This cross-section is dominated at small momentum transfers by the ρ trajectory so that $d\sigma/dt \sim (s)^2 \alpha_p(t)^{-2}$ with $\alpha_p(t) \simeq 0.55 + t$ [t in $(\text{GeV}/c)^2$].

GLAUBER THEORY

The fact that the reactions (1)-(2) involve charge exchange assures at most only weak coherence at small momentum transfer. We do not propose using these reactions for the smallest momentum transfers but rather propose consideration of the region of $qR \gtrsim 1$ where incoherent reactions dominate. Here R is the nuclear radius. Incoherence corresponds to summing the matrix elements for reactions (1) or (2) over all final nuclear states ^{2),3)}. One must impose the constraint that no particles are produced other than the secondary involved in (1) or (2). The final nucleus may of course be neutron or proton unstable. These considerations require an energy resolution of the order of 100 MeV, not worse, in order to prevent other particle production from being included, and not better so that closure can be used.

The $qR \gtrsim 1$ condition and the energy loss restriction limit the useful range of momentum transfer to $0.05 \lesssim q^2 \lesssim 0.10 (\text{GeV}/c)^2$. The discussion below assumes that reactions (1) and (2) proceed predominantly in a one step process preceded and followed only by reactions which do not lead to either the initial or the final state. This is expected to be accurate to a few per cent for reactions (1) and (2) at multi GeV energies and for momentum transfers in the range given above.

To the extent that this is not true, possible corrections are of second order smallness, since we consider ratios of cross-sections for reactions (1) and (2). At larger momentum transfers, multiple step processes including initial and final state scattering will contribute in a more important way because of the smaller slope in momentum transfer for their differential cross-section.

Under these conditions Glauber theory ^{2),3)} gives results equivalent to a classical attenuation calculation. In what follows, we neglect correlations between nucleons in nuclei, identifying the ground state single particle nuclear wave functions squared with the appropriate nucleon density function. The justification for such an approximation has been discussed by Glauber ¹⁾.

To be specific, let the mass, charge and neutron density distributions of a nucleus A_ZN be $A \rho_A(\vec{r})$, $Z \rho_Z(\vec{r})$ and $N \rho_N(\vec{r})$ respectively with the normalization conventions

$$\int \rho_X(\vec{r}) d^3r = 1 \quad X = A, Z, N \quad (3)$$

Let

$$\varepsilon_Z(\vec{r}) = \rho_A(\vec{r}) - \rho_Z(\vec{r}) \quad (4)$$

and note that

$$A \rho_A(\vec{r}) = Z \rho_Z(\vec{r}) + N \rho_N(\vec{r}) \quad (5)$$

We now calculate rates for reactions (1a), (1b), (2a) and (2b). Let the total cross-section of the incident pion or kaon on neutrons be σ_{iN} and on protons σ_{iP} . We then have a charge exchange reaction with the differential cross-section $d\sigma_{CH}$. Let the total cross-section for the produced meson be σ_{fN} on neutrons

and σ_{fp} on protons. The necessary data for a discussion of each of reactions (1)-(2) are thus : $d\sigma_{CH}$, σ_{iN} , σ_{iP} , σ_{fN} and σ_{fP} . Some of these cross-sections are not too well known, but using the well-known values and isospin invariance leads to the set described in the Table.

The yield Y of any one of the reactions (1)-(2) is proportional to $d\sigma_{CH}$ (integrated over the acceptance range) and to the effective neutron number given by

$$N(A,N) \equiv 2\pi \int_0^\infty b db \left[\int_{-\infty}^\infty e^{-\int_{-\infty}^z (N\rho_N \sigma_{iN} + Z\rho_Z \sigma_{iP}) dz'} \right. \\ \left. \cdot N\rho_N e^{-\int_z^\infty (N\rho_N \sigma_{fN} + Z\rho_Z \sigma_{fP}) dz'} dz \right] \quad (6)$$

for charge exchange on neutrons and to $N(A,Z)$ given by the equivalent of Eq. (6) (with the factor $N\rho_N$ replaced by $Z\rho_Z$ between the two attenuation exponentials) for charge exchange on protons. In Eq. (6), b is the impact parameter.

For any given model of nuclear distributions and given total cross-sections $N(A,N)$ and $N(A,Z)$ are easily calculated numerically. In order to obtain estimates of orders of magnitudes involved we may, however, consider contributions from ϵ_Z and from cross-section differences as small, and expand Eq. (6) accordingly. The result is

$$N(A,N) \approx \frac{N}{A} \cdot N(A; \frac{N}{A} \sigma_{iN} + \frac{Z}{A} \sigma_{iP}, \frac{N}{A} \sigma_{fN} + \frac{Z}{A} \sigma_{fP}) \\ + \frac{Z}{A} \cdot E(A; \frac{N}{A} \frac{\sigma_{iN} + \sigma_{fN}}{2} + \frac{Z}{A} \frac{\sigma_{iP} + \sigma_{fP}}{2}) \quad (7a)$$

$$\begin{aligned}
 N(A, Z) \approx \frac{Z}{A} \cdot N(A; \frac{N}{A} \sigma'_{iN} + \frac{Z}{A} \sigma'_{iP}, \frac{N}{A} \sigma'_{fN} + \frac{Z}{A} \sigma'_{fP}) \\
 + \frac{Z}{A} \cdot E(A; \frac{N}{A} \frac{\sigma'_{iN} + \sigma'_{fN}}{2} + \frac{Z}{A} \frac{\sigma'_{iP} + \sigma'_{fP}}{2})
 \end{aligned}
 \tag{7b}$$

where the cross-sections σ and σ' must be taken from the Table for the reaction under consideration,

$$E(A; \sigma) \equiv 2\pi A \int_0^\infty b db e^{-\sigma \int_{-\infty}^\infty A \rho_A dz'} \int_{-\infty}^\infty \epsilon_Z dz \tag{8}$$

and $N(A; \sigma_1, \sigma_2)$ tabulated by Kölbig and Margolis ⁴⁾. Equations (7a)-(7b) neglect second and higher order terms for pions and third and higher order terms for kaons in symmetrical nuclei $\rho(\vec{r}) = \rho(b, +z) = \rho(b, -z)$. We note the symmetry property $N(A; \sigma_1, \sigma_2) = N(A; \sigma_2, \sigma_1)$ and the notation for these magnitudes introduced in the Table. Furthermore, we shall see shortly that ratios

$$E = E(A; \frac{\sigma_1 + \sigma_2}{2}) / N(A; \sigma_1, \sigma_2) \tag{9}$$

are rather insensitive to the cross-sections in the range of cross-section values of interest. For this reason we shall also neglect differences in E values among individual reactions (1) and (2) respectively and use the symbol E_π for the four reactions (1) and E_K for the four reactions (2).

The yield of any one of the reactions over a certain q^2 range, Y_i , is proportional to the product of $d\sigma$ integrated over this q^2 range, notation $d\sigma^*$, and the effective nucleon number in question. The expressions for the eight yields are given in the last column of the Table in the notation specified in the Table and adapted

to an illustration of the symmetries involved. Each yield is of course sensitive to ϵ_Z . However, for obvious reasons, ratios of yields are to be preferred. In this way, neglected theoretical corrections will be minimized, geometrical factors and counter efficiencies can be eliminated completely and proper combinations of the yields lead to amplification of the expected effect and to consistency checks. As the most ambitious example, the combination

$$\frac{Y_1}{Y_3} \frac{Y_4}{Y_2} \frac{Y_5}{Y_7} \frac{Y_8}{Y_6} \simeq \left(\frac{N}{Z}\right)^4 \left(1 + 2 \frac{A}{N} (E_\pi + E_K)\right) \quad (10)$$

would be very sensitive to a possible charge and matter density difference in nuclei, roughly four times as sensitive as any one of the yield ratios entering, each of which is quite sensitive to this difference. In order to evaluate further this sensitivity a model is needed.

SAXON-WOODS DISTRIBUTION INTERPRETATION

Let us assume that ρ_A and ρ_Z are given by Saxon-Woods distributions

$$\rho_A = c_{\text{NORM}} / (1 + \exp[(r - c)/a]) \quad (11a)$$

$$\rho_Z = c'_{\text{NORM}} / (1 + \exp[(r - (c - \Delta c))/(a - \Delta a)]) \quad (11b)$$

where ϵ_Z is described through small variations Δc and Δa of the radius and surface thickness parameters c and a , and where c_{NORM} and c'_{NORM} satisfy the normalization condition (3). It then

follows from the definition of $N(A; \sigma_1, \sigma_2)$ ⁴⁾ and Eqs. (4), (8), (9) and (11) that

$$E = \frac{\Delta c}{c} \beta(A) + \frac{\Delta a}{a} \alpha(A) \quad (12a)$$

with

$$\alpha(A) = a \frac{\partial}{\partial a} \log(N(A; \sigma, \sigma)) \quad (12b)$$

$$\beta(A) = c \frac{\partial}{\partial c} \log(N(A; \sigma, \sigma)) \quad (12c)$$

Numerical results for $N(A; \sigma, \sigma)$ and for $\alpha(A)$ and $\beta(A)$ are shown in Figs. 1 and 2 respectively for the parameter choice mentioned in the figure captions in rough correspondence to high energy pion and kaon data. Since α_π and α_K and similarly β_π and β_K are not very different we see that (as mentioned above) E is not very sensitive to the precise cross-section values and consequently the yield equations in the Table are good approximations. For the very lightest nuclei like helium and deuterium, E approaches zero. For heavy nuclei $\beta \gg \alpha$ and the method discussed can therefore be said to be very sensitive to differences between mass and charge radii for heavy nuclei and considerably less sensitive to variations in surface thickness. For a nucleus like lead the magnitude $2 \frac{A}{N} (\beta_\pi(A) + \beta_K(A))$ involved in Eq. (10) through Eq. (12) and multiplying $\Delta c/c$ has the value 14.5 which is the total possible amplification factor relative to other methods like the comparison of electromagnetic form factors to strong interaction form factors from, e.g., elastic scattering data.

SUGGESTED EXPERIMENTS

A set of four experiments is suggested, each of which is quite sensitive to a possible difference between the charge and mass distributions of heavy nuclei and their combinations are very sensitive to such a difference and permit consistency checks. The individual yield ratios Y_1/Y_3 , Y_4/Y_2 , Y_5/Y_7 and Y_8/Y_6 depend on the said difference and easily calculated factors. The combinations $(Y_1/Y_3)(Y_4/Y_2)$ and $(Y_5/Y_7)(Y_8/Y_6)$ depend to first order on the difference alone.

The measurement of charge exchange reactions with incoming neutral pions can probably not be accomplished because of the short π^0 lifetime.

Measurements of Y_1/Y_3 or Y_5/Y_7 with incoming meson and identification of this meson as π or K (in order to discriminate against nucleons and kaons and pions respectively) in a Cerenkov counter followed by a fast time gate. This part of the equipment may be different for positive and negative mesons. The remaining instrumentation should detect and measure neutral mesons, the decay gammas for π^0 and, e.g., the two charged pions from decay of K_S . Both energy and transverse momentum must be determined. For π^0 this may be done in a suitably shaped very large scintillator ⁵⁾, for K^0 by vertex reconstruction and energy determination for the two pions. These measurements must ensure that the $0.05 \leq q^2 \leq 0.1 \text{ (GeV/c)}^2$ restriction and the energy loss $\Delta E < 140 \text{ MeV}$ restriction are fulfilled. This part of the instrument and its selectivity should be the same for incoming beams of both signs. Under these conditions all efficiency factors and geometrical factors cancel when the yield ratios Y_1/Y_3 or Y_5/Y_7 are formed.

The measurement of Y_8/Y_6 would demand a K_L beam with time of flight measurement between production and appearance of K^+ or K^- in order to determine the K_L momentum and in order to distinguish it from γ and n . The K^+ or K^- daughters must be

identified by a Cerenkov counter and a magnetic field and energy and direction of emission must be determined in order to fulfil the q^2 and ΔE limitations. The efficiency for K^+ and K^- observation must be identical and this identity might be further ensured by repeating the experiment with the magnetic field reversed.

It is natural to suggest that all experiments be carried out on several nuclei.

The yields of charge exchange reactions at high energy are not large and the restrictions imposed mean that the integral $d\sigma^*$ accepted is of the order of magnitude $0.005 \cdot N(A; \sigma, \sigma) \text{mb}$. Using Glauber theory and the optical theorem we estimate the total cross-section to be $\sigma_T \simeq \sigma \cdot N(A; 0, \frac{\sigma}{2})$ with σ the meson nucleon total cross-section. This means that even for a thick target (optimized to maximum yield) the relative yield of genuine events will be of the order of 10^{-5} per incoming meson. Although low this relative yield does not seem to be prohibitive even for the necessary extrapolation to very thin target (necessary to maintain the same detector efficiency for the two types of outgoing mesons).

DISCUSSION AND CONCLUSION

A method for the measurement of a possible difference in radius of the charge and mass distribution of nuclei has been suggested and three or the four possible experiments have been shown to be feasible experimentally. The method is very sensitive to just this difference with an amplification several times that obtained from direct comparison of nuclear radii deduced from elastic scattering experiments using electromagnetic and strong interactions respectively.

Our starting point, formula (6), is valid when the reactions (1)-(2) proceed predominantly in a one step process and consequently the limits on q^2 and ΔE are essential. The evaluation of Eq. (6) neglects both centre-of-mass correlation and other correlations in nuclei. However, both centre-of-mass correlations and the Pauli correlation can be incorporated in the computations if deemed necessary. The effects can be expected to be small. More detailed nuclear models and more detailed scattering theory should perhaps be considered but until experimental data become available such procedures would be premature. The main effect to be expected rests on simple arguments involving attenuation and symmetry only.

Our final conclusion is thus to urge that experiments of the type outlined be carried out utilizing the sensitivity obtainable and the specific ease of interpretation especially of the double ratio $(Y_5/Y_7)(Y_8/Y_6)$ for the kaon charge exchange reactions (2) in order to settle the important discussion point concerning a possible difference between nuclear charge and mass distributions.

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Incoming particle	Reaction	$d\sigma_{CH}$	σ_{iN}	σ_{iP}	σ_{fN}	σ_{fP}	$N(A; \sigma_1, \sigma_2)$	Yield *)
π^+	1 a $\rightarrow$	$d\sigma_{\pi}$	$\sigma_1 + \Delta$	σ_1	$\sigma_1 + \Delta/2$	$\sigma_1 + \Delta/2$	$N \pi^+$	$Y_1 = d\sigma_{\pi}^* \frac{N}{A} \pi^+ (1 + \frac{Z}{N} E_{\pi})$
π^0	1 a $\leftarrow$	$d\sigma_{\pi}$	$\sigma_1 + \Delta/2$	$\sigma_1 + \Delta/2$	$\sigma_1 + \Delta$	σ_1	$N \pi^+$	$Y_2 = d\sigma_{\pi}^* \frac{Z}{A} \pi^+ (1 - E_{\pi})$
π^-	1 b $\rightarrow$	$d\sigma_{\pi}$	σ_1	$\sigma_1 + \Delta$	$\sigma_1 + \Delta/2$	$\sigma_1 + \Delta/2$	$N \pi^-$	$Y_3 = d\sigma_{\pi}^* \frac{Z}{A} \pi^- (1 - E_{\pi})$
π^0	1 b $\leftarrow$	$d\sigma_{\pi}$	$\sigma_1 + \Delta/2$	$\sigma_1 + \Delta/2$	σ_1	$\sigma_1 + \Delta$	$N \pi^-$	$Y_4 = d\sigma_{\pi}^* \frac{N}{A} \pi^- (1 + \frac{Z}{N} E_{\pi})$
K^+	2 a $\rightarrow$	$d\sigma^+$	σ_{+0}	σ_{++}	σ_{++}	σ_{+0}	$N K^+$	$Y_5 = d\sigma_{+}^* \frac{N}{A} K^+ (1 + \frac{Z}{N} E_K)$
K^0	2 a $\leftarrow$	$d\sigma^+$	σ_{++}	σ_{+0}	σ_{+0}	σ_{++}	$N K^+$	$Y_6 = d\sigma_{+}^* \frac{Z}{A} K^+ (1 - E_K)$
K^-	2 b $\rightarrow$	$d\sigma^-$	σ_{-0}	σ_{-+}	σ_{-+}	σ_{-0}	$N K^-$	$Y_7 = d\sigma_{-}^* \frac{Z}{A} K^- (1 - E_K)$
$\bar{K}^0$	2 b $\leftarrow$	$d\sigma^-$	σ_{-+}	σ_{-0}	σ_{-0}	σ_{-+}	$N K^-$	$Y_8 = d\sigma_{-}^* \frac{N}{A} K^- (1 + \frac{Z}{N} E_K)$
) $d\sigma^$ stands for $d\sigma$ integrated over the q^2 range considered.								

Notation for yields and effective nucleon numbers together with cross-section identifications based on isospin invariance. The difference between the total cross-section for π^+p and π^-p is termed Δ .

FIGURE CAPTIONS

Figure 1 The figure shows $N(A; \sigma, \sigma)$ v. A for the Saxon-Woods distribution $\rho = c_{\text{NORM}} / (1 + \exp[(r-c)/a])$ with $a = 0.545$ fermi and $c = 1.14 A^{1/3}$ fermi together with $\sigma = 26$ mb for pions (index π) and $\sigma = 17$ mb for kaons (index K).

Figure 2 The figure shows $\alpha = a \frac{\partial}{\partial a} \log(N(A; \sigma, \sigma))$ and $\beta = c \frac{\partial}{\partial c} \log(N(A; \sigma, \sigma))$ v. A for the Saxon-Woods distribution $\rho = c_{\text{NORM}} / (1 + \exp[(r-c)/a])$ with $a = 0.545$ fermi and $c = 1.14 A^{1/3}$ fermi together with $\sigma = 26$ mb for pions (index π) and $\sigma = 17$ mb for kaons (index K).

FIG. 1

FIG. 2