ON SIXTH ORDER RADIATIVE CORRECTIONS TO THE MUON \( g \)-FACTOR

B.E. Lautrup *)

Brookhaven National Laboratory, Upton, New York

and

F. de Rafael
CERN - Geneva

ABSTRACT

The contribution to the anomalous magnetic moment of the muon from the Feynman diagrams shown in Fig. 1 is found to be

\[
\left( \frac{\alpha}{\pi} \right)^3 \left\{ \left( \frac{119}{27} - \frac{4}{9} \pi^2 \right) \frac{g}{\left( m_e^2 \right)} - \frac{61}{162} + \frac{1}{27} \pi^2 + O \left( \frac{m_e^2}{m_\mu^2} \right)^2 \right\}
\]

*) Address from 1 September 1969:
CERN, Theory Division, 1211 Geneva 23.
We have calculated the contribution to the anomalous magnetic moment of the muon, \( \frac{1}{2} (g_\mu - 2) \), from the Feynman diagrams shown in Fig. 1. If we denote by \( \mu_{\Pi e}^{(2)} \) the total contribution to \( \frac{1}{2} (g_\mu - 2) \) from these diagrams, we find that

\[
\mu_{\Pi e}^{(2)} = \left( \frac{\alpha}{\pi} \right)^3 \left\{ \left( \frac{119}{27} - \frac{4 \pi^2}{9} \right) \ln \left( \frac{m_e}{m_\mu} \right) - \frac{61}{162} + \frac{1}{27} \pi^2 + O\left( \frac{m_e^2}{m_\mu^2} \right) \right\}
\]

(1)

Numerically,

\[
\mu_{\Pi e}^{(2)} = 0.10 \left( \frac{\alpha}{\pi} \right)^3.
\]

(2)

The Feynman diagrams shown in Fig. 1 are the electron-positron vacuum polarization correction to the Feynman diagram shown in Fig. 2. The contribution to \( \frac{1}{2} (g_\mu - 2) \) from this diagram was called \( \mu_{\Pi e} \) by Karplus and Kroll who first calculated it to be \( 1),2) \)

\[
\mu_{\Pi e} = \left( \frac{\alpha}{\pi} \right)^2 \left( \frac{119}{36} - \frac{1}{3} \pi^2 \right).
\]

(3)

It can be seen from the expressions given in Eqs. (1) and (3) that the coefficient of \( \log \left( \frac{m_\mu}{m_e} \right) \) in Eq. (1) is precisely \( \left( \frac{\alpha}{\pi} \right)^3 \mu_{\Pi e} \). This agrees with a result already obtained by Kinoshita, Refs. 3), 4), using the techniques of the renormalization group.

To our knowledge, the term \( -\frac{61}{162} + \frac{1}{27} \pi^2 \) in Eq. (1) is the only constant term (i.e., which does not involve the ratio of electron to muon masses) of second order electron-positron vacuum polarization corrections to the fourth order contributions to \( \frac{1}{2} (g_\mu - 2) \) which so far has been calculated exactly \( 5) \). It has been suggested by Kinoshita \( 4) \) that contributions from these constant terms to \( \frac{1}{2} (g_\mu - 2) \) are probably small compared to the corresponding logarithm terms.
The result obtained in Eq. (1) supports this conjecture, the contribution from the logarithm term is $0.1115(\alpha/\pi)^{3}$ while the contribution from the constant term is $-0.0110(\alpha/\pi)^{3}$.

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REFERENCES


2) For a review of the fourth order contributions to the anomalous magnetic moments of the electron and of the muon, see:
   A. Petermann, Fortschr.Phys. 6, 505 (1956).


5) The constant terms of the fourth order electron-positron vacuum polarization corrections to the second order contribution to $\frac{1}{2}(g - 2)$ have been calculated by Lautrup and de Rafael: