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STATE BETHE-SALPETER WAVE FUNCTION

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A SIMPLE DERIVATION OF THE NORMALIZATION CONDITION FOR THE BOUND STATE
BETHE-SALPETER WAVE FUNCTION ^{*)}

We consider the renormalized inhomogeneous Bethe-Salpeter (B-S) equation for the two-body propagator ¹⁰⁾

$$\text{---} \bigcirc \text{---} = \text{---} + \text{---} \boxed{V} \bigcirc \text{---} = \text{---} + \bigcirc \boxed{V} \text{---} \quad (1)$$

$\boxed{V}$ is the sum of all irreducible graphs or "potential". If there is a bound state pole, then in momentum space :

$$\text{---} \bigcirc \text{---} = \text{---} \bigcirc \text{---} + \text{terms regular near the pole} \quad (2.a)$$

$$= \frac{i \bigcirc \bigcirc}{(2\pi)^4 (K^2 - M_B^2)} + \text{terms regular near the pole} \quad (2.b)$$

K is the total four momentum of the system, M_B is the bound state mass, $\bigcirc$ is referred to as the B-S wave function and $\bigcirc$ as its conjugate. Equation (2.b) represents the factorization of the residue into parts depending on the incoming and outgoing particles respectively ¹¹⁾. Substituting Eq. (2) into Eq. (1), and equating residues at the pole, gives the homogeneous B-S equation for the bound state ^{10), 11)} :

$$\bigcirc = \bigcirc \boxed{V} \quad (3.a)$$

$$\bigcirc = \boxed{V} \bigcirc \quad (3.b)$$

$(K^2 = M_B^2)$

*) This paper describes notation and results which are essential in the accompanying paper ¹⁾. The papers have been severed since the derivation of the normalization condition has a more general interest, although the result is not new ²⁾⁻⁹⁾.

2.

α and β are, of course, completely determined by solving Eq. (1) for all K^2 and explicitly finding the residues at any poles. The normalization is therefore determined by the inhomogeneous term in Eq. (1) and the factors associated with the bound state propagator near the pole in Eq. (2.b); if $\text{---}\bigcirc\text{---}$ is regarded as the sum of all renormalized Feynman graphs, then the factors chosen are the natural ones. In this case amplitudes constructed from α and β will be invariant renormalized Feynman amplitudes ^{*}).

The solution of Eq. (1) is a formidable task and it is usually more convenient to solve Eqs. (3) and use an explicit normalization condition. A bilinear equation is necessary to fix the normalization and we consider

$$\text{---}\bigcirc\text{---}\bigcirc\text{---} = \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}\boxed{V}\text{---}\bigcirc\text{---}, \quad (4)$$

which follows immediately from Eq. (1), and rewrite it thus :

$$\text{---}\bigcirc\text{---} \left[\left(\text{---}\bigcirc\text{---} \right)^{-1} - \boxed{V} \right] \text{---}\bigcirc\text{---} = \text{---}\bigcirc\text{---}, \quad (5)$$

where

$$\left(\text{---}\bigcirc\text{---} \right)^{-1} \left(\text{---}\bigcirc\text{---} \right) = \left(\text{---}\bigcirc\text{---} \right) \left(\text{---}\bigcirc\text{---} \right)^{-1} = 1. \quad (6)$$

Equating the residues at the pole in the variable K_0 in equation (5) and using Eqs. (3) in the form

$$\alpha \left[\left(\text{---}\bigcirc\text{---} \right)^{-1} - \boxed{V} \right] = \left[\left(\text{---}\bigcirc\text{---} \right)^{-1} - \boxed{V} \right] \beta = 0 \quad (K^2 = M_B^2) \quad (7)$$

^{*}) Mandelstam ³⁾ has $(K_0 - \sqrt{\vec{K}^2 + M_B^2})$ in place of $(K^2 - M_B^2)$ in Eq. (2.b). Wave functions with his normalization lead to non-invariant matrix elements.

we obtain

$$\frac{i}{(2\pi)^4 2K_0} \left\langle \left(\frac{\partial}{\partial K_0} \left[(E)^{-1} - [V] \right] \right) \right\rangle = 1 \quad (K^2 = M_B^2), \quad (8)$$

or, in covariant form

$$\frac{i}{(2\pi)^4} \left\langle \left(\frac{\partial}{\partial K_\mu} \left[(E)^{-1} - [V] \right] \right) \right\rangle = 2K_\mu \quad (K^2 = M_B^2), \quad (9)$$

which is the normalization condition.

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R E F E R E N C E S

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