CM-P00058270

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Ref.TH.1047-CERN

REGGE FITS WITH VENEZIANO RESIDUES

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A B S T R A C T

We explore how the Regge limit of the Veneziano formula works experimentally. Absorptive cuts are essential. The resulting fit to πN , KN and $\bar{K}N$ has fewer parameters than any previous Regge fit. Though not perfect, we argue that discrepancies come from the lowest partial waves where absorptive models are not quantitatively reliable. This leads to a scheme for Reggeized phase shift analysis.

Ref.TH.1047-CERN

25 June 1969

1. - INTRODUCTION

The starting point of this work was the following property of the Regge limit of the Veneziano formula ¹⁾: consider the most general (s,t) Veneziano formula with satellites ²⁾

$$\sum_{N=N_0}^{\infty} \sum_{M=M_0}^{\infty} \sum_{\substack{R \leq N+M \\ R \geq N,M}} (-1)^{M+N-R} \beta_{R,N,M} \frac{\Gamma(N-\alpha_s) \Gamma(M-\alpha_t)}{\Gamma(R-\alpha_s-\alpha_t)} \quad (1.1)$$

and take its limit as $s \rightarrow \infty$, $\arg \alpha_s \neq 0$, to get

$$(-1)^{M_0} \cdot \Gamma(M_0 - \alpha_t) \cdot \gamma_t(\alpha_t) \cdot [-\alpha_s]^{\alpha_t} \quad (1.2)$$

where

$$\gamma_t(\alpha_t) = \sum_{M=M_0}^{\infty} \frac{(\alpha_t - M_0)!}{(\alpha_t - M)!} \cdot \sum_{N \geq N_0, M} \beta_{N,N,M}^{N,M} \quad (1.3)$$

will be a polynomial if the number of satellite terms is finite. The (s,u) Veneziano term will also contribute in the Regge limit, the two combining to give the usual signature factor if $\alpha_s = \alpha^0 + \alpha'$ s. We must therefore compare (1.2) with the standard Regge formula

$$\frac{\beta(t)}{\sin \pi \alpha_t} \cdot (-s/s_0)^{\alpha_t}, \quad (1.4)$$

showing that $s_0 = (\alpha')^{-1}$ and

$$\begin{aligned} \beta(t) &= \pi (-1)^{M_0} \cdot \gamma_t(\alpha_t) / \Gamma(\alpha_t + 1 - M_0) \\ &= 0 \quad \text{at} \quad \alpha_t = M_0 - 1, M_0 - 2, \dots \end{aligned} \quad (1.5)$$

Thus the Regge residue predicted by the Veneziano formula always chooses nonsense. Finkelstein ³⁾ has pointed out that this follows from exchange degeneracy, which in turn is necessary to forbid exotic resonances in any theory with duality.

In particular, any $\pi\pi$ Veneziano formula predicts that $\beta_p(t) = 0$ when $\alpha_p(t) = 0, -2, \text{ etc.}$ By factorization, the same must be true for πN scattering in both spin states. Thus, at first sight ⁴⁾, it is a prediction of the Veneziano formula that the πp charge-exchange cross-section should vanish at the dip, $t = -0.55 \text{ GeV}^2$. This is in contradiction with experiment.

The answer was known before Veneziano. Arnold and Blackmon ⁵⁾ and Chiu and Finkelstein ⁶⁾ showed that a Regge formula which chooses nonsense in both spin states can be reconciled with experiment, provided Regge cuts are included by the absorptive model. This also has various other advantages such as predicting charge-exchange polarization. Hitherto absorptive Regge models have been considered as an alternative to models with secondary Regge trajectories. However, if we believe in exchange degeneracy and a fortiori in any Veneziano formula, there is no alternative - the existing data already force us to the absorptive model. (Though the Veneziano formula also contains secondary trajectories, they are much lower than those needed in phenomenological fits without cuts, and also choose nonsense.) As we shall see, the cuts have to be of opposite sign to the generating pole if Veneziano residues are to work. This corresponds to the absorptive sign of Finkelstein and Jacob ⁷⁾. In contrast, the multiperipheral bootstrap of Chew et al. ⁸⁾ predicts the rescattering of AFS sign ⁷⁾ for the cut. At least in its present form, the multiperipheral bootstrap is therefore in contradiction to the Veneziano formula rather than complementary ⁹⁾ to it. Hopefully, more elaborate theoretical treatments of Regge cuts may explain the absorptive sign ¹⁰⁾.

The present fits were inspired by the work of Arnold and his collaborators ^{5), 11)}. We go further in the following respects :

- (1) Because we use the Regge limit of Veneziano, all nonsense zeros are present, whereas Arnold et al. only included that at $\alpha_p(t) = 0$.

- (2) We fit over a larger range of t and s .
- (3) We have even fewer parameters, since trajectories and scale parameter are both fixed a priori.
- (4) We relate the remaining parameters in the residues to low energy resonances.
- (5) We show that the experimental discrepancies arise from the lowest partial waves, where it is unreasonable to expect any simple absorptive model to be quantitatively correct. This leads to a scheme for Reggeized phase shift analysis.

I do not claim that this is the best of all Regge fits, but it is certainly the one with fewest parameters.

2. - VENEZIANO FORMULA FOR $\bar{K}N$

Here we use Veneziano formulae to parametrize Regge residues in accord with exchange-degeneracy (EXD) and duality. Veneziano is used heuristically to derive results which are almost certainly more general, rather than as a genuine model of baryons.

In the present paper, we only consider πN , KN and $\bar{K}N$ scattering at fixed t . Duality then implies that the t channel trajectories are the EXD set $\rho - f - \omega - A_2$, since the φ decouples from nucleons¹²⁾. To obtain the residues, it is sufficient to write down Veneziano formulae for $\bar{K}N$. This system is particularly convenient since there are no resonances in the u channel, which leads immediately to simple EXD requirements on all baryon trajectories with appreciable $\bar{K}N$ coupling¹³⁾. For πN in contrast, the way in which the baryon trajectories have to enter the Veneziano formula is less obvious.

A previous attempt to write down $\bar{K}N$ Veneziano formulae was made by Igi and Storow¹⁴⁾. This paper seems to me wrong. Besides the missing terms tacked on in an erratum and other local errors, there is a pervasive neglect of parity doublets in their equations for resonance widths [see the remarks after Eq. (2.13) below]. As a result the

Y^* widths generated by their Veneziano formulae are nothing like those they claim to have fitted, either in the original or revised version ¹⁵⁾. The present work was contemporaneous with that of Igi and Storrow, and known to them when they wrote their paper (see their Ref. ¹⁰⁾). I delayed its publication mainly to check the reason for the numerous discrepancies with their results. There is also a paper by Inami ¹⁶⁾ whose formulae are at least incomplete.

The most general Veneziano formulae for the A and B amplitudes are

$$A^I(\lambda, t) = \sum_{N=j_0-\frac{1}{2}}^{\infty} \sum_{M=1}^{\infty} \sum_{\substack{R \leq N+M \\ R \geq N, M}}^{\infty} (-1)^{M+N-R} \times \\ \times \delta_{R,I}^{N,M} \cdot \frac{\Gamma(N + \frac{1}{2} - \alpha_{\lambda}^I) \Gamma(M - \alpha_t)}{\Gamma(R + \frac{1}{2} - \alpha_{\lambda} - \alpha_t)}, \quad (2.1)$$

$$B^I(\lambda, t) = \sum_{N=j_0-\frac{1}{2}}^{\infty} \sum_{M=0}^{\infty} \sum_{\substack{R \leq N+M \\ R \geq N, M}}^{\infty} (-1)^{M+N-R} \times \\ \times \varepsilon_{R,I}^{N,M} \cdot \frac{\Gamma(N + \frac{1}{2} - \alpha_{\lambda}^I) \Gamma(M + 1 - \alpha_t)}{\Gamma(R + \frac{3}{2} - \alpha_{\lambda} - \alpha_t)}. \quad (2.2)$$

Here α_t is the EXD $\rho - f - \omega - A_2$ trajectory. For the $\bar{K}N$ s channel isospin 0 state, α_s^0 is the EXD $\Lambda_{\alpha} - \Lambda_{\gamma}$ trajectory and $j_0 = \frac{1}{2}$ because the $j = \frac{1}{2}$ particle is present at the beginning. For the isospin 1 state, α_s^1 is the EXD $\Sigma_{\delta} - \Sigma_{\beta}$ trajectory and $j_0 = \frac{3}{2}$. Following Schmid ¹³⁾ we neglect other Y^* trajectories because they are weakly coupled or at daughter levels. The highest powers of $\cos \vartheta_s$ in the pole term at $\alpha_s^I = j$ are

$$A^I \approx \left(\frac{1}{j - \alpha_\lambda} \right) \cdot \frac{\alpha_\lambda^I(j)}{(j - j_0)!} \cdot [\alpha_t]^{j - \frac{1}{2}}, \quad (2.3)$$

$$B^I \approx \left(\frac{1}{j - \alpha_\lambda} \right) \cdot \frac{b_\lambda^I(j)}{(j - j_0)!} \cdot [\alpha_t]^{j - \frac{1}{2}}, \quad (2.4)$$

where

$$\alpha_\lambda^I(\alpha_\lambda^I) = \sum_{N=j_0-\frac{1}{2}}^{\infty} \frac{(\alpha_\lambda^I - j_0)!}{(\alpha_\lambda^I - \frac{1}{2} - N)!} \cdot \sum_{M \geq 1, N}^{\infty} \delta_{M, I}^{N, M}, \quad (2.5)$$

$$b_\lambda^I(\alpha_\lambda^I) = \sum_{N=j_0-\frac{1}{2}}^{\infty} \frac{(\alpha_\lambda^I - j_0)!}{(\alpha_\lambda^I - \frac{1}{2} - N)!} \cdot \sum_{M \geq 0, N}^{\infty} \epsilon_{M, I}^{N, M}. \quad (2.6)$$

As in any relativistic baryon theory, parity doublets will occur. If we define

$$d_\lambda^I(W) = \alpha_\lambda^I(\alpha_\lambda^I) + (W - M) b_\lambda^I(\alpha_\lambda^I) \quad (2.7)$$

where $W = s^{\frac{1}{2}}$ and M is the nucleon mass, then by standard formulae for meson-baryon partial wave analysis¹⁷⁾

$$f_1^I = \frac{E + M}{8\pi W} [A^I + (W - M) B^I], \quad (2.8)$$

$$f_2^I = \frac{E - M}{8\pi W} [-A^I + (W + M) B^I], \quad (2.9)$$

$$f_{\ell \pm}^I = \frac{1}{2} \int_{-1}^1 d \cos \vartheta \{ P_{\ell}(\cos \vartheta) f_1 + P_{\ell \pm 1}(\cos \vartheta) f_2 \} \quad (2.10)$$

and we get the elastic width of the particle on the leading trajectory with $j = \ell \pm \frac{1}{2}$ and mass W as

$$\Gamma_{j \pm}^I = \pm \frac{(j - \frac{1}{2})! \sqrt{\pi} [\alpha' k^2]^j (E \pm M) d_{\lambda}^I(\pm W)}{(j - j_0)! \Gamma(j+1) 8\pi (\alpha')^{3/2} \lambda} \quad (2.11)$$

(note that $\pm$ is not the parity here). Here k is the centre-of-mass momentum and $E = (M^2 + k^2)^{\frac{1}{2}}$. For a finite number of satellites, we see from (2.5)-(2.7) that $d_S^I(W)$ will be a polynomial in W . As first pointed out by Virasoro¹⁸⁾, duality suggests that the reduced width $d_S^I(W)$ be at least smoothly varying.

Inverting (2.7),

$$a_{\lambda}^I(\alpha_{\lambda}^I) = \frac{1}{2W} [(W+M) d_{\lambda}^I(W) + (W-M) d_{\lambda}^I(-W)], \quad (2.12)$$

$$b_{\lambda}^I(\alpha_{\lambda}^I) = \frac{1}{2W} [d_{\lambda}^I(W) - d_{\lambda}^I(-W)]. \quad (2.13)$$

We see that, if we were to omit either half of (2.13), a pole would result at $W = 0$. This is precisely what Igi and Storrow did¹⁴⁾, on the excuse that parity doublets are not observed experimentally. Then they tried to determine the parameters of their model by averaging $-d_S^0(-W)/W$ along the Λ trajectory and $d_S^1(W)/W$ along the Y_1^* (see their Table I). However, parity doublets are certainly present in their model (except for the lowest), and it is necessary to use both the A and B amplitudes as in (2.8)-(2.10) to disentangle them. In fact so far from fitting the observed hyperon widths, they end up with Veneziano formulae that predict Y_1^* widths 3-6 times larger than experiment. In the original version, their Y_1^* widths are actually negative.

The Λ coupling constant is

$$g_{\Lambda}^2 = - \frac{\alpha_s^0(-m_{\Lambda})}{16\pi \alpha' m_{\Lambda}} \quad (2.14)$$

Here we have used the definition of Queen et al. ¹⁹⁾. To get that of Kim ²⁰⁾ multiply by m_{Λ}/M . Experimental values of (2.14) range from 3.9 ± 0.8 ²¹⁾ to 13.5 ± 2.1 ²⁰⁾.

The leading power of $\cos \vartheta_t$ at the t channel pole $\alpha_t = j$ is given by

$$A^I \approx \left(\frac{1}{j - \alpha_t} \right) \cdot \frac{a_t^I(j)}{(j-1)!} \cdot [\alpha_s]^j, \quad (2.15)$$

$$B^I \approx \left(\frac{1}{j - \alpha_t} \right) \cdot \frac{b_t^I(j)}{(j-1)!} \cdot [\alpha_s]^{j-1}, \quad (2.16)$$

where

$$a_t^I(\alpha_t) = \sum_{M=1}^{\infty} \frac{(\alpha_t - 1)!}{(\alpha_t - M)!} \cdot \sum_{N \geq M, j_0 - \frac{1}{2}}^{\infty} \delta_{N,I}^{N,M}, \quad (2.17)$$

$$b_t^I(\alpha_t) = \sum_{M=0}^{\infty} \frac{(\alpha_t - 1)!}{(\alpha_t - 1 - M)!} \cdot \sum_{N \geq M, j_0 - \frac{1}{2}}^{\infty} \varepsilon_{N,I}^{N,M}. \quad (2.18)$$

These reduced widths are also very convenient for the Regge limits. For forward $\bar{K}N$ scattering at large s and fixed t , we get

$$A^I(s, t) \approx - \Gamma(1 - \alpha_t) \cdot [\alpha_s e^{-i\pi}]^{\alpha_t} \cdot a_t^I(\alpha_t), \quad (2.19)$$

$$B^I(s, t) \approx \Gamma(1-\alpha_t) \cdot [\alpha_s e^{-i\pi}]^{\alpha_t-1} \cdot b_t^I(\alpha_t) \quad (2.20)$$

while for backward KN at large u and fixed s

$$f_{1,2}^I(\sqrt{s}, u) \approx \frac{M \pm E}{8\pi W} \cdot \Gamma(j_0^I - \alpha_s^I) \cdot (-1)^{j_0^I - \frac{1}{2}} \times \\ \times [\alpha' u]^{\alpha_s - \frac{1}{2}} \cdot d_s^I(\pm \sqrt{s}), \quad (2.21)$$

which can then be converted into u channel helicity and isospin amplitudes by the usual crossing relations.

We fix the t channel trajectories at

$$\alpha_t = 0.482 + 0.9 t, \quad (2.22)$$

where the intercept is chosen to give the soft pion constraint ²²⁾. The s channel trajectories must have the same slope ²³⁾, so we take

$$\alpha_s^0 = -0.62 + 0.9 s, \quad (2.23)$$

$$\alpha_s^1 = -0.22 + 0.9 s. \quad (2.24)$$

Now we construct the minimal Veneziano formulae required for sensible widths on the leading s channel trajectories. Equation (2.11) shows that $d_s^I(W)$ must be a polynomial of odd order in W to avoid ghosts (resonances of negative width). Experimentally the S_{01} and P_{03} particles are missing from the Λ trajectory, while the D_{05} is weakly coupled to the $\bar{K}N$ channel ²⁴⁾. For our subsequent high energy fits, we need a Λ coupling constant $\frac{2}{3}$ of Kim's ²⁰⁾, thus getting

$$d_s^0(W) = 25.71 (W - 1.1156) \times \\ \times (W - 1.5348) \cdot (W - 1.8619). \quad (2.25)$$

The predicted $\bar{K}N$ widths of the D_{03} and F_{05} particles are still about twice too large (see Table I). However this is probably because the singlet and octet Λ_8 trajectories have to be degenerate in a Veneziano model ²⁵⁾, so that our $\Lambda(1535)$ actually corresponds to a sum of $\Lambda(1519)$ and $\Lambda(1690)$. Barger's fit to K^+p backward scattering ²⁶⁾ implies

$$\alpha_s^0(0) = \pm 86.7 \quad (2.26)$$

in good agreement with

$$\alpha_s^0(0) = -82.0 \quad (2.27)$$

from (2.25).

A Veneziano formula for the $\bar{K}N$ $I = 0$ state which implements these resonance widths is

$$A^0 = -\gamma_a^0 \frac{\Gamma(\frac{3}{2} - \alpha_s) \Gamma(1 - \alpha_t)}{\Gamma(\frac{3}{2} - \alpha_s - \alpha_t)} + \gamma_b^0 \frac{\Gamma(\frac{1}{2} - \alpha_s) \Gamma(1 - \alpha_t)}{\Gamma(\frac{3}{2} - \alpha_s - \alpha_t)} \quad (2.28)$$

$$B^0 = \beta_a^0 \frac{\Gamma(\frac{1}{2} - \alpha_s) \Gamma(1 - \alpha_t)}{\Gamma(\frac{3}{2} - \alpha_s - \alpha_t)} - \beta_b^0 \frac{\Gamma(\frac{3}{2} - \alpha_s) \Gamma(2 - \alpha_t)}{\Gamma(\frac{5}{2} - \alpha_s - \alpha_t)} \quad (2.29)$$

where

$$\gamma_a^0 = -102.16, \quad \gamma_b^0 = -35.98, \quad (2.30)$$

$$\beta_a^0 = 202.90, \quad \beta_b^0 = 28.62,$$

all in GeV units. The second term in (2.29) is required to make $d_s^0(W)$ cubic, and thus avoid ghosts at the top of the parity doubled trajectory [see Eq. (2.11)].

For the Y_1^* trajectory, there is no D_{13} state, while the coupling constant of the P_{13} is given by

$$\frac{H^2}{4\pi} = \frac{M^2 \cdot d_s(m_{Y_1})}{4\pi m_{Y_1}} \quad (2.31)$$

= 2.4 in $SU(3)$, according to Warnock and Frye²⁷⁾. As in the Λ case, we again take $\frac{2}{3}$ of the $SU(3)$ value to get

$$d_s^1(w) = 11.4 (w + 1.3822). \quad (2.32)$$

The $\bar{K}N$ widths of the D_{15} and F_{15} particles are then 55 and 0.4 MeV, to be compared with the experimental 46 and 6 MeV²⁴⁾.

We implement these widths, and also cancel the S_{11} and P_{11} daughters of $Y_1^*(1385)$, with the Veneziano formulae

$$A^1 = -\gamma_a^1 \frac{\Gamma(\frac{3}{2} - \alpha_s) \Gamma(1 - \alpha_t)}{\Gamma(\frac{3}{2} - \alpha_s - \alpha_t)} + \gamma_b^1 \frac{\Gamma(\frac{3}{2} - \alpha_s) \Gamma(1 - \alpha_t)}{\Gamma(\frac{5}{2} - \alpha_s - \alpha_t)}, \quad (2.33)$$

$$B^1 = \beta_a^1 \frac{\Gamma(\frac{3}{2} - \alpha_s) \Gamma(1 - \alpha_t)}{\Gamma(\frac{5}{2} - \alpha_s - \alpha_t)} - \beta_b^1 \frac{\Gamma(\frac{3}{2} - \alpha_s) \Gamma(2 - \alpha_t)}{\Gamma(\frac{5}{2} - \alpha_s - \alpha_t)}, \quad (2.34)$$

where

$$\begin{aligned} \gamma_a^1 &= 26.50, & \gamma_b^1 &= 17.28 \\ \beta_a^1 &= -65.82, & \beta_b^1 &= 11.42 \end{aligned} \quad (2.35)$$

all in GeV units. Only the first term of (2.33) and the second of (2.34) contribute to the leading s channel trajectory.

Tables I and II show the complete predictions of these Veneziano formulae for parent and daughter widths in the $\bar{K}N$ channel. Though the parents are reasonable, the daughters include many ghosts, so that still more satellites are needed. In contrast to the $\pi\pi$ case ²⁸⁾, baryon Veneziano formulae are therefore not to be taken too seriously. They are useful for parametrizing Regge residues and fulfilling FESR, but judged as models of baryon they are hopelessly uneconomic when compared to $SU(6)$ ℓ excitation ²⁹⁾.

Taking the Regge limit of these Veneziano formulae, we get the following parametrization of meson-exchange Regge terms in $\bar{K}N$

$$A^I(s, t) \approx -\gamma_a^I \cdot \Gamma(1-\alpha_t) \cdot [\alpha_s e^{-i\pi}]^{\alpha_t}, \quad (2.36)$$

$$B^I(s, t) \approx [\beta_a^I + \beta_b^I (\alpha_t - 1)] \cdot \Gamma(1-\alpha_t) \times \\ \times [\alpha_s e^{-i\pi}]^{\alpha_t}. \quad (2.37)$$

The isospin indices are still for s channel, the (s,t) crossing matrix being

$\begin{array}{c c} & I_t \\ \hline I_s & \end{array}$	0	1
0	$\frac{1}{2}$	$\frac{3}{2}$
1	$\frac{1}{2}$	$-\frac{1}{2}$

(2.38)

The $N\bar{N}$ reduced residues of the ρ , f, etc., as defined by (2.15) and (2.16), are thus predicted to be constant for the A amplitude (not A') and linear for B. That B must contain a higher power of t than A follows from the fact that $d_s^I(W)$ must be an odd order polynomial, which in turn is needed to avoid hyperon ghosts.

3. - ABSORPTIVE REGGE MODELS

For comparison with experiment, the Veneziano formula has to be unitarized. In elastic $\pi\pi$ scattering below 1 GeV, K matrix unitarization gives very good results ²⁸⁾. Essentially this means that we project the Veneziano formula into partial waves $R_\ell(s)$, and then take the partial wave T matrix to be

$$T_\ell(s) = R_\ell(s) / [1 - ik R_\ell(s)], \quad (3.1)$$

The justification was that the Veneziano formula has poles on the real axis, which is permitted for the K matrix. If we now take $R_\ell(s)$ to be the Regge limit of the Veneziano formula, instead of the Veneziano formula itself, then we get one form of the absorptive Regge model. (The use of such models to interpolate between high and low energies was recently discussed by Cohen-Tannoudji et al. ³⁰⁾.) We postpone till Section 5 the difficulty that the Veneziano formula only converges to its Regge limit in the complex plane, and not on the positive real axis.

Expanding (3.1) to second order

$$T_\ell(s) \approx R_\ell(s) + ik [R_\ell(s)]^2. \quad (3.2)$$

When summed over partial waves, the second term generates a Regge cut ³¹⁾. This has the experimentally correct absorptive sign ⁷⁾, because we interpreted $R_\ell(s)$ as a complex K matrix, obtained by analytic continuation from the real K matrix at low energies. If instead we were to take the modulus in the second term $ik |R_\ell(s)|^2$, the AFS or rescattering sign would have resulted, which is known to be wrong experimentally ⁷⁾. Other possible absorptive models, for example,

$$T_\ell(s) = [e^{2ik R_\ell(s)} - 1] / (2ik), \quad (3.3)$$

agree with (3.2) to second order. Equation (3.3) corresponds to Arnold's eikonal model ^{32),5),11)}, except that he performs the partial wave projection approximately by means of the Bessel function limit of the Legendre polynomials. Though (3.3) is a possible unitarization of the asymptotic Regge formula, it could not be used for the Veneziano expression because of its poles.

So far we have not considered the Pomeron. Cohen-Tannoudji et al. ^{30),33)} have suggested that it multiplies the Regge terms in the S matrix, instead of adding to them in the T matrix, i.e.,

$$S_\ell(s) = S_\ell^P(s) \cdot S_\ell^R(s), \quad (3.4)$$

where

$$S_\ell^x(s) = 1 + 2ik \cdot T_\ell^x(s), \quad (3.5)$$

giving for the T matrix

$$T_\ell(s) = T_\ell^P(s) + T_\ell^R(s) + 2ik \cdot T_\ell^P(s) \cdot T_\ell^R(s). \quad (3.6)$$

Just as in (3.2), the second term of (3.6) generates a Regge cut when the partial wave series is summed. If we were to interpret $R_\ell(s)$ in Eqs. (3.1) or (3.3) as the sum of the Pomeron and the true Regge terms, then we would again get the same result to second order.

In the present work the Pomeron was taken as a fixed pole at $J = 1$, with related cuts generated by unitarization. We suppose the original pole to be coupled only to the B amplitude and not to A. Because

$$A'(s, t) = A(s, t) + \omega B(s, t) / [1 - t/4M^2], \quad (3.7)$$

where

$$\omega = (s-u)/4M \quad (3.8)$$

this corresponds to the $A' \approx \omega B$ ansatz supported by FESR and by experiment for the Pomeron ³⁴⁾. Equations (2.8)-(2.10) show that it also implies no parity splitting in meson-baryon scattering asymptotically, as is physically reasonable. For the Pomeron residue, I found the most economical parametrization which gives a decent fit to the data, to be a generalized dipole formula of the type used by Arnold et al. ^{5), 11)}

$$B(s, t) = i / [(a + bt)^2 + ct]^2 \quad (3.9)$$

If conjectures ³⁵⁾ relating the Pomeron residue to products of electromagnetic form factors are true, then the number of parameters could be reduced still further by using the recently suggested Veneziano expressions for form factors ³⁶⁾

$$t \Gamma(1 - \alpha_t) / \Gamma(\frac{3}{2} + N - \alpha_t), \quad (3.10)$$

where $N = 2$ for the nucleon but could well be different for pion or kaon. However, I did not learn of (3.10) till after the present fits were complete.

If the Pomeron is pure imaginary before unitarization, then (3.1) and (3.3) show that the T matrix will also be pure imaginary after unitarization. So, at least asymptotically, the unitarization of the Pomeron could be considered merely as a reparametrization. The really essential cut is that between the Pomeron and the Regge terms. This is responsible for the partial filling in of the charge-exchange dip, cross-over effects, etc.

In K^+p scattering at low energies, the Veneziano formula predicts a large repulsive S_1 and at higher energies a similar P_1 with other waves much smaller, in qualitative agreement with experiment ³⁷⁾.

This makes it plausible that the similar repulsion in the $\pi N S_{31}$ state is also due to the (t,u) Veneziano term. It combines with the $\pi N S_{31}$ resonance by multiplying it in the S matrix ³⁸⁾. The resonance and background do not add in the T matrix nor in the K matrix (which would force the phase to ascend through 90° or descend through 0°). The resonance, by duality, must belong to the $(s,t) + (s,u)$ Veneziano terms. There is therefore some reason for investigating more complicated absorptive models, in which the direct and crossed Regge terms are unitarized separately. This might be associated with the fixed poles in the J plane due to the (t,u) Veneziano term ³⁹⁾.

In the present work a whole class of absorptive models were tried out. Let us introduce the following notations : P represents the Pomeron contribution, D the direct Regge terms arising from the limit of (s,t) Veneziano, C the crossed Regge terms arising from the limit of (u,t) Veneziano, $+$ means that two contributions are added in the T matrix, $\times$ that they are multiplied in the S matrix after projection into partial waves as in (3.4), a bar on top of a set of terms means that they are to be interpreted as the K matrix and unitarized as in (3.1), and "exp" means unitarization as in (3.3). Some typical prescriptions for absorptive models are then

$$\begin{aligned} \bar{P} \times (D+C), & \quad \overline{P + D + C}, & \exp(P+D+C), & \quad (\overline{P+C}) \times D, \\ \bar{P} \times \bar{D} \times \bar{C}, & \quad \bar{P} \times D \times C, & \quad \bar{P} \times D \times \bar{C}. \end{aligned}$$

All of these were experimented with, at one stage or another.

To see the physical significance of this variety, consider the analogy of pp scattering at low energies. It is well known that one-boson exchange models explain the small high partial waves ⁴⁰⁾. When unitarized by various possible prescriptions - K matrix, N/D equations, equivalent potentials - they also explain the moderately large medium waves. However, no simple model can account for the pp S waves. We can expect the same to be true for absorptive Regge models, and as we shall see later there is experimental indication that this is so.

The Regge contribution is a Born term, as in the work of Arnold ³²⁾ where it corresponds to a complex optical potential. This Born term is only a good approximation for the small high partial waves. The various absorptive models are different prescriptions for improving the Born term so that it can be applied to moderately large waves. They will agree in second order, and where a partial wave is so large that they give appreciably different results, I see no a priori reason to believe any of them. If one absorptive model works better than another, this is a piece of phenomenological information, analogous to the discovery in low-energy pp scattering that K matrix unitarization is better than equivalent potential ⁴⁰⁾. When a partial wave projected from the Born term is really large, it is unreasonable to expect any simple unitarization prescription to work quantitatively. However, as we shall discuss later, this does not mean that the absorptive model must fail for resonant waves. Because of duality ⁴¹⁾, the Regge exchange term itself actually contains resonances, and for the high waves these have such small coupling to the elastic channel as to make the absorptive corrections negligible.

4. - COMPARISON WITH πN , KN AND $\bar{K}N$ EXPERIMENTS

Because of the small number of adjustable parameters, I found it sufficient to fit by eye. For the Pomeron I got (in GeV units)

$$B_{\pi N} = 64 i / [(1 + .7t)^2 + .3t]^2, \quad (4.1)$$

$$B_{KN} = 53 i / [(1 + .32t)^2 + t]^2, \quad (4.2)$$

before unitarization. The magnitude definitely has to be different in πN and KN , corresponding to asymptotic cross-sections of 21 and 17 mb., respectively. This is not surprising if the Pomeron represents effects of coupled channels. The shape of its residue also seems to be different,

though this is less certain because of the inadequate high energy KN data. Its trajectory was fixed at 1 for all t values, though KN would probably fit better with a small shrinkage.

For the $KN-\bar{K}N$ Regge terms, we have six parameters as defined by (2.36) and (2.37). However, it is nice to assume

$$\beta_b^0 = \beta_b^1 \quad (4.3)$$

since this makes the Y^* widths asymptotically independent of the isospin. Also it is known from previous Regge fits ^{5),34)} that f exchange satisfies $A' \approx \sqrt{2} B$ like the Pomeron, and so also decouples from the A amplitude [see (3.7)]. I thus assumed

$$\gamma_a^0 = -3\gamma_a^1 \quad (4.4)$$

which by (2.38) means that the A amplitude is pure $I = 1$ in the t channel. It is interesting that this automatically gives the signs and relative magnitudes needed in the Veneziano formula to cancel the P_{03} particle on the Λ trajectory, but the D_{13} on the Y_1^* [see (2.30) and (2.35)]. My high energy KN and $\bar{K}N$ fits then gave (GeV units)

$$\begin{aligned} \gamma_a^0 &= -92.4, & \beta_a^0 &= 214.6, \\ \gamma_a^1 &= 30.8, & \beta_a^1 &= -13.4, \\ \beta_b^0 &= \beta_b^1 & &= 18.8. \end{aligned} \quad (4.5)$$

These are in reasonable agreement with the low energy values (2.30) and (2.35), except for β_a^1 which is pure satellite in the s channel. For πN , I took the f and ρ Regge terms to be 1.3 times those in KN (with appropriate signature factor changes of course). However, the evidence for $SU(3)$ breaking in the Regge terms is much weaker

than for the Pomeron - it comes essentially from the rather poor $\bar{K}N$ charge-exchange data and is sensitive to the absorptive corrections. In fact Jackson ⁴²⁾ without absorptive corrections found no evidence for $SU(3)$ breaking. Also comparison of the K^* and ρ widths suggests breaking in the other direction.

In general the predictions were surprisingly insensitive to the particular absorptive model, provided only that the second order Pomeron-Regge term is the same. Even the wide angle scattering is not much affected. The various absorption models seem to differ only where they all disagree with experiment, especially for the shape of the charge-exchange dip and the position of the K^-p/K^+p cross-over. In 2-5 GeV/c experiments the latter can be seen to occur at $t \approx -0.2 \text{ GeV}^2$. Although the $K^\pm p$ experiments are not sufficiently accurate to trace it to higher energies, the same cross-over occurs in $\bar{p}p/pp$ for similar exchanged trajectories, showing that it is a genuinely asymptotic feature. By exchange degeneracy ¹¹⁾, the Regge pole terms in K^+p are pure real and those in K^-p all have the same phase

$$- e^{-i\pi\alpha_t} \quad (4.6)$$

Thus constructive interference with the pure imaginary Pomeron makes K^-p larger in the forward direction. In the absence of Regge cuts, the cross-over would occur at $t = -0.55$, where $\alpha_t = 0$ and the K^-p Regge terms also become pure real. The shift of the cross-over for various absorption models (see Section 4 for notation) is

$$\begin{array}{ll} \bar{P} \times D \times C & -0.54 \\ \overline{P + D + C} & -0.46 \\ \exp(P+D+C) & -0.42 \end{array} \quad (4.7)$$

I found no simple absorptive model able to move it far enough. The exponential (eikonal) model will be used in the subsequent graphs, because it comes nearest.

On the other hand I found no difficulty in giving the π^-p/π^+p cross-over its correct position. This is because, as the total cross-section differences illustrate, the A' Regge pole contribution is relatively much smaller in this case, and therefore more easily cancelled by the cut.

The partial wave projection was done by Gaussian integration. I tested its accuracy by changing the numbers of mesh-points and also with soluble examples. I found 60 l values and 64 mesh-points adequate even at the highest energies. It takes 2.5 seconds on a CDC 6600 to project the resulting 242 waves from the Regge formula, unitarize them, and resum to get observables as functions of t . The huge literature on impact parameter approximations and similar "short cuts" does not seem to me worth while. It is easier to be exact.

The resulting model has been compared with many high energy πN , KN and $\bar{K}N$ experiments. Typical results are shown in Figs. 1-4. These have not been selected to flatter the model, but to illustrate interesting points.

I found no evidence for the large deviation from EXD trajectories, which have been claimed in certain previous Regge fits. The energy dependence is always satisfactory. I also agree with Jackson ⁴²⁾ and disagree with Höhler et al. ⁴³⁾ that the soft pion intercept ²²⁾ $\alpha_p(0) = 0.482$ is compatible with the total and forward cross-sections. The πp elastic differential cross-sections are in quantitative agreement for $t \gtrsim -0.7 \text{ GeV}^2$, and rough agreement even out to $t = -2.5$ from 5 GeV/c up. The πp charge-exchange ⁴⁴⁾ at a typical energy is illustrated in Fig. 1. The agreement is good at small angles, but the dip is the wrong shape and the secondary peak too large. These discrepancies occur at all energies, and are therefore due to the residue (perhaps the scale factor) or the absorptive corrections, but not to the trajectory. The charge-exchange polarization has the right sign at small angles, but is somewhat too small. This is sensitive to the absorptive model, however. Like Arnold and Blackmon ⁵⁾, I found that all my absorptive models gave a sharp negative spike in the dip.

Figure 2 shows the K^+p DCS at 5 GeV/c ⁴⁵⁾. The prediction is lower than the data. This reflects the disagreement in the position of the K^-p/K^+p cross-over. Some shrinkage of the Pomeron would probably help here, since it would produce interference with the Regge terms. In the present model the K^+p Regge terms are pure real and the Pomeron pure imaginary, so Regge effects die out rather rapidly. At 3 GeV/c, there is considerably more wide angle K^+p scattering than predicted. This is so even when the full Veneziano formula is used instead of its Regge limit.

Figure 3 shows the K^-p DCS at 5 GeV/c ⁴⁶⁾. Here the agreement is very good even out to $t = -4 \text{ GeV}^2$. The linear term in the B residue (2.37) contributes significantly to this. By (4.6) the Regge terms will all become negative imaginary where $\alpha_t = -\frac{1}{2}$ ($t \sim -1.1 \text{ GeV}^2$), causing destructive interference with the positive imaginary Pomeron. The corresponding dip is clearly seen in the data. The polarization also changes sign near here for the same reason, as has been observed at lower energies ⁴⁷⁾. However, the no-compensation mechanism ⁴⁸⁾ would give an extra zero in the main Regge residues at $t = -0.55$, thus converting this dip into a secondary peak. This seems to me a strong argument against this mechanism. The structure of the K^-p DCS is still qualitatively correct down to 2 GeV/c, but its absolute magnitude is too small at wide angles at the lower energies.

$\bar{K}N$ charge-exchange is correct, except for being somewhat too big at wide angles, like the πp . KN charge-exchange is almost identical to $\bar{K}N$, as is correct experimentally at 5 GeV/c but not below ⁴⁹⁾.

The solid line in Figure 4 shows the predictions for π^+p polarization at 5.9 GeV/c, compared with experiments at 6.0 ⁵⁰⁾ and 5.15 GeV/c ⁵¹⁾. The reflection symmetry between π^+p and π^-p is due as usual to the decoupling of f from the A amplitude. The fact that each polarization keeps the same sign at wide angles constitutes the main evidence for the so-called no-compensation mechanism ⁴⁸⁾. We see that this feature is predicted by the present model without any violation of

exchange-degenerate residues. There seems to me no reason left to believe this mechanism, which would be decidedly unpleasant for duality.

However, though the solid line in Fig. 4 has the right signs, it is obviously not quantitatively correct. Now at 5 GeV/c there is so much data, in addition to FESR constraints, that it ought to determine the spin decomposition. It seems plausible that, because of their huge number of adjustable parameters, conventional Regge fits may give a correct account of the amplitudes in a limited energy region where the experiments are good, even though their dip mechanisms are wrong and they have neglected cuts. I therefore analysed a recent fit of Barger and Phillips⁵²⁾ into partial waves and compared with my model, to see what was responsible for the remaining disagreement in Fig. 4. The dotted line shows the result of a composite model, in which the partial waves up to H ($\ell \leq 5$) have been taken from Barger and Phillips, and the higher ones from the absorptive Veneziano model. This combines pieces of two incompatible fits, and so cannot be optimal. Nevertheless it shows clearly that the disagreement between the present model and the polarization data arises from the lowest partial waves. As we argued in the last Section, this is where any simple absorptive model is expected to break down.

5. - CONCLUSIONS

According to the Veneziano formula, the number of waves containing resonances increases like $\ell \sim s$. However, from the usual impact parameter argument, the number of waves inside the diffraction peak increases only like $\ell \sim s^{\frac{1}{2}}$. So at high energies there will be more and more resonances outside the diffraction region. These have exponentially decreasing coupling to the elastic channel, as has been verified in the Veneziano formula⁵³⁾. For these "peripheral resonances", absorptive corrections will not be important. Failure to realize that resonances can occur outside the diffraction region seems

to be the root of the wide-spread fallacy (apparently still held in some quarters ⁵⁴⁾) that Regge trajectories cannot increase faster than $\alpha(s) \sim s^{\frac{1}{2}}$. Indeed, according to duality ⁴¹⁾, these peripheral resonances are not only outside the diffraction region, they are actually contained in the Born term, which is the crossed channel Regge exchange amplitude. The equivalence of the s and t channel binomial expansions of the Veneziano formula gives a clear illustration of how a Born term (sum of t channel poles) can contain resonances (s channel poles).

In the example of πp scattering at 5 GeV/c, which we discussed in connection with Fig. 4, at the end of the last Section, resonances should occur in all waves up to $\ell = 9$, diffraction up to $\ell = 7$, while the absorptive model with Veneziano residues seems to be good for $\ell > 5$. In usual phase shift analyses it is necessary to include at least one ℓ value beyond the highest resonance. Thus, by taking higher waves from Regge formulae, the number of parameters in a phase shift analysis at 5 GeV/c could probably be cut by half. This should also greatly reduce the number of alternative solutions, which is the main difficulty in high energy phase shift analysis. This scheme for Reggeized phase shift analysis, which I consider the most important conclusion of the present work, is closely analogous to what is done in pp scattering at low energies ⁴⁰⁾. The Regge exchange terms here correspond to the one-boson-exchange Born terms there, and the absorptive corrections are analogous to the unitarization used to extend downwards the domain in ℓ for which Born terms are valid. The central waves in both cases have to be deduced phenomenologically, and are left as a subject for future dynamical theories.

The scheme can in principle be experimentally tested : in processes where duality diagrams predict ⁵⁵⁾ pure real amplitudes and therefore no polarization, absorptive models predict only that the polarization ^{*)} should have little structure, since it comes entirely from the lowest partial waves.

*) More precisely, it is (polarization $\times$ DCS), which should have little structure. The negative spike in the πp charge-exchange polarization predicted in the dip region, is due to the nearly vanishing DCS.

In this scheme the Regge pole terms will be essentially fixed in, having been almost completely determined from exchange-degeneracy, t channel meson masses, and s channel FESR, as in the present paper. Instead the experiments are to be used to provide information on Regge cuts. This seems to me more rational than the traditional Regge phenomenology, where cuts are ignored and parameters piled into the pole residues until they could fit anything. As we have seen, even the trajectories deduced from old-fashioned Regge fits are suspect. They may well be determined by such model-dependent features as shapes of dips, instead of the experimental energy dependence.

It is clearly Regge poles rather than Regge cuts which are the fundamental objects. A theory in which the poles take the simple and elegant form suggested by duality diagrams, therefore also seems to me more aesthetic than one in which cuts vanish at the expense of vast complications in the pole residues. The present fits show that, if reasonable account is taken of cuts, there is no experimental evidence against exchange-degenerate meson trajectories and residues in πN , KN or $\bar{K}N$.

It might be objected that the scheme for Reggeized phase shift analysis is model-dependent, since when we cut off the expansions (2.17), (2.18) for the reduced residues after one or two terms, we used special properties of the Veneziano formula with a limited number of satellites, rather than just exchange-degeneracy. However, these equations show that higher terms will not contribute to the first poles in the t channel. By the usual Q function arguments, the highest partial waves should be determined by the nearest singularities in t . So, by using the Veneziano formula only to determine the highest partial waves, we become less sensitive to satellite ambiguities.

The Harari-Freund hypothesis ⁵⁶⁾ that the Pomeron is independent of the resonances, requires some modification in the absorptive model. Because the Pomeron multiplies the resonances in the S matrix, as in (3.4), it will make each more inelastic, and therefore contribute to them in the T matrix. The Harari-Freund hypothesis would only be strictly valid for the phase.

We end with a difficulty. So far we have only considered the Regge limit of the Veneziano formula. However, this limit is not valid if α_s is pure real, while making α_t complex would give ancestors. What happens if we repeat our calculations with the Veneziano formula itself instead of its Regge limit? Severe difficulties arise :

(a) Because we have unitarized, the poles will no longer be quite on the real axis. However, apart from a slight damping from the Pomeron, the resonances will still be elastic, and therefore give huge structure in the total cross-section at high energies. The obvious way to eliminate this is to make α_s complex from the first inelastic threshold. However :

(b) Equation (1.2) shows that if α_s is complex, the signature factor for t channel Regge exchange becomes

$$\exp[(\arg \alpha_s / s - i\pi) \alpha_t] \pm 1 \quad (5.1)$$

instead of the usual

$$e^{-i\pi \alpha_t} \pm 1. \quad (5.2)$$

This will move dips far away from the nonsense points, which would be unfortunate experimentally. In fact it turns out that dips are upset long before $\text{Im} \alpha_s$ gets big enough to wipe out the total cross-section structure.

(c) Viewed from the s channel, the absence of a K^-p backward peak is due to cancellation between even and odd parity resonances. Veneziano with a complex trajectory will indeed produce some backward decrease if the imaginary part is big enough to make the resonances overlap. However, the cancellation has to be so precise as to bring the backward cross-section down by at least six orders of magnitude. This is both difficult to achieve and easy to upset by unitarization.

Thus it seems to me that the complex trajectory approach to the Regge limit is inadequate - some stronger averaging procedure like a Cesaro limit seems to be required.

I am grateful to Professor R.C. Arnold, Professor V. Barger, Dr. G. Cohen-Tannoudji, Professor M. Jacob, Dr. C. Michael, Dr. A. Morel, Dr. P. Salin and Dr. C. Schmid for discussions.

- Table I -

γ_0^* WIDTHS (MeV) PREDICTED BY Eqs. (2.28 - (2.30)

[illegible]

- Table II -

Y_1^* WIDTHS (MeV) PREDICTED BY Eqs. (2.33)-(2.35)

MASS	1382	1738	2033	2290	2521	2732	2929	3113
S ₁₁	0	11.7	9.2	1.5	6.6	2.1	5.6	2.6
P ₁₁	0	-87	20	- 5.0	3.8	1.8	2.5	3.5
P ₁₃	(-13.6)	-72	-115	- 46	- 60	- 36	-43	-31
D ₁₃	0	-26	-140	15	- 39	1.3	-19	- 1.3
D ₁₅		54.9	- 96	-108	- 58	- 70	-44	-52
F ₁₅		0.4	- 53	-149	1.4	- 65	- 2.9	-38
F ₁₇			88	- 91	- 79	- 63	-67	-49
G ₁₇			1.9	- 63	-136	- 12	-80	- 8.5
G ₁₉				96	- 77	- 46	-63	-55
H ₁₉				3.7	- 61	-115	-22	-85
H _{1,11}					91	- 62	-17	-59
I _{1,11}					5.1	- 54	-92	-28
I _{1,13}						80	-48	3.3
J _{1,13}						5.9	-45	-71
J _{1,15}							67	-36
K _{1,15}							6.0	-37
K _{1,17}								54
L _{1,17}								5.8

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FIGURE CAPTIONS

- Figure_1 Absorptive Veneziano model compared with π^+p charge-exchange at 10 GeV/c ⁴⁴⁾.
- Figure_2 Absorptive Veneziano model compared with K^+p at 5 GeV/c ⁴⁵⁾.
- Figure_3 Absorptive Veneziano model compared with K^-p at 5-6 GeV/c ⁴⁶⁾.
- Figure_4 π^+p polarization at 5-6 GeV/c ^{50),51)} compared with absorptive Veneziano model (solid line), and with changed lower waves (dotted line).

FIG.1

FIG. 2

FIG. 4