CM-P00058260

Archives.

Ref.TH.1037-CERN

SMOOTHING VENEZIANO

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A B S T R A C T

An averaging procedure is applied to the Veneziano formula. The resulting amplitude has poles on the second sheet and no more poles in the physical region. Crossing symmetry and positivity are preserved. Ways of enforcing unitarity are discussed.

In this note I would like to suggest, to whom it may concern, a possible way of healing the most obvious diseases of the fashionable Veneziano formula ¹⁾, without destroying its nicest features, crossing symmetry and Regge behaviour in both channels with rising trajectories. What one would like, first of all, is to remove poles in the physical region and send them in the second sheet, not only because resonances have a width, but also because one wants to have true Regge behaviour at high energies, and not only in slightly complex directions.

The procedure I propose is directly inspired from my contacts with field theorists ²⁾. In their eyes the Veneziano amplitude is a distribution. A standard procedure to make it smooth is to make a convolution of this amplitude with a smooth function. This can be done in hundreds of ways, and what I want to present here is just one of them.

We want to preserve crossing symmetry. We want also to preserve positivity, if it exists.

In the case of one-term Veneziano formula, Wagner ³⁾, and Sivers and Yellin ⁴⁾, have made it extremely likely that positivity is satisfied for a certain range of the parameters of the linear trajectories. We shall provisionally take positivity for granted in our input function. To establish that the smooth function has positivity, we shall use the following theorem :

let $f(z) = \sum (2n+1)c_n P_n(z)$, with $c_n \geq 0$, then $f(\lambda z)$ with $\lambda > 1$ has also positive coefficients.

Proof :

$$f(\lambda z) = f(z) + \sum_1^{\infty} \frac{(\lambda-1)^n z^n}{n!} \left(\frac{d}{dz}\right)^n f(z)$$

$(d/dz)^n P_e(z)$ is a sum with positive coefficients of Legendre polynomials, and $z^n P_q(z)$ is also a sum of Legendre polynomials with positive coefficients. This is sufficient to get the desired result.

Now let $V(s,t,u)$ be the initial Veneziano term, for instance

it becomes

$$C_n(\sigma, \tau) = \int_{x_m}^1 \frac{\phi(x) \rho_n(x\tau)}{n - \alpha_0 - Bx\sigma} dx$$

where the expression is first defined on the physical sheet, i.e., $\text{Im } \sigma > 0$. First of all notice that $C_n(\sigma, \tau)$ is a polynomial in τ of degree n . The pole at $n = \alpha_0 + B\sigma$ has been cleaned away. We would like to keep it, however, on the second sheet. This can be done by taking $\phi(x)$ to be the restriction of a meromorphic function of x for instance

$$\phi(x) = \frac{(1-x)^{\uparrow} (x-x_m)^{\uparrow}}{(x-x_0)^2 + \Gamma^2}$$

then a typical contribution to $C_n(\sigma, \tau)$ will be

$$\psi_{r,n} = \int_{x_m}^1 \frac{\tau^r (1-x)^{\uparrow} (x-x_m)^{\uparrow} x^n}{[(x-x_0)^2 + \Gamma^2][n - \alpha_0 - Bx\sigma]} dx$$

This expression has a cut starting at $\sigma = (n - \alpha_0)/B$ and can be continued to the second sheet and has a pole on the second sheet given by

$$\sigma = \frac{(n - \alpha_0)(x_0 - i\Gamma)}{B(x_0^2 + \Gamma^2)}$$

We have therefore a new pole trajectory where

$$\bar{\alpha}(\sigma) = \alpha_0 + B(x_0 + i\Gamma)\sigma$$

which is the kind of thing one likes.

Again, $\bar{\alpha}(\sigma) = n$ corresponds to poles in n angular momentum states as before. One could worry about the trajectory not being real below threshold. However, this does not affect the amplitude which is

manifestly real if $n - \alpha(\sigma)$ does not vanish for $\sigma < 8/3$. A more serious trouble is that we get cuts $\alpha(\sigma) = n$, $\alpha(x_m \sigma) = n$ and we have therefore infinitely many cuts. However, these cuts can be weakened by taking p and q large and, anyway, we begin to become accustomed to cuts.

Now what about Regge behaviour? We know that in the initial model we have it for $|s| \rightarrow \infty$, $\text{Arg } s \neq 0$. We still have $\bar{W} \sim s^{\alpha(t)}$ for $t = 4/3$. For $t \neq 4/3$, we are averaging on different t 's and different s 's so that we shall not get exactly Regge. However, the leading term for large s [since $\alpha(t)$ is increasing] will remain

$$\psi(s) s^{\alpha[\chi_m(t - 4/3) + 4/3]}$$

where $\psi(s)$ is a weakly varying function (less than a power). The "effective" value of α will be slightly lower. This small disadvantage is compensated by the fact that we will get a smooth behaviour for s real and large.

Finally let us say a few things about unitarity :

- i) one can restrict parameters entering in $\phi(x)$ in such a way that at all energies the partial wave projections are bounded by one;
- ii) one could then vary these parameters within the allowed range to optimize the elastic unitarity condition for an important partial wave (for instance the p wave in pion-pion scattering). To solve completely the elastic unitarity problem, one should either consider more general averaging procedures or try to start from the smoothed Veneziano term and insert it as an input in the Atkinson procedure ⁵⁾, following the direction recently proposed by Kupsch ⁶⁾, for the case of non-polynomially bounded functions.

R E F E R E N C E S

- 1) G. Veneziano, Nuovo Cimento 57, 190 (1968);
C. Lovelace, CERN preprint TH.950 (1968).
- 2) I am grateful to my friends H. Epstein and V. Glaser for making me familiar with these things.
- 3) F. Wagner, CERN preprint TH.978 (1968).
- 4) D. Sivers and J. Yellin, UCRL-18784 report (1969).
- 5) D. Atkinson, Nuclear Phys. B7, 375 (1968) and B8, 377 (1968).
- 6) J. Kupsch, private communication, to be submitted to Nuovo Cimento.