CM-P00058251

Archives

Ref.TH.1028-CERN

ON THE TWO BARYON SECTOR OF SOLUBLE MODELS

WITH INDEFINITE METRIC. I

O. Nachtmann
CERN - Geneva

A B S T R A C T

Systems containing two unphysical particles with complex energies are studied. It is shown that states which lie on the real axes, if one has no interaction between the two unphysical particles, go to the complex plane in presence of interaction.

Ref.TH.1028-CERN

9 May 1969

1. - INTRODUCTION

Recently, Lee and Wick ¹⁾⁻³⁾ proposed to cure divergence difficulties in quantum field theory by introducing an indefinite metric into the Hilbert space. All states on the real axes are assumed to have positive norm. This guarantees the unitarity of the S matrix between physical states. Besides these states, there exist unphysical ones with complex energy eigenvalues. Since there are states above as well as below the real axes one may wonder if there exist states containing two unphysical particles with energy again on the real axes. Consider for instance the soluble model ⁴⁾ with indefinite metric as discussed in Ref. ¹⁾. The energy spectrum is shown in Fig. 1.

Besides the physical states of type $N\bar{\psi}$ on the real axes there are two unphysical states $V_{\pm}$ with complex energies $E_{\pm}$.

Suppose we have a second identical system V', N' very far away from the first, so that there is no interaction between them and we can simply add the energies. The energy spectrum in the one-baryon sector is still given by Fig. 1, but now each eigenvalue is degenerate. In the two baryon sector, we have the spectrum shown in Fig. 2.

Besides the states of type $NN'\bar{\psi}\bar{\psi}$ on the real axes from 2μ to infinity there exist also two unphysical states for real $E = E_+ + E_-$

$$\frac{1}{\sqrt{2}} \{ |V_+\rangle |V'_-\rangle - |V_-\rangle |V'_+\rangle \} \quad (1.1)$$

$$\frac{1}{\sqrt{2}} \{ |V_+\rangle |V'_-\rangle + |V_-\rangle |V'_+\rangle \} \quad (1.2)$$

It is easy to see that (1.1) has negative norm. However, this does not lead to a contradiction to unitarity as long as there is no interaction between the two systems NV and $N'V'$.

The purpose of our work is to investigate the behaviour of the states (1.1) and (1.2) when the two systems approach and interact with each other. It turns out that in the model we will study below, the state (1.1) with negative norm gives two unphysical states on the first sheet of the energy plane, whereas the state (1.2) with positive norm goes to the second sheet. This is of some interest since in our case the part of the Hamiltonian describing the interaction between the VN and $V'N'$ systems will be Hermitian, so that from first order perturbation theory one would conclude that (1.1) and (1.2) remain on the real axes.

So we find that our model has no unphysical states on the real axes. The formal proof of unitarity for physical states runs along the same lines as in Ref. ³⁾ and will not be repeated here.

2. - DEFINITION OF THE MODEL

Let us start by considering a soluble model of the type invented by Lee ⁴⁾, in one dimension but with two sources, a distance L apart (see Fig. 3). V and V' will have negative metric.

The Hamiltonian of this model is

$$\begin{aligned}
 H = & m (V^\dagger V + V'^\dagger V') \\
 & + \int dk \, \omega(k) a^\dagger(k) a(k) \\
 & + g \int dk \, f(k) [e^{ikL/2} V^\dagger N a(k) - h.c.] \\
 & + g \int dk \, f(k) [e^{-ikL/2} V'^\dagger N' a(k) - h.c.]
 \end{aligned}
 \tag{2.1}$$

$\omega(k) = \sqrt{k^2 + \mu^2}$; $f(k) = f(-k)$ real ; $m > \mu$ so that V and V' are unstable.

The commutation relations and metric η are

$$[a(k), a^+(k')] = \delta(k-k') \quad (2.2)$$

$$[N, N^+] = [N', N'^+] = [V, V^+] = [V', V'^+] = 1 \quad (2.3)$$

$$\eta = (-1)^{V^+V + V'^+V'} \quad (2.4)$$

Now it is convenient to introduce new operators

$$a_+(k) = \sqrt{\frac{\omega}{2|k|}} (a(k) + a(-k)) \quad (2.5)$$

$$a_-(k) = i\sqrt{\frac{\omega}{2|k|}} (a(k) - a(-k)) \quad (2.6)$$

$$a_{\pm}(\omega) = a_{\pm}(k) \quad k > 0 \quad (2.7)$$

$$\rho_{1,2}(\omega) = 2\sqrt{\frac{\omega}{|k|}} f(k) \begin{cases} \sin |k| L/2 \\ \cos |k| L/2 \end{cases} \quad (2.8)$$

The Hamiltonian expressed in these new operators is

$$\begin{aligned} H = & m (V^+V + V'^+V') \\ & + \int d\omega \omega (a_+^+(\omega) a_+(\omega) + a_-^+(\omega) a_-(\omega)) \\ & + g/\sqrt{2} \int d\omega \{ [\rho_2(\omega) a_+(\omega) + \rho_1(\omega) a_-(\omega)] V^+ N - h.c \} \\ & + g/\sqrt{2} \int d\omega \{ [\rho_2(\omega) a_+(\omega) - \rho_1(\omega) a_-(\omega)] V'^+ N' - h.c \} \end{aligned} \quad (2.9)$$

From (2.8) we see that $\rho_{1,2}(\omega)$ are not strictly positive. It turns out that this leads for special values of L to bound states, corresponding to standing waves between the two sources. These states which are peculiar of one dimension become mixed up with the states we want to study, thus making the situation rather unclear. Therefore we will consider in the following the Hamiltonian (2.9) as our starting point with arbitrary but strictly positive functions $\rho_{1,2}(\omega)$ *).

For $\rho_1(\omega) = \rho_2(\omega)$, the Hamiltonian (2.9) is the sum of two terms corresponding to two non-interacting $V \leftrightarrow N$ systems. (V, N are only coupled to $a_+ + a_-$, $V'N'$ only to $a_+ - a_-$.) This is the case discussed in Section 1. We will study small departures from the symmetrical case. To be specific, we will set

$$\begin{aligned}\rho_1^2(\omega) &= \rho^2(\omega) \\ \rho_2^2(\omega) &= \rho^2(\omega) (1 + \alpha)\end{aligned}\tag{2.10}$$

and consider $\alpha \rightarrow 0$.

The conserved quantities of (2.9) are

$$B = V^\dagger V + N^\dagger N \tag{2.11}$$

$$B' = V'^\dagger V' + N'^\dagger N' \tag{2.12}$$

$$Q = N^\dagger N + N'^\dagger N' - \int d\omega \{a_+^\dagger(\omega) a_+(\omega) + a_-^\dagger(\omega) a_-(\omega)\} \tag{2.13}$$

$$\begin{aligned}P : \quad & N \leftrightarrow N' \quad V \leftrightarrow V' \\ & a_+ \leftrightarrow a_+ \quad a_- \leftrightarrow -a_-\end{aligned}\tag{2.14}$$

*) We may, however, remark that for all but the above-mentioned special values of L the result concerning the absence of unphysical states on the real energy axes remains true also in the original model.

Since the sector with one baryon presents nothing new compared to Ref. ¹⁾, we will immediately pass to the two baryon sector.

3. - SOLUTION IN THE SECTOR $B = B' = 1, Q = 0, P = -1$

This sector of the total Hilbert space is spanned by the following basis vectors

$$\begin{aligned} & a_+^+(\omega) \frac{1}{\sqrt{2}} \{ |NV'\rangle - |N'V\rangle \} \\ & a_-^+(\omega) \frac{1}{\sqrt{2}} \{ |NV'\rangle + |N'V\rangle \} \\ & a_+^+(\omega) a_-^+(\omega') |NN'\rangle \end{aligned} \quad (3.1)$$

We can write an expansion for the eigenvector $|E\rangle$:

$$\begin{aligned} |E\rangle = & \int d\omega u_1(\omega) a_+^+(\omega) \frac{1}{\sqrt{2}} \{ |NV'\rangle - |N'V\rangle \} \\ & + \int d\omega' u_2(\omega') a_-^+(\omega') \frac{1}{\sqrt{2}} \{ |NV'\rangle + |N'V\rangle \} \\ & + \int d\omega d\omega' v(\omega, \omega') a_+^+(\omega) a_-^+(\omega') |NN'\rangle \end{aligned} \quad (3.2)$$

The eigenvalue equation

$$(H - E) |E\rangle = 0 \quad (3.3)$$

gives the following integral equations

$$\begin{aligned} u_1(\omega) (m + \omega - E) - g \int d\omega' p_1(\omega') v(\omega, \omega') &= 0 \\ u_2(\omega') (m + \omega' - E) + g \int d\omega p_2(\omega) v(\omega, \omega') &= 0 \\ v(\omega, \omega') (\omega + \omega' - E) + g \{ u_1(\omega) p_1(\omega') - u_2(\omega') p_2(\omega) \} &= 0 \end{aligned} \quad (3.4)$$

Let us look for normalizable states. Then we have :

$$v(\omega, \omega') = - \frac{g}{\omega + \omega' - E} \{ u_1(\omega) p_1(\omega') - u_2(\omega') p_2(\omega) \} \quad (3.5)$$

$$\begin{aligned} & \{ m + \omega - E + g^2 \int d\omega' \frac{p_1^2(\omega')}{\omega' + \omega - E} \} u_1(\omega) \\ & - g^2 p_2(\omega) \int d\omega' \frac{p_1(\omega') u_2(\omega')}{\omega' + \omega - E} = 0 \\ & \{ m + \omega - E + g^2 \int d\omega' \frac{p_2^2(\omega')}{\omega' + \omega - E} \} u_2(\omega) \\ & - g^2 p_1(\omega) \int d\omega' \frac{p_2(\omega') u_1(\omega')}{\omega' + \omega - E} = 0 \end{aligned} \quad (3.6)$$

Define

$$h_{1,2}(z) = z - m - g^2 \int d\omega \frac{p_{1,2}^2(\omega)}{\omega - z} \quad (3.7)$$

$h_{1,2}(z)$ has a cut $\mu \leq z < \infty$ and exactly two zeros in the complex plane (recall that $m > \mu$) (Fig. 4).

$$\begin{aligned} h_i(E_{\pm}^i) &= 0 & i &= 1, 2 \\ (E_+^i)^* &= E_-^i & \text{Im } E_+^i &> 0 \end{aligned} \quad (3.8)$$

Introducing $\tilde{u}_i(\omega)$ by

$$\begin{aligned} u_1(\omega) &= p_2(\omega) \tilde{u}_1(\omega) \\ u_2(\omega) &= p_1(\omega) \tilde{u}_2(\omega) \end{aligned} \quad (3.9)$$

we can rewrite (3.6) as

$$\begin{aligned} \tilde{u}_1(\omega) h_1(E - \omega) + g^2 \int d\omega' \frac{p_1^2(\omega') \tilde{u}_2(\omega')}{\omega' + \omega - E} &= 0 \\ \tilde{u}_2(\omega) h_2(E - \omega) + g^2 \int d\omega' \frac{p_2^2(\omega') \tilde{u}_1(\omega')}{\omega' + \omega - E} &= 0 \end{aligned} \quad (3.10)$$

To discuss the solutions of (3.6), respectively (3.10), we distinguish three cases.

A - E real $E > 2\mu$

In this case we have from (3.4) the condition

$$\tilde{u}_1(\omega) - \tilde{u}_2(\omega') = \theta \quad (3.11)$$

for $\omega + \omega' = E$. Also instead of Eq. (3.10) we have

$$\tilde{u}_i(\omega) h_i^P(E-\omega) + g^2 \int d\omega' \frac{\rho_i^2(\omega') \tilde{u}_j(\omega')}{(\omega + \omega' - E)_P} = \theta \quad (3.12)$$

$i, j = 1, 2 \quad i \neq j$

where P means principal value.

Define the following functions :

$$\tilde{\tilde{u}}_i(z) = - \frac{g^2}{h_i(E-z)} \int d\omega' \frac{\rho_i^2(\omega') \tilde{u}_j(\omega')}{\omega' + z - E} \quad (3.13)$$

$\tilde{\tilde{u}}_i(z)$ are analytic except for a cut $-\infty < z \leq E - \mu$ and poles at $z = E - E_{\pm}^1$. For $z = \omega > E - \mu$ $\tilde{\tilde{u}}_i(\omega) = \tilde{u}_i(\omega)$. But we have also

$$\tilde{\tilde{u}}_i(\omega \pm i\varepsilon) = \tilde{u}_i(\omega) \quad (3.14)$$

for $\mu < \omega < E - \mu$, since :

$$\begin{aligned} \tilde{\tilde{u}}_i(\omega \pm i\varepsilon) h_i(E - \omega \mp i\varepsilon) &= -g^2 \int d\omega' \frac{\rho_i^2(\omega') \tilde{u}_j(\omega')}{(\omega + \omega' - E)_P} \\ &\quad \mp i\pi \rho_i^2(E - \omega) \tilde{u}_j(E - \omega) \end{aligned} \quad (3.15)$$

Using (3.11) and (3.12) gives

$$\tilde{\tilde{u}}_i(\omega \pm i\varepsilon) h_i(E - \omega \mp i\varepsilon) = \tilde{u}_i(\omega) h_i(E - \omega \mp i\varepsilon) \quad (3.16)$$

from which follows (3.14). Therefore, by the reflection principle, $\tilde{u}_i(\omega)$ is analytic also for $\mu < \omega < E - \mu$ and then by (3.11) can have no cut at all on the real axes. There remain the poles which, again by (3.11), must satisfy

$$E = E_+^1 + E_-^2 = E_+^2 + E_-^1 \quad (3.17)$$

in order to give a solution of (3.12). Equation (3.7) is certainly fulfilled for $\rho_1 = \rho_2$, when this solution is just the state (1.1). For $\rho_1 \neq \rho_2$, (3.17) could be met accidentally. We will exclude such special values of ρ_1, ρ_2 from our considerations.

B - E real E < 2μ

To study this case, we write Eq. (3.6) in matrix form :

$$\int d\omega' K(\omega, \omega') \begin{pmatrix} u_1(\omega') \\ u_2(\omega') \end{pmatrix} = 0 \quad (3.18)$$

$$K(\omega, \omega') = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} (m + \omega - E) \delta(\omega - \omega') + g^2 R(\omega, \omega') \quad (3.19)$$

$$R(\omega, \omega') = \begin{pmatrix} \int d\omega'' \frac{\rho_1^2(\omega'')}{\omega'' + \omega - E} \delta(\omega - \omega') & - \frac{\rho_2(\omega) \rho_1(\omega')}{\omega + \omega' - E} \\ - \frac{\rho_1(\omega) \rho_2(\omega')}{\omega + \omega' - E} & \int d\omega'' \frac{\rho_2^2(\omega'')}{\omega'' + \omega - E} \delta(\omega - \omega') \end{pmatrix} \quad (3.20)$$

We will show by a method similar to ³⁾ that $K(\omega, \omega')$ is a positive definite operator. Therefore (3.18) has only the trivial solution $u_1 = u_2 = 0$ for these values of E .

The first term in Eq. (3.19) is obviously positive definite. If we can show that $R \geq 0$, then $K > 0$. Thus we must show :

$$\mathcal{Y} = \int d\omega d\omega' (u_1^*(\omega), u_2^*(\omega)) R(\omega, \omega') \begin{pmatrix} u_1(\omega') \\ u_2(\omega') \end{pmatrix} \geq 0 \quad (3.21)$$

$$\begin{aligned} \mathcal{Y} = & \int d\omega d\omega' \frac{|p_1(\omega')|^2 |u_1(\omega)|^2}{\omega + \omega' - E} \\ & + \int d\omega d\omega' \frac{|p_2(\omega')|^2 |u_2(\omega)|^2}{\omega + \omega' - E} \\ & - \int d\omega d\omega' \frac{p_2(\omega) u_1^*(\omega) p_1(\omega') u_2(\omega')}{\omega + \omega' - E} \\ & - \int d\omega d\omega' \frac{p_1(\omega) u_2^*(\omega) p_2(\omega') u_1(\omega')}{\omega + \omega' - E} \end{aligned} \quad (3.22)$$

Now we use

$$\frac{1}{\omega + \omega' - E} = \int_0^\infty d\lambda e^{-\lambda(\omega + \omega' - E)} \quad (3.23)$$

and introduce a positive definite scalar product :

$$(a, b)_\lambda = \int d\omega e^{-\lambda(\omega - E/2)} a^*(\omega) b(\omega) \quad (3.24)$$

$$\|a\|_\lambda = \sqrt{(a, a)_\lambda} \quad (3.25)$$

Then $\mathcal{Y}$ becomes :

$$\begin{aligned} \mathcal{Y} = & \int_0^\infty d\lambda \left\{ \|u_1\|_\lambda^2 \|p_1\|_\lambda^2 + \|u_2\|_\lambda^2 \|p_2\|_\lambda^2 \right. \\ & \left. - (u_1, p_2)_\lambda (p_1, u_2)_\lambda - (u_2, p_1)_\lambda (p_2, u_1)_\lambda \right\} \end{aligned} \quad (3.26)$$

An application of the Cauchy-Schwartz inequality gives finally :

$$\mathcal{Y} \geq \int_0^\infty d\lambda \left\{ \|u_1\|_\lambda \|p_1\|_\lambda - \|u_2\|_\lambda \|p_2\|_\lambda \right\}^2 \geq 0 \quad (3.27)$$

C - E complex

The solution in this case is analogous to the N $\mathcal{D}\mathcal{D}$ sector of the usual Lee model. By the same arguments as presented in Refs. 3), 5) we find that the following expression (3.28) is a solution of (3.10)

$$\tilde{u}_i(z) = (-1)^j \int_{C_j} \frac{d\xi}{2\pi i} \frac{1}{h_j(\xi) h_i(E-\xi) (\xi-z)} \quad (3.28)$$

$$i, j = 1, 2 \quad i \neq j$$

$[C_j]$ encloses only the singularities of $h_j(\xi)$ (see Fig. 5), provided

$$U(E) = \int_{C_1} \frac{d\xi}{2\pi i} \frac{1}{h_1(\xi) h_2(E-\xi)} = \int_{C_2} \frac{d\xi}{2\pi i} \frac{1}{h_2(\xi) h_1(E-\xi)} = 0 \quad (3.29)$$

The singularities of $U(E)$ are easily found from (3.29); they are (see Fig. 6) :

$$\begin{aligned} \text{poles at} \quad & E = E_{\pm}^1 + E_{\pm}^2 \\ & E = E_{\pm}^1 + E_{\mp}^2 \\ \text{cuts at} \quad & E_{\pm}^1 + \mu < E < \infty \\ & E_{\pm}^2 + \mu < E < \infty \\ & 2\mu < E < \infty \end{aligned}$$

Now it turns out to be more convenient to study, instead of $U(E)$, the function

$$W(E) = (E - E_+^1 - E_-^2)(E - E_-^1 - E_+^2) U(E) \quad (3.30)$$

$W(E)$ has the same singularities as $U(E)$ except for the two poles at $E_{\pm}^1 + E_{\pm}^2$ and also the same zeros if $E_{+}^1 + E_{-}^2 \neq E_{-}^1 + E_{+}^2$. If $E_{+}^1 + E_{-}^2 = E_{-}^1 + E_{+}^2$, $W(E)$ has a zero at that point, corresponding to the state (3.17). In the following we will set

$$\begin{aligned} \rho_1^2(\omega) &= \rho^2(\omega) \\ \rho_2^2(\omega) &= \rho^2(\omega) (1 + \alpha) \end{aligned} \quad (3.31)$$

$$\begin{aligned} E_{\pm}^1 &\equiv E_{\pm} \quad \text{etc.} \\ E_{\pm}^2 &\equiv E_{\pm}(\alpha) \quad \text{etc.} \end{aligned} \quad (3.32)$$

$$W(E) \equiv W(E, \alpha)$$

Let us study $W(E, \alpha)$ above the real axes near $E = E_{+} + E_{-}$. By displacing the cut, we see that $W(E, \alpha)$ is analytic for $E_0 = E_{+} + E_{-}$, $\alpha = 0$. Moreover

$$\begin{aligned} W(E_0, \alpha = 0) &= \theta \\ \frac{\partial W}{\partial E}(E_0, \alpha = 0) &= - \frac{2}{|h'(E_{+})|^2} \neq \theta \end{aligned} \quad (3.33)$$

Therefore by the theorem of implicit functions

$$W(E, \alpha) = \theta \quad (3.34)$$

has a unique solution in the neighbourhood of $E = E_0$, $\alpha = 0$:

$$\begin{aligned} E &= E(\alpha) \\ W(E(\alpha), \alpha) &= \theta \end{aligned} \quad (3.35)$$

$E(\alpha)$ represents the trajectory of the state (1.1) in the energy plane and is exactly the quantity we are looking for. Since $E(\alpha)$ is analytic at $\alpha = 0$, we can expand in powers of α :

$$E(\alpha) = E_0 + \alpha \frac{dE}{d\alpha}(\alpha=0) + \frac{\alpha^2}{2} \frac{d^2E}{d\alpha^2}(\alpha=0) + \dots \quad (3.36)$$

$dE/d\alpha$ and $d^2E/d\alpha^2$ can be calculated from the partial derivatives of $W(E, \alpha)$. $dE/d\alpha$ turns out to be real, whereas $d^2E/d\alpha^2$ has an imaginary part :

$$\frac{dE}{d\alpha} = - \frac{\frac{\partial W}{\partial \alpha}(E_0, 0)}{\frac{\partial W}{\partial E}(E_0, 0)} = \frac{1}{2} \frac{\partial}{\partial \alpha} (E_+(\alpha) + E_-(\alpha))_{\alpha=0} \quad (3.37)$$

$$= \operatorname{Re} \left\{ \frac{E_+ - m}{h'(E_+)} \right\}$$

$$\operatorname{Im} \frac{d^2E}{d\alpha^2} = \pm |h'(E_+)|^2 \left(\operatorname{Im} \frac{\partial E_+}{\partial \alpha} \right)^2.$$

$$g^4 \pi \int d\omega d\omega' \frac{\rho^2(\omega) \rho^2(\omega') \delta(\omega + \omega' - E)}{|h(\omega + i\varepsilon)|^2 |h(\omega' + i\varepsilon)|^2} \quad (3.38)$$

where the $\pm$ sign in (3.38) is valid if we approach the real axes from above or below. This shows that the state (1.1) gives two states, with complex energy eigenvalue when the interaction is turned on, which remain on the first sheet of the energy plane.

4. - SOLUTION IN THE SECTOR $B = B' = 1, Q = 0, P = +1$

Here the basis vectors are

$$|VV'\rangle$$

$$a_-^+(\omega) \frac{1}{\sqrt{2}} \{ |NV'\rangle - |N'V\rangle \}$$

$$a_+^+(\omega) \frac{1}{\sqrt{2}} \{ |NV'\rangle + |N'V\rangle \}$$

(4.1)

$$a_-^+(\omega) a_-^+(\omega') |NN'\rangle$$

$$a_+^+(\omega) a_+^+(\omega') |NN'\rangle$$

$$\begin{aligned}
 |E\rangle &= c |VV'\rangle \\
 &+ \int d\omega u_1(\omega) a_-^+(\omega) \frac{1}{\sqrt{2}} \{ |NV'\rangle - |N'V\rangle \} \\
 &+ \int d\omega u_2(\omega) a_+^+(\omega) \frac{1}{\sqrt{2}} \{ |NV'\rangle + |N'V\rangle \} \\
 &+ \int d\omega d\omega' v_1(\omega, \omega') a_-^+(\omega) a_-^+(\omega') |NN'\rangle \\
 &+ \int d\omega d\omega' v_2(\omega, \omega') a_+^+(\omega) a_+^+(\omega') |NN'\rangle
 \end{aligned} \tag{4.2}$$

$$v_i(\omega, \omega') = v_i(\omega', \omega)$$

The eigenvalue equation gives the following integral equations :

$$\begin{aligned}
 c(2m-E) + g \int d\omega \rho_1(\omega) u_1(\omega) + g \int d\omega \rho_2(\omega) u_2(\omega) &= \theta \\
 u_1(\omega)(m+\omega-E) - 2g \int d\omega' \rho_1(\omega') v_1(\omega, \omega') - c g \rho_1(\omega) &= \theta \\
 u_2(\omega)(m+\omega-E) + 2g \int d\omega' \rho_2(\omega') v_2(\omega, \omega') - c g \rho_2(\omega) &= \theta \\
 v_1(\omega, \omega')(\omega+\omega'-E) + g/2 [u_1(\omega) \rho_1(\omega') + u_1(\omega') \rho_1(\omega)] &= \theta \\
 v_2(\omega, \omega')(\omega+\omega'-E) - g/2 [u_2(\omega) \rho_2(\omega') + u_2(\omega') \rho_2(\omega)] &= \theta
 \end{aligned} \tag{4.3}$$

Let us again look for bound states and set

$$u_i(\omega) = \rho_i(\omega) \tilde{u}_i(\omega) \quad i=1,2 \tag{4.4}$$

This leads to the equations

$$\begin{aligned}
 \tilde{u}_i(\omega) h_i(E-\omega) - g^2 \int d\omega' \frac{\rho_i^2(\omega') \tilde{u}_i(\omega')}{\omega' + \omega - E} &= -c g \\
 i &= 1, 2
 \end{aligned} \tag{4.5}$$

For E real $E > 2\mu$ one finds by an analysis similar to 3-A that (4.5) has a solution only if

$$E = E_+^1 + E_-^1 = E_+^2 + E_-^2 \quad (4.6)$$

This will in general only happen if $\rho_1 = \rho_2$. Then (4.6) corresponds to the state (1.2).

For other values of E one finds the following transcendental equation for the eigenvalues [compare (3.29)] :

$$2m - E + \sum_{i=1,2} \frac{1}{U_i(E)} \int_{C_i} \frac{d\xi}{2\pi i} \frac{\xi - m - h_i(\xi)}{h_i(\xi) h_i(E - \xi)} = 0 \quad (4.7)$$

where

$$U_i(E) = \int_{C_i} \frac{d\xi}{2\pi i} \frac{1}{h_i(\xi) h_i(E - \xi)} \quad (4.8)$$

and C_i as indicated in Fig. 5.

Equation (4.7) can be simplified considerably. One can verify that

$$\int_{C_i} \frac{d\xi}{2\pi i} \frac{\xi - m - h_i(\xi)}{h_i(\xi) h_i(E - \xi)} \quad (4.9)$$

and

$$\frac{1}{2} (E - 2m) U_i(E) \quad (4.10)$$

have the same singularities in the E plane. Therefore they differ only by an entire function which is fixed by the behaviour at infinity. In this way, one finds

$$\int_{C_i} \frac{d\xi}{2\pi i} \frac{\xi - m - h_i(\xi)}{h_i(\xi) h_i(E - \xi)} = \frac{1}{2} (E - 2m) U_i(E) + 1 \quad (4.11)$$

Inserting this into (4.7) we get

$$\frac{1}{u_1(E)} + \frac{1}{u_2(E)} = 0 \quad (4.12)$$

From (4.12) we see first of all that also in this sector there are no ghost states on the real axes for $E < 2\mu$: the discussion in 3-B shows that $u_i(E)$ have no zeros for $E < 2\mu$. Since they are real and finite there and positive at $E = -\infty$, they must be positive throughout this region, therefore also the sum (4.12) is positive and $\neq 0$ for $E < 2\mu$.

Next we look at the case $\rho_1 = \rho_2 = \rho$, $u_1 = u_2 = u$. Then the solutions of (4.12) are the poles of $u(E)$ which are [compare Section 3]

$$E = 2E_{\pm} \quad E = E_+ + E_- \quad (4.13)$$

The last one of course corresponds to the state (1.2) we want to study.

Since for general $\rho_1 \neq \rho_2$ the poles of $u_1(E)$ and $u_2(E)$ are different, the following function

$$u_1(E) + u_2(E) \quad (4.14)$$

has the same zeros as (4.12). The function (4.14) has a structure very similar to (3.29). Its singularities are

$$\text{poles at } E_+^1 + E_-^1, E_+^2 + E_-^2 \quad (4.15)$$

$$2E_{\pm}^1, 2E_{\pm}^2 \quad (4.16)$$

$$\begin{aligned} \text{cuts at } & E_{\pm}^1 + \mu < E < \infty \\ & E_{\pm}^2 + \mu < E < \infty \\ & 2\mu < E < \infty \end{aligned} \quad (4.17)$$

Again it is convenient to cancel the poles (4.15) by multiplication with $(E - E_+^1 - E_-^1)(E - E_+^2 - E_-^2)$.

$$V(E) = (E - E_+^1 - E_-^1)(E - E_+^2 - E_-^2) \{u_1(E) + u_2(E)\} \quad (4.18)$$

Let us set, as previously,

$$\begin{aligned} \rho_1^2(\omega) &= \rho^2(\omega) \\ \rho_2^2(\omega) &= \rho^2(\omega) (1 + \alpha) \end{aligned} \quad (4.19)$$

By using the same arguments as in Section 3, we find that the equation

$$V(E, \alpha) = 0 \quad (4.20)$$

has a unique solution $E = E(\alpha)$ with $E(0) = E_0 = E_+ + E_-$, in the neighbourhood of $E = E_0, \alpha = 0$. Again we can write an expansion

$$E(\alpha) = E_0 + \alpha \frac{dE}{d\alpha}(\alpha=0) + \frac{\alpha^2}{2} \frac{d^2E}{d\alpha^2}(\alpha=0) + \dots \quad (4.21)$$

$$\frac{dE}{d\alpha}(\alpha=0) = \operatorname{Re} \frac{\partial E_+(\alpha)}{\partial \alpha} \Big|_{\alpha=0} \quad (4.22)$$

$$\begin{aligned} \operatorname{Im} \frac{d^2E}{d\alpha^2}(\alpha=0) &= \mp |h'(E_+)|^2 \left(\operatorname{Re} \frac{\partial E_+(\alpha)}{\partial \alpha} \right)^2 \\ g^4 \pi \int d\omega d\omega' &\frac{\rho^2(\omega) \rho^2(\omega') \delta(\omega + \omega' - E)}{|h(\omega + i\varepsilon)|^2 |h(\omega' + i\varepsilon)|^2} \end{aligned} \quad (4.23)$$

where we have to take $\mp$ if we approach the real axes from above or below. This shows that the trajectory of the state (1.2) passes into the second sheet of the energy plane.

ACKNOWLEDGEMENTS

The author wants to thank Professor T.D. Lee and Professor W. Thirring for suggesting this investigation and for a critical reading of the manuscript.

REFERENCES

- 1) T.D. Lee and G.C. Wick - Nuclear Phys. B9, 209 (1969).
- 2) T.D. Lee - Proceedings of the Topical Conference on Weak Interactions, CERN, CERN Report 69-7 (1969).
- 3) T.D. Lee and G.C. Wick - To be published.
- 4) T.D. Lee - Phys.Rev. 95, 1329 (1954).
- 5) Many authors have investigated the $N \bar{N}$ sector of the Lee model. We will only list the papers we have used :
A. Pagnamenta - J.Math.Phys. 6, 955 (1965) ; 7, 356 (1966) ;
G.L. Tindle - Nuovo Cimento 45A, 619 (1966) ;
For the model with indefinite metric, see Ref. 3).

FIGURE CAPTIONS

- Figure_1 Energy spectrum for the model of Ref. 1).
- Figure_2 Energy spectrum in the two baryon sector.
- Figure_3 Lee model with two sources.
- Figure_4 Analytic properties of $h_1(z)$.
- Figure_5 Path for the integral (3.28).
- Figure_6 Singularities of $U(E)$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6