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ON THE TWO-BARYON SECTOR OF SOLUBLE MODELS
WITH INDEFINITE METRIC. II

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A B S T R A C T

A soluble model with internal symmetry group is shown to have an unphysical state on the real energy axes. If one can freely add further interactions, this state can be made to go to the complex plane.

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1. INTRODUCTION

This work continues the investigation of the accompanying paper, hereafter referred to as (I). We will describe a model with internal symmetry group $SU(2)$ which has a stable state of negative metric on the real axes of the energy plane in the two-baryon sector. This will lead to violation of unitarity in higher sectors. However, if one is free to add further fields and further interactions, this state disappears.

2. THE MODEL

Our model has two V , two N and three $\vec{a}$ particles: V , N and $\vec{a}$ are all assumed to be bosons.

$$V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \quad N = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} \quad (2.1)$$

$$\vec{a}(\omega) = (a_1(\omega), a_2(\omega), a_3(\omega)) \quad (2.2)$$

These particles are coupled in an $SU(2)$ invariant way:

$$H = m V^\dagger V + \int_{-\infty}^{\infty} d\omega \, \omega \, \vec{a}^\dagger(\omega) \vec{a}(\omega) + g \int_{-\infty}^{\infty} d\omega \, f(\omega) [V^\dagger \vec{\tau} N \vec{a}(\omega) - h.c.] \quad (2.3)$$

$\vec{\tau}$ are the usual Pauli matrices. $f(\omega)$ is real, $m > \mu$, V is assumed to have negative metric.

$$\eta = (-1)^{V^\dagger V} \quad (2.4)$$

The conserved quantities are

$$B = V^+ V + N^+ N \quad (2.5)$$

$$Q = N^+ N - \int d\omega \vec{a}^+(\omega) \vec{a}(\omega) \quad (2.6)$$

$$\vec{T} = \frac{1}{2} V^+ \vec{\epsilon} V + \frac{1}{2} N^+ \vec{\epsilon} N - i \int d\omega \vec{a}^+(\omega) \times \vec{a}(\omega)$$

$$\vec{T}^2 = T(T+1) \quad (2.7)$$

The sector of interest for us has the quantum numbers:

$B=2$, $Q=0$, $T=0$. It is spanned by

$$\frac{1}{\sqrt{6}} \vec{a}^+(\omega) N^+ \vec{\epsilon} \tilde{V} |0\rangle \quad (2.8)$$

$$\frac{1}{\sqrt{12}} \vec{a}^+(\omega) \times \vec{a}^+(\omega') N^+ \vec{\epsilon}_{1/2} \tilde{N} |0\rangle \quad (2.9)$$

where we have set

$$\tilde{N} = \begin{pmatrix} N_2^+ \\ -N_1^+ \end{pmatrix} \quad \tilde{V} = \begin{pmatrix} V_2^+ \\ -V_1^+ \end{pmatrix} \quad (2.10)$$

The eigenvector $|E\rangle$ is expanded as

$$\begin{aligned} |E\rangle = & \int d\omega u(\omega) \frac{1}{\sqrt{6}} \vec{a}^+(\omega) N^+ \vec{\epsilon} \tilde{V} |0\rangle \\ & + \int d\omega d\omega' v(\omega, \omega') \frac{1}{\sqrt{12}} \vec{a}^+(\omega) \times \vec{a}^+(\omega') N^+ \vec{\epsilon}_{1/2} \tilde{N} |0\rangle \end{aligned} \quad (2.11)$$

$$v(\omega, \omega') = -v(\omega', \omega) \quad (2.12)$$

The eigenvalue equation

$$(H - E) |E\rangle = 0 \quad (2.13)$$

leads to the following integral equations

$$\begin{aligned} u(\omega) (m + \omega - E) + 2ig\sqrt{2} \int d\omega' v(\omega, \omega') f(\omega') &= 0 \\ v(\omega, \omega') (\omega + \omega' - E) + ig\sqrt{2} [u(\omega) f(\omega') - u(\omega') f(\omega)] &= 0 \end{aligned} \quad (2.14)$$

Introduce

$$h(z) = z - m - 4g^2 \int d\omega \frac{f^2(\omega)}{\omega - z} \quad (2.15)$$

$$h(E_{\pm}) = 0 \quad E_+^* = E_- \quad (2.16)$$

$$u(\omega) = f(\omega) \tilde{u}(\omega) \quad (2.17)$$

We are looking for bound states with energy E real, $E > 2\mu$, therefore, we have

$$v(\omega, \omega') = - \frac{ig\sqrt{2} f(\omega) f(\omega')}{\omega + \omega' - E} (\tilde{u}(\omega) - \tilde{u}(\omega')) \quad (2.18)$$

and the constraint

$$\tilde{u}(\omega) - \tilde{u}(\omega') = 0 \quad \text{for } \omega + \omega' = E \quad (2.19)$$

This leads to the following equation for $\tilde{u}(\omega)$:

$$\tilde{u}(\omega) h(E - \omega) + 4g^2 \int d\omega' \frac{f^2(\omega') \tilde{u}(\omega')}{\omega' + \omega - E} = 0 \quad (2.20)$$

(2.20) can be solved by an ansatz:

$$\tilde{u}(\omega) = \frac{1}{\omega - E_+} - \frac{1}{\omega - E_-} \quad (2.21)$$

if $E = E_+ + E_-$. Therefore we get the solution

$$E = E_+ + E_- \quad \text{real}$$

$$u(\omega) = 2g f(\omega) \left\{ \frac{1}{\omega - E_+} - \frac{1}{\omega - E_-} \right\}$$

$$v(\omega, \omega') = -i\sqrt{2} g^2 f(\omega) f(\omega') \left\{ \frac{1}{(\omega - E_+)(\omega' - E_-)} - \frac{1}{(\omega - E_-)(\omega' - E_+)} \right\}$$

(2.22)

This state is of negative norm:

$$\begin{aligned} \langle E | \eta | E \rangle &= - \int d\omega |u(\omega)|^2 + \int d\omega d\omega' |v(\omega, \omega')|^2 \\ &= - |h'(E_+)|^2 < 0 \end{aligned}$$

(2.23)

So we have found a state with negative norm on the real axes which will lead to a non-unitary S matrix in the higher sectors.

3. REMOVAL OF THE UNPHYSICAL STATE

If we can freely introduce further fields and interactions we can remove this unphysical state from the real axes. For this purpose we introduce a scalar field $b(\omega)$ and the following total Hamiltonian $[H_0]$ as in (2.3)

$$H = H_0 + \int d\omega \, \omega \, b^\dagger(\omega) b(\omega) + g' \int d\omega \, f(\omega) (V^\dagger N b(\omega) - \text{h.c.}) \quad (3.1)$$

The details of the calculation are now quite similar to I, Section 3, and will not be repeated here. The result is that for $g \neq g'$ the unphysical state disappears from the real axes.

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