Supersymmetry at Linear Colliders: 
The Importance of Being $e^-e^-$

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Advantages of the $e^-e^-$ option at linear colliders for the study of supersymmetry are highlighted. The fermion number violating process $e^-e^- \rightarrow \tilde{e}^-\tilde{e}^-$ provides unique opportunities for studies of slepton masses and flavor mixings. In particular, slepton mass measurements at the 100 MeV level through threshold scans of scalar pair production may be possible. Such measurements are over an order of magnitude better than those possible in $e^+e^-$ mode, require far less integrated luminosity, and may lead to precise, model-independent measurements of $\tan \beta$. Implications for studying gauginos and the importance of accurate beam polarimetry are also discussed.

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1. Introduction

The possible role of supersymmetry (SUSY) in stabilizing the electroweak scale is cause for optimism in the search for SUSY at current and planned colliders. If SUSY is discovered, detailed studies of superpartner properties will likely become a long-term focus of high energy physics and the primary goal of future colliders.

In recent years, our appreciation for the variety of possible superpartner mass spectra, flavor structures, and SUSY breaking mechanisms has grown dramatically. At future colliders, it will therefore be important to seek \textit{model-independent} measurements of all possible superpartner properties. Such studies will yield constraints on SUSY parameters that ultimately could shed light on a variety of mysteries, including the physics at or near the Planck scale.

What contributions might an $e^−e^−$ collider make toward this goal? The replacement of a beam of positrons with electrons is straightforward at linear colliders, and the option of colliding electrons in the $e^−e^−$ mode is therefore, for the most part, a simple extension for any linear collider program. However, when considering the physics promise of the $e^−e^−$ mode, it is, of course, important first to recall the potential of the more conventional hadron or $e^+e^−$ colliders. In particular, a direct comparison can be made to the $e^+e^−$ mode of linear colliders, where luminosities of 50 $\text{fb}^{-1}/\text{yr}$, center of mass energies of up to 1.5 TeV, and highly polarizable $e^−$ beams have been shown to be powerful tools for model-independent studies of SUSY particles. At $e^+e^−$ colliders, superparticles may be discovered essentially up to the kinematic limit, and their couplings may be measured at the percent level to determine if they are, in fact, supersymmetric partners of standard model particles.\textsuperscript{1−6}

Detailed studies of the chargino and neutralino sectors,\textsuperscript{7,2,8} sleptons,\textsuperscript{9,7,3,8,11} and squarks\textsuperscript{10,8,11} find that the masses of most of these particles may be measured to a few percent, and mixings, such as gaugino-Higgsino mixing\textsuperscript{7,2} and left-right scalar mixing,\textsuperscript{3,11} may also be determined.

What, then, can an $e^−e^−$ collider add? At first sight, there appear to be only disadvantages. In $e^−e^−$ mode, pair production of almost all superpartners is forbidden by total lepton number and charge conservation:

$$e^−e^− \not\rightarrow \chi^−\chi^+, \chi^0\chi^0, \tilde{q}\tilde{q}^*, \tilde{\nu}\tilde{\nu}.$$ \hfill (1)

It is therefore clear that the general purpose potential of $e^+e^−$ colliders cannot be matched by $e^−e^−$ colliders. In fact, the only possible superpartner pair production is the fermion number violating process\textsuperscript{12}

$$e^−e^− \rightarrow \tilde{e}^-\tilde{e}^−,$$ \hfill (2)

which is allowed through the $t$-channel Majorana gaugino exchange of Fig. 1. The advantages of the $e^−e^−$ mode over the $e^+e^−$ mode for SUSY studies are almost certainly confined to those derived from this reaction.

The process $e^−e^− \rightarrow \tilde{e}^-\tilde{e}^−$, however, is particularly well-suited to precision studies. First, backgrounds may be highly suppressed. Second, selectrons are typically...
Fig. 1. The contribution to $e^- e^- \rightarrow \tilde{e}^- \tilde{e}^-$ from $t$-channel Majorana neutralino exchange.

expected to be among the lighter superparticles, and they are therefore likely to be kinematically accessible. As we will see, the cross sections for $e^- e^- \rightarrow \tilde{e}^- \tilde{e}^-$ are then typically large, and so statistical errors are small. Third, the properties of selectrons are largely determined by quantum numbers, and so selectron production and decay have strong dependences on only a few SUSY parameters. Theoretical systematic errors arising from unknown SUSY parameters are therefore also typically small.

In fact, the only selectron properties not determined by quantum numbers are their masses and flavor mixings, and, as we will see in the following two sections, $e^- e^-$ colliders provide unparalleled potential for detailed studies of both of these properties. Note, however, that the simple characteristics of selectrons also make them ideal for probing other sectors. A few comments on implications for gaugino mass measurements will be given below. In addition, high precision measurements of selectron couplings may be used to constrain very massive sparticle sectors through the super-oblique corrections introduced in Ref. 4 — this possibility is described in the contribution of H.-C. Cheng to these proceedings.

2. Slepton Masses

Let us consider first the case of $\tilde{e}_R$ pair production in the absence of flavor mixing. At an $e^+ e^-$ collider, this takes place through $s$-channel $\gamma$ and $Z$ processes and $t$-channel neutralino exchange. Assuming that the selectron decays directly to a stable neutralino $\chi$, the signal is $e^+ e^- \rightarrow e^+ e^- \chi \chi$, where the neutralinos go undetected. The dominant backgrounds are $W^+ W^-$, which can be nearly eliminated by right-polarizing the $e^-$ beam, and $e^\pm \nu_e W^\mp$ and $\gamma \gamma \rightarrow W^+ W^-$, which cannot.

As the reaction requires a right-handed electron and a left-handed positron, the initial state has spin 1, leading to the well-known $\beta^3$ behavior of scalar pair production at threshold. Measurements of scalar masses through threshold scans are therefore impossibly poor, and one must resort to kinematic endpoints. For example, $e^+ e^-$ the upper and lower endpoints of the energy distributions of the final state $e^+$ and $e^-$ are determined by $m_{\tilde{e}_R}$ and $m_{\chi}$, and by measuring these endpoints,
Fig. 2. Cross sections $\sigma(e_R^- e_R^- \rightarrow \tilde{e}_R^- \tilde{e}_R^-)$ and $\sigma(e^+ e_R^- \rightarrow \tilde{e}_R^+ \tilde{e}_R^-)$ for $m_{\tilde{e}_R} = 200$ GeV and $m_{\tilde{B}} = 100$ GeV. The inset is a magnified view for $\sqrt{s}$ near threshold. Effects of initial state radiation, beamstrahlung, and the selectron width are not included.

$m_{\tilde{e}_R}$ may be constrained to a few GeV with an integrated luminosity of 20 to 50 fb$^{-1}$.a

In the $e^- e^-$ mode, selectron pair production takes place only through $t$-channel neutralino exchange. The signal is $e^- e^- \rightarrow e^- e^- \chi\chi$. However, among the potential backgrounds, $W^- W^-$ is forbidden by total lepton number conservation, $\gamma\gamma \rightarrow W^+ W^-$ does not produce like-sign electrons, $e^- e^- Z$ may be eliminated by kinematic cuts,17 and the remaining backgrounds $e^- \nu_e W^-$ and $\nu_e \nu_e W^- W^-$ may be completely eliminated, in principle, by right-polarizing both beams.

In addition, the initial state $e^+_R e^-_R$ required for $e_R^- e_R^- \tilde{e}_R^- \tilde{e}_R^-$ production has spin 0, and the threshold cross section therefore has the $\beta$ behavior more commonly associated with fermion pair production. The $\sqrt{s}$ dependence of the cross section is shown in Fig. 2 for $m_{\tilde{e}_R} = 200$ GeV, where, for simplicity, we have assumed gaugino-like neutralinos, and the effects of initial state radiation, beamstrahlung, and selectron width have been neglected. For comparison, the $e^+ e^-$ cross section is also plotted; it is barely visible near threshold.

As the $e^- e^-$ cross section rises sharply at threshold, let us now consider what precision might be expected from a threshold mass measurement. The 1σ statistical error on the mass from a measurement of the cross section is

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aIn such analyses, the information contained in the fact that electrons and positrons come paired in events is lost. Using kinematic variables that are sensitive to this correlation,15,16 slepton mass measurements may be improved, sometimes very significantly.15,16 These improved analyses do not reduce the required integrated luminosities, however, and measurements much below the GeV level still appear to be rather challenging.
\[
\Delta m = \Delta \sigma \left( \frac{\partial \sigma}{\partial m} \right)^{-1},
\]

where \( \Delta \sigma = \sqrt{\sigma/L} \), and \( L \) is the total integrated luminosity. At \( \sqrt{s} = 2m_{\tilde{e}_R} + 0.5 \) GeV, where the cross section is \( \sigma = 200 \) fb, an integrated luminosity of \( L = 1 \) fb\(^{-1} \) gives a cross section measurement of \( \Delta \sigma = 14 \) fb, and the resulting 1\( \sigma \) statistical uncertainty on the mass is \( \Delta m = 40 \) MeV. This result contrasts sharply with results from the \( e^+e^- \) mode, which, as noted above, are typically more than an order of magnitude worse. Note also that the necessary integrated luminosity can be collected in a matter of weeks, even given the possible factor of 2 to 3 reduction in luminosity for the \( e^-e^- \) mode relative to the \( e^+e^- \) mode.\(^{18} \)

In the above, effects of background have been neglected. The dominant background arises from imperfect beam polarization, and is \( e^-\nu_eW^- \) with cross section \( B = 43 \times 2P(1 - P) + 400 \times (1 - P)^2 \) fb.\(^{19} \) The beam polarization \( P \) is defined here as the fraction of right-handed electrons in each individual beam: \( P = N(e_R)/[N(e_L) + N(e_R)] \). Polarizations of \( P = 90\% \) are already available, and higher polarizations may be possible for future colliders.\(^{20} \) For \( P = 90\% \) (95\%), the background is \( B = 12 \) (5) fb and is negligible, assuming it is well-understood and so contributes only to the the uncertainty through statistical fluctuations. While the difference between 90\% and 95\% polarization is not critical for this study, one might worry that the systematic uncertainty from beam polarization measurement might be significant. For example, to take an extreme case, if \( P = 90 \pm 5\% \), the \( e^-\nu_eW^- \) background is constrained only to the range 5 to 20 fb. However, if the projected beam polarization uncertainties of \( \Delta P \sim 1\% \) are achieved,\(^{21} \) the systematic uncertainty does not significantly degrade these results.

The analysis above is clearly highly idealized, and more concrete estimates require a number of refinements.\(^{22} \) In particular, effects of the selectron width, initial state radiation, and beamstrahlung must be included, and other experimental systematic errors, such as uncertainties in the beam energy, will also be important at this high level of precision. In addition, theoretical systematic errors from uncertainties in the masses and gaugino purity of the neutralinos also enter. Finally, the entire scan must be optimized once all these effects are included. It is clear, however, that the \( e^-e^- \) mode offers an exceptionally promising method for measuring selectron masses.

Although the analysis for right-handed selectrons is the most elegant, other slepton masses may also be measured using the \( e^-e^- \) and \( e\gamma \) modes and similar strategies. For example, \( m_{\tilde{e}_L} \) can be measured through \( e_L\tilde{e}_L \rightarrow \tilde{e}_L\tilde{e}_L \). In this case, beam polarization may not be used to remove the dominant backgrounds, but again, if systematic uncertainties are small, the \( \beta \) behavior may be exploited to obtain a precise measurement. (Note that \( e^+e^- \rightarrow \tilde{e}_R\tilde{e}_L \) also has \( \beta \) behavior at threshold.) Finally, along similar lines, the cross sections for chargino pair production \( e^+e^- \rightarrow \chi^+\chi^- \) and the \((-,-)\) helicity component of \( e^-\gamma \rightarrow \tilde{e}_R\chi^- \) also rise as \( \beta \) near threshold, and, as noted in Refs. 23 and 24, this behavior may be exploited to
Fig. 3. Contours giving the upper and lower limits on $\tan \beta$ for a given underlying $\tan \beta$ and experimental uncertainty in mass difference $\Delta m \equiv m_{\tilde{\tau}_L} - m_{\tilde{\nu}_e}$ as indicated (in GeV), for fixed $m_{\tilde{\nu}_e} = 200$ GeV.

determine $m_{\chi^\pm}$ and $m_{\tilde{\nu}_e}$ accurately.\(^6\) In this way, all first generation slepton masses may be measured to high precision.

It is appropriate to ask what use such high accuracy measurements might be. One important application is to loop-level SUSY studies.\(^1,3\)–\(^6,14\) Another is to the measurement of $\tan \beta$, which has important implications for Yukawa couplings, unification scenarios, and a wide variety of other SUSY measurements. At tree level, the relation

$$m_{\tilde{\tau}_L}^2 - m_{\tilde{\nu}_e}^2 = -M_W^2 \cos 2\beta$$  \hspace{1cm} (4)

provides a model-independent measurement of $\tan \beta$. If these slepton masses are measured and their mass splitting is highly constrained, bounds on $\tan \beta$ may be obtained. As an example, in Fig. 3, upper and lower bounds are given as a function of the underlying value of $\tan \beta$ for fixed $m_{\tilde{\nu}_e} = 200$ GeV and uncertainties in $m_{\tilde{\tau}_L} - m_{\tilde{\nu}_e}$ as indicated. For moderate and large $\tan \beta$, $\cos 2\beta \approx -1$, and so constraints from Eq. (4) require high precision measurements of the mass splitting. We see that if the mass difference is known to, say, 200 MeV, the mass splitting provides a powerful determination of $\tan \beta$ for $\tan \beta \lesssim 10$. Note that model-independent measurements of $\tan \beta$ in the intermediate range $4 \lesssim \tan \beta \lesssim 10$ are extremely difficult; previous suggestions have been limited to those exploiting processes involving heavy Higgs scalars.\(^{25,26}\)

\(^6\)Note that this method may be used to measure $m_{\tilde{\nu}_e}$ even if $\tilde{\nu}_e$ decays invisibly.
3. Slepton Flavor Mixings

In SUSY theories, there are generically many new sources of flavor violation. In the standard model, there is no flavor violation at neutral gauge boson vertices $V^\nu f\bar{f}$. However, this is not the case for neutral gaugino vertices $\bar{f}f\bar{V}$, as the fermion- and sfermion-diagonalizing matrices need not be identical. There are therefore 7 new Cabibbo-Kobayashi-Maskawa-like matrices, one for each fermion species $f = u_L, u_R, d_L, d_R, e_L, e_R, \nu$, all of which are worthwhile to explore at future colliders. For simplicity here, let us consider right-handed lepton flavor violation, and let us simplify still further to the case of only $\tilde{e}$ colliders. For simplicity here, let us consider right-handed lepton flavor violation, and let us simplify still further to the case of only $\tilde{e}_R - \tilde{\mu}_R$ mixing, which may be parametrized by a single mixing angle $\theta_R$.

The mixing of $\tilde{e}_R - \tilde{\mu}_R$ induces decays $\mu \to e\gamma$ at low energies, and so is already constrained by the rather stringent bound $B(\mu \to e\gamma) < 4.9 \times 10^{-11}$. With the simplifying assumptions above, $\mu \to e\gamma$ receives contributions from two diagrams, which interfere destructively. Both are proportional to $(\Delta m_R^2/m_R^2) \sin 2\theta_R$, where $\Delta m_R^2 \equiv m_R^2 - m_{\tilde{\mu}_R}^2$, and $m_R^2 \equiv (m_{\tilde{e}_R}^2 + m_{\tilde{\mu}_R}^2)/2$, and one has an additional dependence on the left-right mass mixing parameter $\tilde{t} \equiv (-A + \mu \tan\beta)/m_R$. Note that the superGIM suppression factor $\Delta m_R^2/m_R^2$ suppresses the rate for $\Delta m_R \lesssim m_R$.

The collider signal of lepton flavor violation is $e^+e^- \to e^+\mu^-\chi\chi$ for the $e^+e^-$ mode, or $e^-e^- \to e^-\mu^-\chi\chi$ for the $e^-e^-$ mode. In $e^+e^-$ mode, the leading backgrounds are once again $W^+W^-, e\nu_e W$, and $\gamma\gamma \to W^+W^-$. The essential virtue of the $e^-e^-$ mode for this study is the absence of analogous backgrounds if both $e^-$ beams are right-polarized.

For the $e^-e^-$ case, the flavor-violating collider cross section takes a form familiar from $B$ physics, and is proportional to $\sin^2\theta \sqrt{s} 2\theta_1 2\theta_2$, where $x \equiv \Delta m_R^2/\Gamma$ and $\Gamma$ is the slepton decay width. Note that this cross section is superGIM suppressed only for $\Delta m_R \lesssim \Gamma$, in contrast to the $\mu \to e\gamma$ signal. There is therefore a large range of mass splittings $\Gamma \lesssim \Delta m_R \lesssim m_R$ where the low energy signal is suppressed below current bounds, but the collider signal can be maximally flavor-violating.

In Fig. 4 we present the reach of an $e^-e^-$ collider in the $(\Delta m_R^2/m_R^2, \sin 2\theta_R)$ plane, where we demand a $5\sigma$ excess, and assume $\sqrt{s} = 500$ GeV, $L = 20$ fb$^{-1}$, and 200 GeV right-handed sleptons. We see that lepton flavor violation may be probed down to mixing angles $\sim 10^{-2}$, far below the Cabibbo angle, and for a wide range of mass splittings. This result is a significant improvement over the $e^+e^-$ case. Note that the discovery of lepton flavor violation would have major consequences for SUSY models. For example, the cases of pure gauge-mediated SUSY and pure minimal supergravity would both be eliminated, as both assume degenerate sleptons at some scale and therefore predict the complete absence of lepton flavor violation. For details, see Refs. 28 and 29.

It is important to note that in presenting these results, we have assumed a right-handed beam polarization of $P = 90\%$, for which the background is $B = 12$ fb and a $5\sigma$ signal is $S = 3.9$ fb, and we have neglected experimental systematic uncertainties in beam polarization. However, for this study, as we are looking for a rare signal and a large background has been eliminated through beam polarization,
accurate polarimetry is absolutely crucial. For example, as noted above, if the beam polarization is \( P = 90 \pm 5\% \), the background is constrained only to the range 5 to 20 fb; the 5\( \sigma \) signal is then overwhelmed by polarimetry uncertainties. In fact, even for \( P = 90 \pm 1\% \), the background ranges from \( B = 10 \) to 13 fb, which is also significant relative to the statistical uncertainty. As these SUSY flavor studies may provide important insights into not only the mixings of superpartners, but also the observed patterns of standard model fermion masses and mixings, they are an important example of studies for which beam polarimetry plays an essential role.

4. Gaugino Mass Measurements

As noted in the introduction, the simplicity of selectrons allows one to use selectrons to probe other sectors. It is possible, for example, to exploit the spin structure of the amplitude of Fig. 1 to study the gaugino sector. In particular, because this amplitude includes a \( t \)-channel neutralino mass insertion,

\[
\sigma(e_R^- e_R^- \rightarrow \tilde{e}_R^- \tilde{e}_R^-) \sim \left| \frac{M_1}{t - M_1^2} \right|^2 \sim \frac{1}{M_1^2}
\]

for large \( M_1 \), where \( M_1 \) is the Bino mass. The exact dependence on \( M_1 \) is given in Fig. 5 for \( \sqrt{s} = 500 \) GeV and \( m_{e_R} = 200 \) GeV. The dependence of \( \sigma(e^+ e_R^- \rightarrow \tilde{\chi}_L^0 \tilde{\chi}_R^0) \)
Cross sections for $\sigma(e_R^- e_R^- \rightarrow \tilde{e}_R^- \tilde{e}_R^-)$ and $\sigma(e^+ e_R^- \rightarrow \tilde{e}_R^+ \tilde{e}_R^-)$ as functions of the Bino mass $M_1$ for $m_{\tilde{e}_R} = 200$ GeV and $\sqrt{s} = 500$ GeV. The $t$-channel mass insertion for the $e^- e^-$ case leads to large cross sections, even for $M_1 \sim \mathcal{O}(1$ TeV).

5. Conclusions

It is clear that the possibilities for general studies of SUSY at an $e^+ e^-$ collider cannot be matched by an $e^- e^-$ collider. However, given that the $e^- e^-$ mode is experimentally a relatively simple extension of any linear collider program and is also motivated by the desire for high energy $e\gamma$ and $\gamma\gamma$ studies, it is certainly worth addressing what additional information the $e^- e^-$ mode might bring to precision SUSY studies.

In this study, we have highlighted two possible applications. First, as a result of the fact that the scalar superpartners present in SUSY theories have an associated handedness, $e^- e^-$ colliders may enable one to measure slepton masses through threshold scans with far greater precision than in the $e^+ e^-$ mode. Such high precision measurements are useful for measuring $\tan\beta$, and, for example, may also allow one to be sensitive to small radiative effects.
It is also worth noting that such studies require far less luminosity than the corresponding studies in the $e^+e^-$ mode. At present, most studies of SUSY at linear colliders assume integrated luminosities of $\gtrsim 20$ fb$^{-1}$. In addition, these studies often assume beam energies and polarizations that are optimized for the particular study at hand. While it is clear that not all of these analyses may be conducted simultaneously, systematic attempts to determine how best to distribute the luminosity have not been undertaken, and, in any case, may be premature, given the strong dependence on the actual superpartner spectrum realized in nature. However, in the event that practical limitations on luminosity become relevant, novel studies requiring only weeks of beam time may prove particularly attractive.

In addition, we have shown that the extraordinarily clean environment of $e^-e^-$ colliders leads to striking sensitivity in probes of supersymmetric flavor structure through lepton flavor violation. In such studies, an accurate knowledge of beam polarization is crucial. Finally, note that, for concreteness, we have concentrated on scenarios with stable neutralinos as the lightest supersymmetric particles. However, in other scenarios, such as gauge-mediated scenarios, the signals typically become much more spectacular, and the results given above only improve.

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