MEMBRANES AND FIVEBRANES WITH LOWER SUPERSYMMETRY AND THEIR AdS SUPERGRAVITY DUALS

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Abstract

We consider superconformal field theories in three and six dimensions with eight supercharges which can be realized on the world-volume of M-theory branes sitting at orbifold singularities. We find that they should admit a $\mathcal{N} = 4$ and $\mathcal{N} = 2$ supergravity dual in $AdS_4$ and $AdS_7$, respectively. We discuss the characteristics of the corresponding gauged supergravities.

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CERN-TH-98-79
March 1998
In recent papers, the close connection between $AdS_{p+2}$ and the dynamics on the world-volume of $p$-branes was explored [1, 2, 3, 4, 5, 6]. The explicit proposal of Maldacena [7] that the large $N$ limit of certain conformal field theories can be described in terms of supergravity paved the way for a novel approach to superconformal theories. In particular, it has been argued that the type IIB supergravity on $AdS_5 \times S^5$ is dual to $D = 3, \mathcal{N} = 4 U(N)$ SYM theory at large $N$. Similarly, it has been proposed that the $(2,0), D = 6$ superconformal field theory with $N$ tensor multiplets is dual for large $N$ to eleven-dimensional supergravity on $AdS_7 \times S^4$ with large radii. It has also been conjectured that these dualities can be elevated to field-theory/string-theory and field-theory/M-theory equivalence for finite $N$.

The proposed duality between large $N$ field theories and anti-de Sitter supergravities has been tested by identifying massless excitations in the bulk with singletons composite operators on the anti-de Sitter boundary [8]. Extending further this relation, it has been argued that in a suitable limit, the generating functional for the boundary correlators of singleton composite field is reproduced by the anti-de Sitter supergravity action [9, 10]. The conjecture was further explored in [11, 12, 13, 14, 15, 16, 17, 18]. Moreover, models with lower world-volume supersymmetries started to be explored [19, 20, 21, 22]. We could ask if the field-theory/string-theory correspondence can be extended in such a way that for any given $\mathcal{N} = 0, 1, 2$ superconformal model in four dimensions there exist a supergravity theory in $AdS_5$, and, similarly, if the field-theory/M-theory duality gives a supergravity theory on $AdS_4$ or $AdS_7$ for any superconformal theory in three and six dimensions, respectively. The conjecture, in the form in which it has been formulated up to now, requires the realization of the superconformal theory in terms of branes, in a setting which is compatible with the limits considered in [7]. Fortunately enough, quite a lot of superconformal models can be realized in this way. It is one of the purpose of this paper, to show superconformal models in $D = 3, 6$ which fulfil the previous requirements.

The maximally supersymmetric theories in three and six dimensions have been explored
recently [23, 24, 25, 26]. Here, we will consider \( D = 3 \) and \( D = 6 \) theories with lower supersymmetries, in particular, \( \mathcal{N} = 4 \) and \( \mathcal{N} = 1 \) supersymmetric theories realized on the world-volume of M2- and M5-branes. Such theories can be obtained by appropriate orbifolds of the transverse space. For the M2-brane, the transverse space is \( \mathbb{C}^4 \) and one may consider orbifolds of the form \( \mathbb{C}^2/\Gamma \times \mathbb{C}^2/\Gamma' \), \( \mathbb{C} \times \mathbb{C}^3/\Gamma \), or \( \mathbb{C}^4/\Gamma \), corresponding to M-theory compactifications on \( K3 \times K3 \), \( CY_3 \) and \( CY_4 \), respectively, giving rise to theories with \( \mathcal{N} = 4 \), \( \mathcal{N} = 2 \) and \( \mathcal{N} = 1 \). For the M5-brane, the transverse space is \( \mathbb{R}^5 \) and one may consider orbifolds \( \mathbb{C}^2/\Gamma \times \mathbb{R} \) that lead to \((1,0)\) theories on the world-volume of the M5-brane. A similar procedure was followed in [19] and further elaborated in [21] for theories on the D3-brane of the type IIB string theory.

Let us start with \( D = 6 \). The M5-brane breaks half of the 32 supersymmetries leaving a \((2,0)\) theory on the M5 world-volume. There exist five scalar fields representing fluctuations transverse to the brane and three additional degrees of freedom coming as collective coordinates associated to the three-form of the eleven-dimensional supergravity. All together, we have 8 degrees of freedom which fill the unique \( \mathcal{N} = 2 \), \( D = 6 \) tensor multiplet of the chiral \((2,0)\), \( D = 6 \) supersymmetry. The eleven-dimensional supergravity solution for \( N \) M5-branes is

\[
\begin{align*}
 ds^2 &= f^{-1/3} \left( -dt^2 + dx^a dx^a \right) + f^{2/3} dy^a dy^a , \\
 f &= 1 + \frac{\pi N \ell_p^3}{r^3} , \quad r^2 = y^a y^a ,
\end{align*}
\]

where \( \ell_p \) is the eleven-dimensional Planck length, \( x^a, y^a (a = 1, \ldots, 5) \) are coordinates on the five-brane and transverse to it, respectively. There is a horizon at \( r = 0 \). The near horizon geometry is given by \( AdS_7 \times S^4 \), namely as the product of a seven-dimensional anti-de Sitter space with a four-sphere of radii \( R_{S^4} = R_{AdS_7}/2 = \ell_p (\pi N)^{1/3} \). The M-theory five-brane interpolates between flat Minkowski space-time at \( r \to \infty \) and \( AdS_7 \times S^4 \) at the horizon. The decoupling limit considered in [7] is \( \ell_p \to 0 \) and \( r/\ell_p^3 \) finite, while supergravity is valid for \( N \gg 1 \).
In order to describe a gauge theory in six dimensions, we should further break the $(2,0)$ supersymmetry to the $(1,0)$ one. The massless sector of the latter contains tensors, vectors and hypermultiplets. In particular, the $(2,0)$ tensor multiplet gives rise to a tensor and a hyper of the $(1,0)$ superalgebra. A $(1,0)$ supersymmetry on the M5-brane can be obtained by taking an appropriate orbifold in the transverse space. Since we want to keep in the near-horizon geometry the $AdS_7$ structure, the orbifolding should be such that it acts on $S^4$ only. Now at $t, x^a = \text{const.}$, the transverse space is topologically $R^5$ with metric

$$ds_\perp = \left(1 + \frac{\pi N_f^3}{r^3}\right)^{2/3} dy^a dy^a.$$  \hspace{1cm} (2)

We may act with a discrete group $\Gamma \subset SU(2)$ on $R^5$ to form $R^5/\Gamma = C^2/\Gamma \times R$ where the modding by $\Gamma$ identifies points which are at equal distance from the origin. Thus, the transverse space to the M5-branes is the product of a flat sixth direction $R$ and the ALE space $C^2/\Gamma$. We will consider here the cases $\Gamma = Z_k$ $(k \geq 2)$ and $\Gamma = D_k$ $(k \geq 4)$ where $D_k$ is the binary extension of the dihedral group. The former gives an $A_{k-1}$ ALE space and the latter a $D_k$ one. The generators of $\Gamma$ act on $C^2$ with coordinates $z_1, z_2$ as

$$Z_k : \quad z_1 \rightarrow e^{\frac{2\pi}{k}} z_1, \quad z_2 \rightarrow e^{-\frac{2\pi}{k}} z_2, \hspace{1cm} (3)$$

$$D_k : \quad z_1 \rightarrow e^{i\pi/(k-2)} z_1, \quad z_2 \rightarrow e^{-2i\pi/(k-2)} z_2 \quad \text{and} \quad z_1 \rightarrow z_2, \quad z_2 \rightarrow -z_1.$$

Since these ALE spaces are of $SU(2)$ holonomy, they break half of the original 32 supersymmetries while half more are broken by the M5-branes. Thus, the eight remaining supercharges lead to a $(1,0)$ world-volume theory in six dimensions. The $(2,0)$ tensor multiplet gives a $(1,0)$ tensor and a hyper. The tensor contains an antiself-dual two-form and a scalar representing fluctuations of the five-brane along the sixth direction $R$, which is not affected by the orbifold. The hyper contains two complex scalars $Z^1, Z^2$ corresponding to the position of the M5-branes in the ALE space. In addition, there exist vector multiplets coming from the wrapping of M-theory membranes along the non-vanishing two-cycles of the ALE space.
We will first consider the case of $N$ M5-branes with world-volume $(0, 1, 2, 3, 4, 5)$ located at $N$ points in the sixth direction $\mathbf{R}$ and at the same point in the $A_{k-1}$ ALE space. To analyze the spectrum, it is more convenient to consider our system in the context of type IIA theory. Let us recall that a metric of the form
\[
d s_{TN}^2 = V^{-1}(dX_{11} + \tilde{\omega}) + V d\vec{X} d\vec{X},
\]
\[
V = 1 + \sum_{i=1}^{k} \frac{R_{11}}{2|\vec{X} - \vec{X}_i|}, \quad \vec{\nabla} \times \tilde{\omega} = -\vec{\nabla} V,
\]
where $\vec{X}$ is a three vector and $X_{11}$ has periodicity $2\pi R_{11}$ describes a Taub-NUT space. In the limit $R_{11} \to \infty$ we can ignore the 1 in the expression of $V$ and the Taub-NUT space approaches the ALE space $\mathbb{C}^2/\mathbb{Z}_k$. By identifying $R_{11}$ with the type IIA string coupling $g^4 = (R_{11}/\ell_p)^{3/2}$, we may replace the ALE space in M-theory with a Taub-NUT one. This can be interpreted as M-theory with $k$ KK monopoles, which is type IIA with $k$ D6-branes located at $\vec{X}_i$ [27, 28, 29, 30]. Thus, the spectrum of $N$ M5-branes with transverse space $\mathbf{R} \times \mathbb{C}^2/\mathbb{Z}_k$ in M-theory for large $R_{11}$ can be analyzed in the type IIA side by considering $k$ D6-branes and $N$ NS5-branes at strong string coupling. One should expect that the spectrum can be evaluated at weak coupling and followed into strong coupling by rescaling appropriate parameters and moduli.

The spectrum of the above system at weak coupling can be analyzed by standard brane technology [31, 32, 33]. We consider a system of $N$ NS5-branes with world-volume $(0, 1, 2, 3, 4, 5)$ and $k$ D6-branes stretched between them with world-volume $(0, 1, 2, 3, 4, 5, 6)$. This configuration breaks the ten-dimensional Lorentz-invariance to $SO(1, 5) \times SO(3)$ and has a $(1, 0)$, D=6 supersymmetry living on the world-volume of the NS5-branes. The latter has an $Sp(1)$ R-symmetry which is geometrically realized as rotations in the three-dimensional space transverse to the D6-brane (the $SO(3)$ group above.) We choose the $N$ NS5-branes to be located at $(x^6_\alpha, x^7, x^8, x^9)$, $\alpha = 1, ..., N$, and coincident with $k$ D6-branes at the same fixed coordinate $(x^7, x^8, x^9)$ in the transverse space. If the NS5-branes were absent, we would have a system of $k$ coincident D6-branes, which in the M-theory
language would be interpreted as \( k \) KK-monopoles. In this case, there would be an \( U(k) \) gauge symmetry. The presence of the NS5-branes implies that the D6-branes are in fact divided in \( N-1 \) pieces of finite length along the sixth direction, each of them stretched between two adjacent NS5-branes. In addition, there are \( k \) semi-infinite D6-branes on the left and right of the first and last NS5-branes, respectively. This has the effect to give rise to the gauge group \( U(k)^{N-1} \) on the NS5-branes world-volume as follows from the KK reduction of the D6 world-volume theory along \( x^6 \). There exist charged hypermultiplets in the representation

\[
(k, \bar{k}, 1, ..., 1) \oplus (1, k, \bar{k}, 1, ..., 1) \oplus \cdots \oplus (1, ..., 1, k, \bar{k}) \oplus k(\bar{k}, 1, ..., 1) \oplus k(1, ..., 1, k)
\]

of the gauge group, where the two last terms arise from the semi-infinite D6-branes. In addition to the massless spectrum coming from the D6-branes, each of the NS5-branes provides a \((2,0)\) tensor multiplet. In addition to the self-dual two-forms, these \( N \) tensor multiplets of the \((2,0)\) superalgebra contain five scalars \( \phi^I_\alpha \) \((I = 6, ..., 10)\). Under the unbroken \((1,0)\) supersymmetry, these multiplets decompose into tensor multiplets that contain the scalars \( \phi^6_\alpha \), while \( \phi^{7,8,9,10}_\alpha \) fill hypermultiplets. The KK reduction to \((5 + 1)\) dimensions shows that \( \phi^6_\alpha \) appears as the effective coupling of each of the \( N-1 \) \( U(k) \) factors

\[
\frac{1}{g^2_\alpha} F^{\alpha}_{\mu \nu}^2 + (\partial \phi^6_\alpha)^2 + \sqrt{c} \phi^6_\alpha F^{\alpha}_{\mu \nu}^2 .
\]

The bare coupling \( g_\alpha \) can be absorbed into \( \phi^6_\alpha \) so that the effective coupling is indeed

\[
\frac{1}{g^{eff}_\alpha} = \sqrt{c} \phi^6_\alpha ,
\]

where \( c \) is the anomaly coefficient \([34]\). \( \phi^{7,8,9}_\alpha \) are \( SO(3) \)-triplets of FI terms for the diagonal \( U(1) \)'s of the gauge group factors. The hypermultiplets they belong to are exactly what is needed to cancel the \( U(1) \) anomalies and, as a consequence of a Green-Schwarz mechanism,
all the diagonal $U(1)$ gauge bosons are massive. As a result, the $(5 + 1)$ world-volume theory we obtain is the $\mathcal{N} = 1$, $SU(k)^{N-1}$ gauge theory with one tensor multiplet and $k$ hypermultiplets in the $k \oplus \bar{k}$ representation for each of the gauge group factors.

Six-dimensional gauge theories are restricted by the gauge anomaly, which is given by

$$Tr_{\text{adj}}F^4 - Tr_{\text{R}}F^4 = a \text{tr}F^4 + \frac{c}{d^2}(\text{tr}F^2)^2$$

for a simple group, where $Tr_{\text{adj}}, Tr_{\text{R}}$ and $\text{tr}$ are traces in the adjoint, $R$ and fundamental representations, respectively, and $d$ is the dimension of the fundamental. When the theory is not coupled to gravity, it makes sense when $\alpha = 0$ and $c \geq 0$. For $c = 0$, the theory is anomaly free, while for $c > 0$ the anomaly can be canceled by a tensor multiplet [35, 34]. In our case, $c = 3k^2$ for each $SU(k)$ factor and the anomaly can indeed be canceled by the tensor multiplet. In fact, as follows from eq.(7), the theory may have a non-trivial fixed point at $\phi_6 = 0$ where it is strongly coupled. After absorbing the bare coupling into $\phi_6$, the terms in (6) are scale invariant, supporting the existence of this non-trivial fixed point. In fact, in the large $N$ limit, the six-dimensional $(1,0) SU(k)^{N-1}$ SYM theory with one tensor and $k$ hypermultiplets in $k \oplus \bar{k}$ for each $SU(k)$ factor should be dual to eleven-dimensional supergravity on $AdS_7 \times S^4/\mathbb{Z}_k$ of radii $R_{S^4} = R_{AdS_7}/2 = \ell_p (\pi N)^{1/3}$ and therefore conformal invariant. Then, it is natural to conjure that for all $N$, the above six-dimensional $(1,0) SU(k)^{N-1}$ SYM theory is M-theory on $AdS_7 \times S^4/\mathbb{Z}_k$.

We will now consider the case where $\Gamma$ is the binary dihedral group which corresponds to a $D_k$ ALE space. It is known that M-theory on such spaces is type IIA with $2k$ D6-branes sitting at an orientifold O6-plane with charge $-4$. Therefore, the system we consider is as in the $A_{k-1}$ case, namely $N$ NS5-branes with world-volume $(0,1,2,3,4,5)$ and D6-branes stretched between them, with world-volume $(0,1,2,3,4,5,6)$, together with an orientifold six-plane. The system is located at a single point in the transverse space. Charge conservation imposes $N$ to be even and the gauge group turns out to be [31, 32, 33] $Sp(k - 4) \times (SO(2k) \times Sp(k - 4))^{N/2 - 1}$. Each factor of the gauge group is coupled to a
(1, 0) tensor multiplet as in eq. (6) and the hypermultiplet content is

\[ \left\{ \frac{1}{2} \left( (2k - 8, 2k, 1, \ldots, 1) \oplus (1, 2k, 2k - 8, 1, \ldots, 1) \oplus \cdots \oplus (1, \ldots, 1, 2k - 8, 2k, 1) \oplus (1, \ldots, 1, 2k, 2k - 8) \right) \oplus 2k \left( 2k - 8, 1, \ldots, 1 \right) \right\} . \]  

(9)

Effectively, each \(SO(2k)\) factor of the gauge group is coupled to \(2k - 8\) hypers in the vectorial representation \(2k\), while each of the \(Sp(k - 4)\) factor is coupled to \(2k\) hypers in the fundamental \(2k - 8\). In this case the anomaly eq. (8) has \(a = 0\) and \(c = 12k^2\), \(c = 12(k - 4)^2\) for each \(SO(2k)\) and \(Sp(k - 4)\) factor, respectively. Thus, the anomaly can be cancelled by the same mechanism as before, namely by the coupling of the scalars in the tensor multiplets with the gauge fields. The theory flows then to a non-trivial fixed point in the IR which can be described by the supergravity on \(AdS_7 \times S^5/D_k\).

Branes at orbifold singularities generally give rise to six-dimensional superconformal fixed points [36]. They can be realized using variations of the previous construction [32, 33]. The ones which admit a consistent description in M-theory (in some cases defined on \(R/Z_2\)–see [20] for a related example) are likely to have a dual description as a supergravity in \(AdS_7\).

The \(\mathcal{N} = 2\) gauged supergravity, coupled to matter, on \(AdS_7\) was constructed in [38]. The gauge group is \(SU(2) \times H\), where the \(SU(2)\) vector fields, gauging the R-symmetry of the superconformal theory on the boundary, live in the supergraviton multiplet, while the vectors in the adjoint of \(H\), gauging the flavour symmetries of the boundary theory, are provided by additional vector multiplets. At the six-dimensional superconformal point discussed above (considering, for simplicity, the \(A_k\) singularity), the global symmetry is \(H = SU(k)\). The scalars in the supergravity theory parametrize the manifold,

\[ R^+ \times \frac{O(3,n)}{O(3) \times O(n)} , \]  

(10)

with \(n = \text{dim} H\) and coordinates a real scalar \(\phi_0\) belonging to the graviton multiplet, and by the scalars \(\phi_i^h\) in the adjoint of \(SU(2)\) and Lie-algebra valued in \(H\). The \(\mathcal{N} = 2\)
supergravity has a potential which admits a stable [37] $SU(2)$ invariant anti-de Sitter vacuum at $\phi_0 = \hat{\phi}_0, \phi_i^A = 0$ [38].

It would be interesting to understand better how the previous theory can be obtained as a KK reduction of the eleven-dimensional supergravity to seven dimensions [39], since the KK states would give information on the spectrum of conformal operator of the superconformal theory living at the boundary [8, 10, 13, 23, 24]. The relevant superconformal algebra is $OSp(6, 2/2)$, which also classifies the particle states in $AdS_7$. Here we simply note that singleton representations of the algebra, corresponding to degrees of freedom living on the boundary of $AdS_7$, would correspond to the fields of the superconformal theory, while the other representations correspond to composite operators [8].

Similar constructions can be repeated for the case of superconformal theories in three dimensions, by orbifolding the $\mathcal{N} = 8$ example discussed in [7]. The theory of $N$ coincident membranes in M theory is supposed to be dual for large $N$ to the eleven-dimensional supergravity on $AdS_4 \times S^7$. We can consider orbifolds of M-theory in which the $AdS_4$ part is not affected by the projection. This corresponds to projecting by a suitable discrete group the transverse space of the membranes $R^8$.

$\mathcal{N} = 4$ superconformal theories in three dimensions have a $OSp(4/4)$ symmetry. Non-trivial superconformal theories can be obtained in the IR limit of three dimensional Yang-Mills theories [40, 41]. We will now determine a large class of three dimensional Yang-Mills theories, which have a brane realization and whose IR limit should admit a dual description in terms of a $\mathcal{N} = 4$ supergravity in $AdS_4$.

The original $\mathcal{N} = 8$ example in [7] can be also understood in the following way. Let us start with $N$ D2-branes in type IIA which give rise to the maximal supersymmetric Yang-Mills theory in three dimensions. The theory is not conformal, having a dimensionful gauge coupling but flows in the IR to a superconformal fixed point which is the same as the one discussed in [7]. The gauge coupling for the D2-branes is determined by the IIA
string coupling in such a way that it becomes very large, and the theory therefore flows to
the IR superconformal fixed point, exactly when the M-theory description takes over and
the system is better described with N M-theory membranes. Note also that the $SO(7)$
global symmetry of the D2-branes theory, which rotates the transverse direction in type
IIA, is promoted at the superconformal point to an $SO(8)$ symmetry [42], which rotates
the directions transverse to the membranes, and which is the appropriate R-symmetry of
the $\mathcal{N} = 8$ superconformal algebra.

Let us now consider the $\mathcal{N} = 4$ case. The simplest example is obtained by putting $N$
membranes near an orbifold singularity of the form $R^4/Z_k \times R^4/Z_n$. The spectrum for this
theory was computed in [28]. In a type IIA description, we have a theory of D2-branes
sitting at an orbifold singularity in the presence of $n$ D6-branes, whose world-volume is,
in part, parallel to the D2 world-volume and, for the remaining part, wrapped around a
four-dimensional singular space $R^4/Z_k$. The corresponding Yang-Mills theory realized on
the world-volume of the D2-branes can be easily derived to be [43, 28] a $SU(N)^k$ theory
with hypermultiplets in the representations:

$$(N, N, 1, ..., 1) \oplus (1, N, N, ..., 1) \oplus \cdots \oplus (N, 1, ..., 1, N) + n(N, 1, ..., 1) . \quad (11)$$

In the IR, these theories flow to a superconformal fixed point which should have a super-
gravity dual description as eleven-dimensional supergravity on $AdS_4 \times S^7/(Z_k \times Z_n)$.

Since there are two ways to get a type IIA description starting from M theory on
$R^4/Z_k \times R^4/Z_n$, which correspond to compactify, in one case, on $R^4/Z_k$ and, in the second
case, on $R^4/Z_n$, we could expect to obtain two Yang-Mills candidates (corresponding to the
exchange of $k$ and $n$) for the same IR fixed point (and the same supergravity description).
However, and this was the original motivation of [28], these two theories are actually three
dimensional mirror pairs in the sense of [41], and therefore flow in the IR to the same
superconformal fixed point. The Yang-Mills theories described above indeed contain and
generalize the example in [41]. The differences between the theory in (11) and its mirror,
with \( k \) and \( n \) interchanged, in fact disappear in the eleven-dimensional description which is the relevant one for capturing the structure of the superconformal fixed point.

The R-symmetry \( SU(2) \times SU(2) \) of the superconformal point is already manifest in the Yang-Mills theory at finite coupling. In the \( \mathcal{N} = 4 \) case, it is the global flavour symmetry which is enhanced at the IR fixed point \([41]\). The manifest \( SU(n) \) flavour symmetry of the theories discussed above is enhanced in the IR to \( SU(n) \times SU(k) \) (a symmetry which is manifest in the M-theory description).

The case in which the orbifold projection is performed with a dihedral discrete group is analogous. The Yang-Mills theories that can be obtained in this way can be found in \([28]\).

These superconformal fixed points should have a description as a \( \mathcal{N} = 4 \) supergravity in \( AdS_4 \), whose supergraviton multiplet contains the \( SU(2) \times SU(2) \) vector fields gauging the R-symmetry, coupled to additional vector multiplets which gauge the flavour symmetries. Such gauged supergravity was constructed in \([44]\). If the gauge group is \( SU(2) \times SU(2) \times H \), the scalars in the theory parametrize the manifold,

\[
\frac{SU(1,1)}{U(1)} \times \frac{SO(6,n)}{SO(6) \times SO(n)},
\]

with \( n = \text{dim}H \). The coset \( SU(1,1)/U(1) \) is parametrized by a \( SU(2) \times SU(2) \) singlet complex scalar \( Z \) in the supergraviton multiplet. Since we are looking for an anti-de Sitter \( SU(2) \times SU(2) \) invariant vacuum, all the remaining scalars, in the adjoint of \( SU(2) \times SU(2) \) and Lie-algebra valued in \( H \), can be set to zero. The potential for \( Z \) admits a stable \( AdS_4 \) vacuum when the coupling constants for the two \( SU(2) \) are equal \([44]\), giving the symmetric gauged \( SO(4) \) supergravity. From the point of view of the Yang-Mills theory which flows to this fixed point, the symmetry between the two \( SU(2) \) is nothing else than the mirror symmetry of \([41]\).

Also in these three-dimensional theories, it would be interesting to have a description in terms of an explicit KK reduction of the eleven-dimensional supergravity to four dimensions, since the KK states would give information on the spectrum of conformal operator of the
superconformal theory living at the boundary [10]. The relevant superconformal algebra is $OSp(4/4)$, which also classifies the particle states in $AdS_4$. The irreducible representations of $OSp(4/4)$ are discussed in [45]. The singleton representations of the algebra, in this case, correspond to a multiplet with a complex scalar and a fermion, transforming as $(1/2, 0) + (0, 1/2)$ under $SU(2) \times SU(2)$, in the superconformal boundary theory, while the other representations correspond to composite operators [8].

In principle, one could construct, by changing the orbifold projection, three-dimensional theories with lower $\mathcal{N} = 2, 1$ supersymmetries, based on the superconformal algebras $OSp(2/4)$ and $OSp(1/4)$. Their supergravity duals would correspond to 4d $\mathcal{N} = 2, 1$ $AdS_4$ supergravities with some additional matter multiplets. The supermultiplets in $AdS_4$ would be composite operators of the singleton representations on the 3d boundary [8].

It would be interesting, both in three and six dimensions, to extend the results of the present paper to cases with lower supersymmetry and eventually to cases with $\mathcal{N} = 0$. This can be achieved by considering different types of orbifold projections. In the dual supergravity side, it is known that the scalar potential for the gauged supergravities has other critical points where supersymmetry is partially or completely broken. We may ask if we can identify a superconformal boundary theory with lower or zero supersymmetry, whose generating functional for composite operators is reproduced by the tree-level supergravity expanded around these other critical points. This would provide an explicit flow between superconformal theories with different supersymmetries.

Acknowledgements

This work is supported in part by the EEC under TMR contract ERBFMRX-CT96-0090. S.F. is supported in part by the DOE under grant DE-FG03-91ER40662, Task C, and by ECC Science Program SCI* -CI92-0789 (INFN-Frascati).
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