Energy Losses of Magnetic Monopoles and of Dyons in the Earth

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Abstract

In this paper we discuss the energy losses of magnetic monopoles and of dyons in the earth core and mantle over a large range of velocities. The calculations are used to compute the maximum fractional geometrical acceptance for a detector located in one of the underground halls at the Gran Sasso laboratory in central Italy.

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1 Introduction

The study of the interactions of magnetic monopoles (MM) with matter is needed to understand their energy losses in matter in general and in particle detectors in particular. It is also important to estimate the possible formation of bound systems with atomic nuclei.

In this paper we evaluate the energy losses of fast and slow monopoles and of dyons in the earth mantle and nucleus. This is needed to evaluate the acceptance of an underground detector, such as MACRO at the Gran Sasso Laboratory (LNGS), to monopoles of different masses and of different magnetic charges.

We recall the Dirac relation between the basic electric charge $e$ and the basic magnetic charge $g$ [1]

$$eg = \frac{n\hbar c}{2}$$

where $n$ is an integer, $n=1, 2, 3, \ldots$. For $n=1$, assuming that the basic electric charge is that of the electron, one has $g = g_D = \frac{\hbar c}{2e} = 3.29 \times 10^{-8}$ u.e.s.

There are predictions for the existence of monopoles with $n=1$ [2], $n=2$ [3], and $n=3$ [4]. Moreover if the basic electric charge is that of the quark $d$ the basic magnetic charge has to be increased by a factor of three.

Bound states of a magnetic monopole with an atomic nucleus may form a system with both magnetic and electric charges; some models predict the existence of elementary particles with both electric and magnetic charges, the so called dyons [5].

GUT theories predict the existence of magnetic monopoles with masses of the order of $10^{17}$ GeV/c$^2$ [2], but other theories predict monopoles with masses of $10^{16}$ GeV/c$^2$ [4], $10^{13} - 10^{14}$ GeV/c$^2$ [3]; pointlike monopoles and dyons with $10^3$ GeV < $m_{MC^2} < 10^7$ GeV may also be possible [6]. We shall consider all these possibilities in the evaluation of the energy losses in the Earth, even if most emphasis will be placed on GUT monopoles.

Preliminary calculations of the monopole energy losses in the Earth were considered in Ref [7].

2 A model of the Earth interior

The density profile of the Earth is shown by the dashed line of Fig. 1 [8]; one may observe three layers: the nucleus, the mantle and the crust. For our purposes it is sufficient to use a simpler model, in which the density and composition of each layer is uniform, as shown by the solid line in Fig. 1. In this model the nucleus is made of iron, with a density of $11.5$ g/cm$^3$ and a conductivity of $1.6 \times 10^{16}$ s$^{-1}$; the mantle is made of Si, with a density of $4.3$ g/cm$^3$. The radius of the nucleus is 0.54 earth radii. The crust may be neglected as long as we consider monopoles and dyons arriving at MACRO from below. The rock
above MACRO is assumed to have the same composition of the mantle and a
density of 2.7 g/cm$^3$.

3 Energy losses of fast monopoles with $\beta > 10^{-1}$

A fast monopole passing through a medium can ionize or excite
the atoms and molecules on its path because of the interaction of the electric field
produced by the moving magnetic charge with the electrons in the medium.

For MM with high velocities the energy losses in the Earth are given by the
Bethe-Bloch formula adapted by Ahlen to magnetic monopoles [9]:

$$
\frac{dE}{dx} = \frac{4\pi N_e g^2 e^2}{m_e c^2} \left[ \ln \left( \frac{2m_e c^2 \beta^2 \gamma^2}{I} \right) - \frac{1}{2} \frac{k}{2} \frac{\delta}{2} - B_m \right]
$$

$N_e$ is the density of electrons, $m_e$ the electron mass, $g$ the magnetic charge of
the monopole, $I$ the mean ionization potential, $\delta$ the density effect correction, $k$
the QED correction, $B_m$ the Bloch correction. For silicon at the density of the
terrestrial mantle ($\rho_{Si} = 4.3$ g/cm$^3$) and iron at the density of the terrestrial
core ($\rho_{Fe} = 11.5$ g/cm$^3$) we have used:

$N_e$(Si) = 1.29 × 10$^{24}$ cm$^{-3}$, $N_e$(Fe) = 3.22 × 10$^{24}$ cm$^{-3}$
$I$(Si) = 172 eV, $I$(Fe) = 285 eV

$$k(|g|) = \begin{cases} 
0.406 & \text{for } |g| = 137e/2 \\
0.346 & \text{for } |g| = 137e \\
0.3 & \text{for } |g| \geq 3 \cdot 137e/2 
\end{cases}$$

$$B_m(|g|) = \begin{cases} 
0.248 & \text{for } |g| = 137e/2 \\
0.672 & \text{for } |g| = 137e \\
1.022 & \text{for } |g| = 3 \cdot 137e/2 \\
1.685 & \text{for } |g| = 6 \cdot 137e/2 \\
2.085 & \text{for } |g| = 9 \cdot 137e/2 
\end{cases}$$

For silicon the formula should be accurate to within about $\pm 3\%$ for $\beta > 0.1$
and $\gamma < 100$. The resulting energy losses of monopoles with one unit, two units
and three units of Dirac charge are shown in Fig. 2 for silicon and Fig. 3
for iron. We assume that Eq. 2 applies for $\beta > 0.1$ (solid lines A).

4 Energy losses of monopoles with $10^{-3} < \beta < 10^{-2}$

For monopoles of medium velocity the energy losses can be computed
assuming that the medium is a degenerate electron gas.
For losses in the earth mantle we use the result of Ahlen and Kinoshita [10] for nonconductors

\[
\frac{dE}{dx} = \frac{2\pi N_e \beta^2 e^2 \beta}{m_e c v_F} \left[ \ln \frac{2m_e v_F \Lambda}{\hbar} - 0.5 \right]
\]

(3)

where \( v_F = \frac{A}{m_e} (3\pi^2 N_e)^{1/3} \approx 3.9 \times 10^8 \text{ cm/s} \) is the Fermi velocity (the numeric value applies to silicon at the earth mantle density, \( \rho_{Si} = 4.3 \text{ g/cm}^3 \)); \( \Lambda = 53 \times 10^{-10} \text{ cm} \) is the Bohr radius; \( N_e \) is the density of electrons. For the earth mantle we have (see Ref. [10])

\[
\frac{1}{\rho} \frac{dE}{dz} = 20 \left( \frac{g}{g_D} \right)^2 \cdot \beta \quad \text{GeV/cm}^2 \cdot \text{g}
\]

(4)

One should also include a contribution to the energy losses from the electron magnetic moment; in [10] there is a rough estimate of the strength of this contribution which means increasing the value given by Eq. 4 by a factor of 1.37; this factor is included in the calculations.

For monopole energy losses in the earth core, we use the relation for conductors [10]

\[
\frac{dE}{dx} = \left( \frac{dE}{dx} \right)_{\text{bulk}} + \left( \frac{dE}{dx} \right)_{\text{conduction}}
\]

(5)

where \((dE/dx)_{\text{bulk}}\) and \((dE/dx)_{\text{conduction}}\) represent the contributions of bulk (non valence) electrons and conduction electrons, respectively. \((dE/dx)_{\text{bulk}}\) is given by Eq. 3 with \( N_e = \text{density of bulk electrons and } v_F = \frac{A}{m_e} (3\pi^2 N_e)^{1/3} \approx 5.2 \times 10^8 \text{ cm/s} = \text{Fermi velocity of bulk electrons (the numeric value applies to iron at the earth core density, } \rho_{Fe} = 11.5 \text{ g/cm}^3 \text{, number of conduction electrons per atom = 1.2) }\); \((dE/dx)_{\text{conduction}}\) is given by Eq. 3 with \( N_e = \text{density of conduction electrons. } v_F \approx 1.9 \times 10^8 \text{ cm/s (Fermi velocity of conduction electrons)}\) and

\[
\Lambda = \frac{50 \cdot aT_m}{T}
\]

(6)

where \( a = \sqrt{\frac{\pi}{N_a}} = 2 \times 10^{-6} \text{ cm} \) is the reticular length, \( N_a \) the Avogadro number, \( T \) the temperature of the nucleus and \( T_m \) the fusion temperature. We have assumed \( T \approx T_m \) (the nucleus is partially solid and partially liquid).

We apply these calculations for poles with \( 10^{-3} < \beta < 10^{-2} \) obtaining the solid lines B of Fig. 2 for the earth mantle and of Fig. 3 for the earth nucleus.

Notice that for \( 10^{-2} < \beta < 10^{-1} \) there are more uncertainties in the energy losses of monopoles; the losses in this \( \beta \)-region are shown as dashed lines in Figures 2 and 3.
5 Energy losses of slow monopoles with $\beta < 5 \times 10^{-4}$

For slow monopoles the main contribution to the energy losses is due to elastic collisions of monopoles with atoms. The interaction is between the magnetic moment of the electron and the magnetic field of the monopole. For hydrogen and helium, for $10^{-4} < \beta < 10^{-3}$, one also has the contribution of the Drell effect [11], which in this note is not considered because its contribution to the energy losses in the Earth is small.

For diamagnetic materials we compute the energy losses with the following procedure [12]. The effect of a magnetic monopole in an atom is to leave the state 1s, $m_j = 0$ unperturbed and to excite the state 1s, $m_j = -1$ by a quantity [13]

$$V_0 = \frac{13.6Z^2n}{n + 1} \text{ eV for } g = n g_D$$

We neglect the effect on the other energy levels. We choose the potential between the magnetic monopole and an atom at distance $R$ to be [14]

$$V(R) = \begin{cases} V_0 e^{-aR} & \text{for } R < 0.3 \\ \frac{9Z^2n^2}{R^4} & \text{for } R > 0.3 \end{cases}$$

where $R$ is expressed in Å, $V_0$ and $V(R)$ in eV, and $n$ is the magnetic charge in units of the Dirac charge. The continuity of the potential requires

$$a = -\frac{1}{0.3} \ln \frac{12Zn^2}{V_0}$$

Assuming the validity of this potential we calculated numerically the classical trajectories of atoms of the medium ($\rho_S = 4.3 \text{ g/cm}^3$) relative to the incident monopole. We then obtained the relation between scattering angle $\theta$ and impact parameter $b$. From this relation, the differential cross section $\sigma(\theta)$ is readily obtained from

$$\sigma(\theta) = -(db/d\theta) \cdot b/\sin \theta$$

The transferred kinetic energy $K$ is related to $\theta$ by the relation

$$K = 4E_{\text{inc}} \sin^2(\theta/2)$$

where $E_{\text{inc}}$ of the atom relative to the monopole in the center of mass system; the energy losses are finally obtained by integrating the transferred energies as

$$-\frac{dE}{dx} = N \int \sigma(K) dK$$

where $N$ is the number density of atoms in the medium, $\sigma(K)$ is the differential cross section as function of the transferred kinetic energy $K$. The energy losses
of magnetic monopoles in Si are shown as the solid line C in Fig. 2 for \( \beta < 5 \times 10^{-4} \). The results of the calculation are compatible with the result of Orito for CR39 [15].

In Fig. 2 are shown the curves for monopole energy losses in the mantle; the dashed lines between solid lines B, C are rough interpolations; also the dashed lines for \( \beta < 10^{-5} \), are rough extrapolations (at these velocities it is no longer possible to consider the atoms as free).

For paramagnetic materials and \( \beta < 10^{-5} \) the energy losses are given by the relation [16]:

\[
\frac{dE}{dx} = \mu \cdot \frac{4\pi \hbar g e N}{cm_e} \cdot 0.6
\]

(13)

\( \mu \) is the magnetic moment of the target atoms, in Bohr's magnetons. For higher \( \beta \) the energy losses decrease because of the finite dimensions of the atoms (Eq. 13 is valid for pointlike atoms) and goes down as \( 1/\beta^2 \) for \( \beta \sim 10^{-4} \). In Fig. 3 are shown the curves (solid lines C) for the energy losses of slow monopoles in iron at the conditions present in the earth nucleus.

In Fig. 3 the dashed lines between the solid lines B, C represent rough interpolations.

6 Energy losses of dyons

Energy losses of dyons with \( \beta > 0.03 \). The energy losses of dyons in this velocity range are assumed to be the sum of the energy losses due to their electric and magnetic charges. The magnetic contribution has already been discussed (see Section 3). Here we consider the contribution due to an electric charge \(+Ze\). For \( Z \leq 1 \) the Bethe-Bloch familiar expression holds:

\[
\frac{dE}{dx} = \frac{4\pi N_e Z_1^2 e^4}{mc^2\beta^2} \left[ \ln \left( \frac{2m_e c^2 \beta^2 \gamma^2}{l} \right) - \beta^2 \frac{\delta}{2} - B_e \right]
\]

(14)

\( N_e \) is the density of electrons, \( l \) the mean ionization potential, \( \delta \) the density-effect correction, \( B_e \) the Bloch correction. Eq. 14 overestimates the energy losses because it does not consider that the effective electric charge of the dyon can be reduced by the capture of electrons; a good agreement with experimental data on the energy losses of fast ions is obtained with [17]

\[
\frac{dE}{dx} = \left( \frac{dE}{dx} \right)_{Z=1} \frac{Z_1^2}{Z} \left( 1 - e^{-0.55Z} \right)
\]

(15)

which is used in the present calculations.

Energy losses of dyons with \( \beta < 0.01 \). If the dyon velocity is low (\( \beta < 0.01 \)) there are two contributions to the energy losses: the losses due to the interactions with the nuclei and those with the electrons of the target
material. We add the two contributions to obtain the electric energy losses for
$10^{-5} < \beta < 10^{-2}$.

Electronic contribution. As the interactions electric charge-electron and mag-
netic charge-electron are longitudinal and transversal respectively, the energy
loss of the dyon is simply the incoherent sum of the energy losses due to
the electric charge and the energy losses due to the magnetic charge. The energy
losses due to the magnetic charge are discussed in Section 4. The energy losses
due to the electric charge have been computed with the following formulee [18]:

\[
\frac{dE}{dx} = \begin{cases} \frac{8\pi e^2 a_0 \delta}{\alpha} \frac{Z_1^{-1/4} N_\gamma}{(Z_1/2 + Z_2/2)^{1/2}} & \text{for } Z_1 \geq 1 \\ \frac{1}{2} \left( \frac{dE}{dx} \right)_{Z_1=1} & \text{for } Z_1 = 1/3 \end{cases}
\]  

(16)

Nuclear contribution. In considering the dyon energy losses due to interac-
tions with the nuclei of the earth core we add again the magnetic and electric
ccontributions. The energy losses due to the magnetic charge are discussed in
Section 5. The electric energy losses due to collisions with the nuclei of the
medium are given by [19]

\[
\frac{dE}{dx} = \frac{\pi a^2 \gamma N E}{e} S_n(\epsilon)
\]  

(17)

where:

\[
a = \frac{0.885 a_0}{(\sqrt{Z_1} + \sqrt{Z_2})^{1/4}}, \quad \gamma = \frac{4 M_2}{M_1}, \quad \epsilon = \frac{a M_2 E}{Z_1 Z_2 e^2 M_1}, \quad S_n(\epsilon) \approx \frac{0.56 \ln(1.2\epsilon)}{1.2\epsilon - (1.2\epsilon)^{0.63}}
\]

The masses $M_1$ and $M_2$ are relative to the incident and target particles, respectively; $Z_1 e$ and $Z_2 e$ are their electric charges; we assume $M_1 \gg M_2$; $a_0$ is the
Bohr radius.

For the energy losses due to collisions with the nuclei of the earth mantle
we adopt a different procedure. Here the interactions magnetic charge-atom
and electric charge-atom are both longitudinal and each can be described by
a potential. We add the two potentials and then we compute the dyon energy
losses with the same procedure used in Section 5 and in [19]. The results are
different by what obtained computing the energy losses of a magnetic monopole
(Section 5) and of an electric charge([19]) and then adding them.

The total energy losses of dyons in the earth mantle and in the earth nucleus
are shown in Fig. 4 assuming $g = g_D$ and $Z=1$ (this is also valid for a monopole
bound with a proton) or $Z=13$ (valid also for a monopole bound to an aluminium
nucleus).
7 Energy losses of monopoles in the earth’s mantle

We considered the earth’s mantle made of Si with constant density. The energy losses of monopoles as a function of $\beta$ are presented in Fig. 2. For $\beta > 0.001$ the solid lines are computed with Eq. 2 and Eq. 4. The solid lines for lower $\beta$ are the result of the numerical calculations described in Section 5. The dashed lines indicate that we have extrapolated a formula outside its interval of validity ($\beta < 10^{-5}$ and $\beta \sim 8 \times 10^{-4}$) or where we have connected two safe regions with a polynomial, requiring continuity of function and derivative ($0.01 < \beta < 0.1$). The energy losses are roughly linear in $g$ at low $\beta$ and quadratic in $g$ at high $\beta$.

8 Energy losses of monopoles in the earth’s nucleus

We assume that the earth core is made of iron with the already noted high density of $\rho = 11.5$ g/cm$^3$. The corresponding energy losses are presented as solid lines in Fig. 3 using Eqs. 2, 5 and 13. The dashed lines are interpolations obtained requiring smooth connections and a dependence of the type $1/\beta^2$ near $10^{-4}$. The energy losses are again roughly linear in $g$ at low $\beta$ and quadratic in $g$ at high $\beta$.

9 Energy losses of dyons in the earth’s interior

The total energy losses of dyons in the earth’s mantle and nucleus are presented in Fig. 4 for $g=g_D$ and $Z=1$ (this applies also to a monopole bound with a proton) or $Z=13$ (it applies also to a monopole bound to an aluminium nucleus).

10 Earth composition

The exact composition of the earth core is somewhat controversial [20], therefore the energy losses in it should be considered as estimates.

As far as the earth mantle is concerned a better approximation would be to use a SiO$_2$ composition instead of Si; we estimated that the difference of the energy losses between the Si and SiO$_2$ compositions to be at most 10%, except for MM with $g=g_D$ and $\beta \sim 5 \times 10^{-4}$ for which the difference is about 30%. This correction has not been used, that is all our curves are relative to a Si composition of the earth mantle.
11 MM and dyon acceptances

Using the energy losses discussed above and presented in Figure 2, 3 and 4 we calculate, for a given mass and a given velocity, the fraction of the total solid angle that is seen by a detector located in one of the underground halls at the Gran Sasso laboratory (i.e. for which the corresponding monopoles arrive at the hall with $\beta > 10^{-5}$). Figures 5 and 6 show the fractional acceptance of an underground detector at LNGS as a function of the monopole mass for five values of the magnetic charge and for four values of $\beta$; Fig. 8 shows the fractional acceptance for dyons.

Starting from the upper right hand corner in the figures, we notice there an ideal geometrical acceptance of $4\pi$; going to lower monopole masses we observe a decrease after which the acceptance is $2\pi$ (the underground detectors are now blind for any upgoing monopole); then there is a second decrease at the end of which one has the lowest mass that can be detected for each $g$ and each $\beta$. To obtain the second part of the curves (relative to the monopoles coming from above), we have used the MACRO position in the mountain and the real rock thickness surrounding the detector (we used 90x45 bins for the solid angle calculation).

Fig. 7 shows the accessible region for a detector at the MACRO location in the plane (mass, $\beta$) for MM with different magnetic charges $g$; Fig. 9 shows the same for dyons. From these plots one sees that the detector is not able to see monopoles or dyons with masses smaller than few $10^5$ GeV.

12 Conclusions

Using a rough model of the earth composition and density profiles, and the energy losses for MM with different magnetic charges (with $g=2g_D$, $3g_D$, $6g_D$, $9g_D$) and for dyons, we have computed the fractional solid angle acceptance for MM and dyons of different initial velocities arriving to a detector like MACRO located in the Gran Sasso underground laboratories.

We gratefully acknowledge the cooperation of many members of the MACRO collaboration, in particular of all the members of the MACRO Bologna group. M.O thanks the ICSC-World Laboratory for providing a fellowship. J.D. M.O and V.P thank INFN for providing FAI grants for non italian citizens. V.T acknowledges a fellowship from Intercast Europe Co., Parma.
Table 1: First column: magnetic charges. In the second column are given the lowest masses (in $GeV/c^2$) of MM with $g = g_D$, $2g_D$, $3g_D$, $6g_D$, $9g_D$, respectively, leading at the underground LNGS location to an isotropic flux, and thus $\Omega = 4\pi$. In the third column are given the minimum masses for which $\Omega$ is still zero (but $\Omega$ starts to increase). In the fourth column are given the minimum masses above which $\Omega = 2\pi$. The Table is relative to MM incident with $\beta=0.1$ at the top of atmosphere.

<table>
<thead>
<tr>
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<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
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Table 2: Same as in Table 1, but for $\beta=0.01$.

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Table 3: Same as Table 1, but for $\beta=0.001$.

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Table 4: Same as in table 1, but with $\beta=0.0001$.

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</table>

References


Figure 1: The dashed line represents the density profile of the Earth according to Ref. 8; in the present paper we use the simple approximation represented by the solid line ($r_0 = \text{radius of the Earth}$).
Figure 2: Energy losses of magnetic monopoles in the earth mantle as function of velocity \( \beta c \) and of magnetic charge \( g \). Notice the solid curves in the regions A, B, C where the calculations are more reliable, and the dashed lines for which there are more uncertainties. The lowest three curves apply to magnetic monopoles with \( n=1, 2, 3 \) and assuming that the basic electric charge is that of the electron; the third curve also applies to MM with \( n=1 \) assuming that the basic electric charge is that of the quark d. The highest two curves apply to magnetic monopoles with \( n=2, 3 \) and basic charge \( \frac{1}{3}e \).
Figure 3: Energy losses of magnetic monopoles in the earth nucleus, assuming that it is made of iron with $\rho = 11.5 \text{ g/cm}^3$, as function of $\beta$ for various magnetic charges. The notation is as in Fig. 2.
Figure 4: Energy losses of dyons with $g = g_D$ and $q = Z e$ in the earth nucleus (solid lines) and in the mantle (dashed lines), as function of $\beta$. $Z = 1$ corresponds also to a MM with $g = g_D$ bound to a proton, $Z = 13$ corresponds to a magnetic monopole ($g = g_D$) bound to an aluminium nucleus.
Figure 5: Fractional geometrical acceptance for magnetic monopoles with $\beta = 10^{-1}, 10^{-2}, 10^{-3}$ and $10^{-4}$ as a function of the MM mass, and for magnetic charges $g = g_D, 2g_D$ and $3g_D$ for a detector in one of the underground halls at the Gran Sasso laboratory.
Figure 6: Fractional geometrical acceptance for magnetic monopoles with $\beta = 10^{-1}, 10^{-2}, 10^{-3}$, and $10^{-4}$ as a function of the MM mass, and for magnetic charges $6g_D$ and $9g_D$ for a detector in one of the underground Gran Sasso halls.
Figure 7: Accessible region in the plane (mass, $\beta$) of monopoles from above for an experiment in one of the underground halls of the Gran Sasso laboratory.
Figure 8: Fractional geometrical acceptance for dyons from above with $Z=1$, $g=g_D$ and with $\beta = 10^{-1}, 10^{-2}, 10^{-3}$ and $10^{-4}$ versus dyon mass for a detector in one of the underground halls Gran Sasso halls.
Figure 9: Accessible region in the plane \((\text{mass}, \beta)\) of dyons with \(Z=1\) and \(g = g_D\) for an experiment in one of the underground Gran Sasso halls.
Figure 10: Accessible region in the plane (mass, $\beta$) of monopoles with $g=g_D$, $2g_D$, $3g_D$, $6g_D$, and $9g_D$ in order to cross 10 cm of iron before reaching from above a detector at ground level.