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SELF-ACTION ON A CURRENT WITH INTERNAL STRUCTURE IN COSMIC STRING SPACETIMES

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Abstract

We calculate the topological self-energy of the stationary linear distribution of a four-current with zero total current and a charge per unit length in the spacetime of a straight cosmic string. It is shown that if the structure of a current is determined in the first approximation by its electric and magnetic dipole moments, then the self-force is not equal to zero and the current feels the action of a moment of force too which has a topological nature. The generalization of the obtained results for the spacetime of multiple cosmic string is discussed.

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1. Conical spaces appear in consideration of physical phenomena at very different space-time scales. Cosmic string network^{1,2} is the most popular but not the only physical system which generates conical singularities in a spacetime. Another examples we can find in the solid state physics³, physics of black holes⁴, cosmology⁵.

In the simplest case of the static straight infinitely thin cosmic string, the spacetime looks like the direct product $M_2 \times Cone$ of the two-dimensional Minkowski space and a cone and the corresponding Riemannian tensor vanishes everywhere except on the symmetry axis, where it has a δ -like singularity⁶. So, infinitely thin cosmic string does not affect the local geometry of the spacetime but change its global properties, and therefore its effect on the matter fields is purely topological. This means that in spite of the local triviality of the spacetime geometry, its global conical structure gives a nonvanishing contribution to the local observables.

In recent years, the interest in cosmic strings has been associated with investigations in the quantum field theory. The vacuum energy has been calculated for the massive and massless fields of different spins at zero⁷ and finite⁸ temperatures. It was shown⁹ that the additional deformation of the vacuum loops due to the multiconical boundary conditions leads to the Casimir-like interaction between the strings, and the strings of the multiple cosmic string spacetime which have no purely gravitational interaction are attracted to each other with a force that decreases with the third power of the distance between them.

The effect of the straight string on quantized fields is purely topological. From the formal point of view it is because of the fact that the solutions of a partial differential equation are sensitive to the global structure of a spacetime. Therefore, similar effects must take place in the classical theory too if we consider classical fields and have to work with the wave equations. The repulsion of a charged particle¹⁰ and the attraction of a gravitating one¹¹ are the well known examples of this type of effects. Some new features of the topological self-action give us the linear sources. It was shown^{12,13} that a linear current which is parallel to a cosmic string interacts with the string as with another parallel current at the same distance in the Euclidean space.

In all the papers mentioned previously the simplest point or linear sources without inner structure were considered. However, from the low-dimensional gravity we know¹⁴ that if the charge distribution has a finite size and can not be described by the total charge only the problem of the topological contribution to an interaction energy arises. It was shown that this contribution is of the same order of magnitude as the topological contribution to the

self-energy and is not equal to zero even in a locally flat space. Moreover, the topological shift of the interaction energy does not tend to zero in the limit of infinitely small source if the structure of the source (dipole moment, in the case of Ref. 14) is fixed.

Others problems have to be solved when we consider the topological effects in the space-time of multiple cosmic strings. Such a consideration is well motivated physically because there are about ten straight cosmic strings under the Hubble horizon¹⁵ and because the number of disclinations per unit volume in a real crystal exceeds the unity by far. So, it is necessary to consider spaces with more than one conical singularities. Problems involving multiple conical singularities are very complicated due to the loss of the azimuthal symmetry which makes impossible to separate the variables in the field equations and to proceed along the same line as in the case of only one string. One possible approach to the problem in this context is associated with the use of perturbation theory⁹, another one is based on the possibility to solve some of the problems explicitly in low-dimensional gravity¹⁴.

Here we consider some effects of the topological self-action on the linear distribution of a four-current with equal to zero total current and a charge per unit length but with an internal structure described by its electric and magnetic dipole moments. The simple enough structure of the current enables us to solve the problem explicitly. Moreover, the method used in this paper gives us the possibility to consider the spacetime of a straight string and some more general cases, the case of a multiple cosmic string space, in particular.

2. Let us start with the metric of background spacetime which is supposed to be given by the line element

$$ds^2 = dt^2 - dz^2 - \gamma_{ab}(x)dx^a dx^b, \quad a, b, c = 1, 2. \quad (1)$$

Note that the spacetime of an infinitely thin cosmic string^{1,2} and its multistring generalization¹⁴ are the particular cases of the space (1) when the subspace $t = \text{const}, z = \text{const}$ is chosen to be a locally flat surface with one or more conical singularities.

On this background we shall consider a stationary distribution of a four-current

$$j^\mu(x) = (j^t(x), 0, 0, j^z(x)), \quad (2)$$

which is supposed to be time independent and invariant under translations along the z axis.

The metric of the spacetime and the the four-current do not depend on z . Due to this fact, the energy-momentum tensor of the electromagnetic field associated with (2) does not depend on z too and the total energy of the field is infinite. So, let us consider the energy per unit length along the current which is given by the expression

$$\frac{E}{\int dz} = \int d^2x \sqrt{\gamma(x)} T_{tt}(x). \quad (3)$$

In the Lorentz gauge the components of a four-potential satisfy the equation

$$\nabla_\nu \nabla^\nu A_\mu - R_\mu^\nu A_\nu = 4\pi j_\mu , \quad (4)$$

where R_μ^ν is the Ricci tensor of the background spacetime.

Lorentz gauge does not fix a four-potential uniquely. In our case, essential simplification comes from the possibility to choose a gauge in which t - and z - components of a four-potential do not depend on t and z and $A^1 = A^2 = 0$. In this gauge the second term in the lhs of the Eq. (4) is equal to zero and the nonzero components of a four-potential satisfy the two-dimensional Poisson equation

$$\frac{1}{\sqrt{\gamma(x)}} \partial_a \left(\sqrt{\gamma(x)} \gamma^{ab}(x) \partial_b A^{t(z)}(x) \right) = -4\pi j^{t(z)}(x) : \quad (5)$$

As the result, the energy per unit length (3) can be written as follows

$$\begin{aligned} \frac{E}{\int dz} = & -2\pi \int d^2 x_1 d^2 x_2 \sqrt{\gamma(x_1)} \sqrt{\gamma(x_2)} \times \\ & \left(j^t(x_1) G^{(2)}(x_1, x_2) j^t(x_2) + j^z(x_1) G^{(2)}(x_1, x_2) j^z(x_2) \right) . \end{aligned} \quad (6)$$

In the expression above $G^{(2)}$ stands for the Green function of the Eq. (5).

In our previous papers^{12,13} we showed that for a simple infinitely thin straight current without internal structure the induced nonlocal force is the same as the force between two parallel four-currents which are proportional to each other in the Euclidean space. So, let us start from Eq. (6) and consider the limit when the supports of the functions $j^t(x)$ and $j^z(x)$ collapse into the point x and the total current and the charge per unit length are equal to zero, that is

$$\int d^2 x \sqrt{\gamma(x)} j^t(x) = \int d^2 x \sqrt{\gamma(x)} j^z(x) = 0 . \quad (7)$$

The simplest way to make the limiting current density a nontrivial one is to demand that the limits of the two-vectors

$$d^a(x) = \int d^2 x' \sqrt{\gamma(x')} \sqrt{2\sigma(x, x')} t^a(x') j^t(x') \quad (8)$$

and

$$\mu^a(x) = \epsilon^{ab}(x) \int d^2 x' \sqrt{\gamma(x')} \sqrt{2\sigma(x, x')} t_b(x') j^z(x')$$

are nonzero. In the last expression $\sigma(x, x')$ is the half of the squared geodesic distance between the points x and x' , $t^a(x')$ be a tangent vector to the geodesic at the point x directed towards the point x' and $\epsilon^{ab}(x) = e^{ab}/\sqrt{\gamma(x)}$ is the unit antisymmetric two-tensor ($e^{12} = -e^{21} = 1$, $e^{11} = e^{22} = 0$).

Both these two-vectors lie in the tangent plane to the Riemannian surface at the same point x and define the electric and magnetic dipole moments per unit length which are measured by an inertial observer located at the current.

In the limit under consideration the functions j^t and j^z are well defined in the sense of distributions and have the following forms

$$j^t(x') = -\frac{(d^a \partial'_a) \delta^2(x - x')}{\sqrt{\gamma(x')}} , \quad j^z(x') = -\frac{(\mu^{*a} \partial'_a) \delta^2(x - x')}{\sqrt{\gamma(x')}} . \quad (9)$$

In Eq. (9) μ^{*a} stands for the vector $\mu^{*a} = -\epsilon^{ab} \mu_b$.

Substituting (9) into the expression for the energy per unit length (6), we obtain

$$\frac{E}{\int dz} = -2\pi(d^a d^b + \mu^{*a} \mu^{*b}) \partial_a \partial_b G^{(2)}(x, x) . \quad (10)$$

This expression is purely formal because $G^{(2)}$ diverges in the coincidence limit and some regularization is needed.

The well known definition of a point dipole generalization of which is used in our paper makes it natural to use the covariant point-splitting method, the direction of splitting being determined by the vectors d^a and μ^{*a} for the first and the second terms in the Eq. (10) correspondingly.

The use of the point-splitting method requires a knowledge of the decomposition of the function $G^{(2)}(x_1, x_2)$ with respect to σ up to the terms of the order σ^2 . In the case of infinite Riemannian surface which is globally conformal to the Euclidean plane corresponding result was obtained by one of us in Ref. 14. It was shown that if it is possible to introduce a global coordinate system in which the metric takes the form

$$\gamma_{ab}(x) = e^{-\Omega(x)} \delta_{ab} \quad (11)$$

and if the requirement (boundary condition) that the field of a point source tends to zero at large conformal distances is physically well motivated, then the Green function reads

$$G^{(2)}(x_1, x_2) = \frac{1}{4\pi} \log 2\sigma(x_1, x_2) + \frac{\Omega(x)}{4\pi} + \quad (12)$$

$$\frac{\sigma}{48\pi} t^a t^b \left(\nabla_a \nabla_b \Omega + \nabla_a \Omega \nabla_b \Omega - \frac{1}{2} \gamma_{ab} \nabla_c \Omega \nabla^c \Omega \right) + O(\sigma^2) .$$

Here t^a is a tangent vector to the geodesic between the points x_1 and x_2 at the middle point x .

We keep in mind that the spaces under consideration are locally flat. So, we can eliminate the divergences by subtracting the Euclidean Green function, which coincides with the first term in the rhs. of the Eq. (12) for $G^{(2)}$. After that we can perform the differentiation with

respect to x and consider the coincidence limit. Proceeding according to the prescriptions of the point-splitting method, see Ref. 17 and references therein, we obtain

$$\left(t^a t^b \partial_a \partial_b G^{(2)}\right)_{reg}(x, x) = \frac{1}{48\pi} \left(2t^a t^b \nabla_{ab}^2 \Omega - (t^a \nabla_a \Omega)^2 + \frac{1}{2} \nabla_a \Omega \nabla^a \Omega \right), \quad (13)$$

where $t^a = d^a/d$ or μ^{*a}/μ^* for the first and the second terms in Eq. (10) correspondingly.

Now we are in a position to consider the particular case of a straight cosmic string. For this purposes the conformal factor in Eq. (11) must be chosen in the form^{16,18}

$$\Omega(x) = 2(1-b) \log \frac{r}{a}, \quad r = (x_1^2 + x_2^2)^{1/2}, \quad (14)$$

where a is an arbitrary scaling parameter and b defines the deficit angle of the string $\Delta\phi = 2\pi(1-b)$.

To make the interpretation of the result more obvious, let us introduce a new radial coordinate ρ which gives the geodesic distance from the string¹⁸

$$\rho = \frac{a}{b} \left(\frac{r}{a} \right)^b. \quad (15)$$

Substituting (13) into Eq. (10) and taking into account Eqs. (14) and (15) we obtain that the topological self-energy of the current (9) in the spacetime of a straight infinitely thin cosmic string is the same as the interaction energy between two similar current distributions with parallel magnetic and antiparallel electric dipole moments in the Euclidean space. Indeed a straightforward calculation leads to the result that from the point of view of the inertial observer which is located at the current, the topological self-energy takes the following form

$$\frac{E_{top}}{\int dz} = -(\mathbf{d}\mathbf{E}_{ind}) - (\boldsymbol{\mu}\mathbf{H}_{ind}). \quad (16)$$

The induced electric and magnetic fields which we introduce in Eq. (16) are the same as the fields produced by a current density of the form (9) with

$$d_{ind}^a = -\frac{(1-b^2)}{12b^2} d^a, \quad \mu_{ind}^a = \frac{(1-b^2)}{12b^2} \mu^a \quad (17)$$

at the distance ρ in the Euclidean space, that is, they have the forms

$$\mathbf{E}_{ind} = \frac{2\rho(\rho\mathbf{d}_{ind}) - \rho^2\mathbf{d}_{ind}}{\rho^4}, \quad \mathbf{H}_{ind} = \frac{2\rho(\rho\boldsymbol{\mu}_{ind}) - \rho^2\boldsymbol{\mu}_{ind}}{\rho^4}. \quad (18)$$

The force can be calculated by taking the negative gradient of the topological self-energy (16). We see that in contrast with the case of a simple current^{12,13} the signs of its electric and magnetic constituents depend on the orientation of the corresponding moment and the sign of the total force depends on the angle between the moments and on their relative

magnitude. But the most interesting feature of the obtained result is the appearance of a topological moment of force. In the frame of a locally inertial observer placed at the current, it has the following form

$$\mathbf{K}_{top} = (\boldsymbol{\mu} \times \mathbf{H}_{ind}) + (\mathbf{d} \times \mathbf{E}_{ind}) = \frac{(1 - b^2)}{6b^2\rho^4} \left((\boldsymbol{\rho} \times \mathbf{d})(\boldsymbol{\rho} \mathbf{d}) - (\boldsymbol{\rho} \times \boldsymbol{\mu})(\boldsymbol{\rho} \boldsymbol{\mu}) \right). \quad (19)$$

This expression can be obtained from Eq. (16) by the differentiation with respect to the angles determining the orientation of the dipole moments.

The electric part of the moment (19) is the same as in the low-dimensional gravity¹⁴. The essentially new feature of our result is the existence of a magnetic self-moment. Both moments are equal to zero if the corresponding dipole is oriented along or perpendicular to the line between the string and the point of location. A simple analysis shows that if the electric dipole tends to be oriented in the perpendicular direction, the magnetic one tends to be parallel or antiparallel to the vector $\boldsymbol{\rho}$.

To conclude this section we should like to stress that the topological self-energy which is given by Eq. (16) consists of the topological contribution to both self- and interaction energy. To understand it, let us suppose that our system consists of a number of infinitely thin parallel simple currents I_i and the conditions (7) hold. In the limit under consideration terms of the order $I_i \sqrt{\sigma(x_i, x)}$ tend to some nonzero values. So, the terms of the order $I_i I_k \sigma(x_i, x_k)$ have a nonzero limits too. Now we see that the first term in the rhs of Eq. (12) is subtracted during the regularization, the second one gives a zero contribution to the interaction energy because of the Eqs. (7). Thus the interaction energy is determined by the third term and is equal to the sum of the expressions of the order $I_i I_k \sigma(x_i, x_k), i \neq k$ which do not tend to zero in the limit under consideration as it was explained above.

3. We considered in detail the spacetime of a single cosmic string because this particular case permits a very interesting interpretation. The results of this paper being combined with the results of Refs. 9-14 give us the possibility to say that in some sense a conical singularity behaves like a mirror in which physical systems see the reflection of their electromagnetic structure.

However, the method used in this paper enables us to investigate some more general cases. In particular, to consider the spacetime of multiple cosmic strings it is necessary to take the conformal factor in the form¹⁶

$$\Omega(x) = \sum_{i=1}^N 2(1 - b_i) \log \frac{r_i}{a}, \quad (20)$$

where b_i stands for the parameter which determines the angle deficit of the i th conical singularity, r_i is a conformal distance from the point x to the i th string.

Since the conformal factor is defined the subsequent calculations are straightforward. Nevertheless one comment is needed. For a point charge in the (2+1)-gravitational theory¹⁴ and for a simple straight current in the multistring space¹² we obtained a conformal principle of superposition according to which the contributions to the self-energy from different conical defects being expressed in terms of the Euclidean space conformally connected with the Riemannian surface add to each other. As it was explained in Refs. 14 and 12 it is not a superposition principle in the explicit sense of this word but it really simplifies the calculations on the multiconical spaces.

Now we see that even the simplest electromagnetic structure which is introduced by the moments (8) leads to the loss of this principle. This fact is a consequence of the nonlinear dependence of the regularized second derivative (13) on Ω and shows once again that selfaction is a global, or in the multistring case it is better to say collective effect.

4. Spaces with conical singularities are of great interest both from the pure theoretical point of view and because of their applications in the phenomena of very different spacetime scales.

In this paper we consider an explicitly solvable problem which enables us to establish some new details in the topological interaction of a cosmic string with a physical system.

The primary goal of our paper was to show that the topological interaction which is induced by the global conical structure of a string spacetime is much more nontrivial and much less studied phenomenon than it seems from the first glance. We showed that a conical singularity has a lot of faces and its interaction with a physical system depends not on the integral properties of the system only but on its internal structure and more complicated is the system more nontrivial will be its interaction with a cosmic string.

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