Baryon Asymmetry of the Universe and the Electroweak Phase Transition

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ABSTRACT

The present baryon asymmetry of the universe has finally been determined at the finite temperature electroweak phase transition. The strength of this transition plays a crucial role. The effective action is presented to higher orders, including wave function correction factors and the full g^4, λ^2 effective potential. An upper bound for the Higgs mass $m_H \approx 70$ GeV is concluded for the reliability of the perturbative approach. The finite temperature electroweak phase transition is studied on the lattice and the most important results of Monte-Carlo simulations are collected.

1 Introduction

In the early universe at high temperatures the electroweak gauge symmetry has been restored. Only many years after the first studies of this symmetry restoration was realized that in the standard model the rates of anomalous baryon- and lepton-number violating processes are unsuppressed at high temperatures. In particular it is conceivable that the observed baryon asymmetry can be understood within the standard model. Clearly, the study of a "electroweak baryogenesis" requires a detailed knowledge of the transition from the symmetric to the broken phase. In order to provide the observed baryon asymmetry, one needs a departure from thermal equilibrium, thus a sufficiently strong first order phase transition via bubble nucleation.

Figure 1: Picture of the effective potential for different temperatures as a function of the Higgs field.

One basic ingredient in understanding this phenomenon is $V_{eff}(\varphi, T)$, the temperature dependent effective potential, which is shown in Fig. 1 as a function

of the background field φ for different temperatures T . At high temperatures the potential has only one global minimum at $\varphi = 0$, the symmetry is restored. As the temperature decreases, a second local minimum develops at $\varphi \neq 0$ which, at a critical temperature T_c , is degenerate with the minimum at $\varphi = 0$. For temperatures $T < T_c$ the local minimum at $\varphi = 0$ becomes metastable, and a barrier appears between the two minima. The phase transition is first order, since the change in the expectation value φ is discontinuous. The physical picture at temperatures below T_c is the decay of the energetically unfavoured false vacuum. Thermal fluctuations produce bubbles of true vacuum (c.f. Fig 2). If the bubble is too small, the gain in volume energy is smaller than the loss in surface energy, the bubble shrinks. For large enough bubbles it is energetically favourable to grow, converting the false vacuum to true vacuum. At a given barrier temperature $T_b < T_c$ the minimum at the symmetric state disappears.

Figure 2: Picture of the bubble nucleation in the false vacuum. Subcritical bubbles shrink, supercritical bubbles grow.

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In order to understand this rather involved process, the electroweak phase transition, intensive theo-

retical work has been devoted to clarify details of $V_{eff}(\varphi, T)$ in the standard model.

In Sect. 2 I would like to show that the most complete calculation to the finite temperature effective action (which contains wave function corrections [2], and potential terms upto g^4, λ^2 [3]) suggests that the perturbative approach breaks down around 70 GeV Higgs boson masses. A sign of this breakdown is given by the inclusion of the wave function correction factor. This will lead to large correction to the nucleation measure, a quantity, which is sensitive to a infrared regulator (e.g. magnetic mass [4]). The most straightforward way to see this breakdown is to determine higher order corrections to different physical quantities [3, 5]. One can calculate latent heat or surface tension from the finite temperature effective potential. These quantities for $m_H > 70$ GeV have $O(100\%)$ corrections compared to the lower order results.

Sect. 3 presents the results of the different lattice approaches including three-dimensional and four-dimensional simulations [6]–[11]. The perturbative results are compared with those of obtained in the four-dimensional Monte-Carlo studies [12]. At $m_H \approx 18$ GeV even the one-loop effective potential gives a reliable answer, however, for $m_H \approx 49$ GeV the two-loop result is needed to obtain an agreement.

2 Perturbation theory

Consider the $SU(2)$ -Higgs model at finite T , described by the lagrangian $\mathcal{L} = W_{\mu\nu}^a W_{\mu\nu}^a/4 + (D_\mu \Phi)^\dagger D_\mu \Phi + V_0(\varphi^2)$, where $V_0(\varphi^2) = m^2 \varphi^2/2 + \mu \lambda \varphi^4/4$, $\varphi^2 = 2\Phi^\dagger \Phi$. D_μ and $W_{\mu\nu}^a$ are the covariant derivative and the Yang-Mills field strength, respectively. In this section Landau gauge is used and the effects of the three generations of fermions ($m_t = f_t v/\sqrt{2}$) have been included.

To get the effective action $\Gamma_\beta[\Phi]$ at finite temperature a systematic expansion is needed where in all propagators the tree-level masses are replaced by one-loop plasma masses to order g^2 and λ .

At one-loop order this improved perturbation theory yields the effective potential to order $g^3, \lambda^3/2$,

$$V_{eff}(\varphi^2, T) = \frac{1}{2} \left(\frac{3g^2}{16} + \frac{\lambda}{2} + \frac{1}{4} f_t^2 \right) (T^2 - T_b^2) \varphi^2 + \frac{\lambda}{4} \varphi^4 - (3m_L^3 + 6m_T^3 + m_\varphi^3 + 3m_\chi^3) \frac{T}{12\pi}, \quad (1)$$

which is equivalent to the result of the ring summation. Here $m_L^2 = 11g^2 T^2/6 + g^2 \varphi^2/4$, $m_T^2 = g^2 \varphi^2/4$, $m_\varphi^2 = (3g^2/16 + \lambda/2 + f_t^2/4)(T^2 - T_b^2) + 3\lambda \varphi^2$, $m_\chi^2 = (3g^2/16 + \lambda/2 + f_t^2/4)(T^2 - T_b^2) + \lambda \varphi^2$ and $T_b^2 = (16\lambda v^2)/(3g^2 + 8\lambda + 4f_t^2)$.

The strength of the electroweak phase transition is rather sensitive to the nonperturbative magnetic mass of the gauge bosons. In Landau gauge the one-loop gap equations yield $m_T = g^2 T/(3\pi)$ at $\varphi = 0$. In order to estimate its effect we will replace [4] the previous

definition of m_T by $m_T^2 = \gamma^2 g^4 T^2/(9\pi^2) + g^2 \varphi^2/4$ and compute sensitive quantities for different values of γ .

V_{eff} has degenerate local minima at $\varphi = 0$ and $\varphi = \varphi_c > 0$ at a critical temperature. The evaluation of the transition rate requires knowledge of a stationary point of the free energy which interpolates between the two local minima. The effective action can be expanded in powers of derivatives. For time-independent fields one has $T \cdot \Gamma_\beta[\Phi] = \int d^3x [V_{eff}(\varphi^2, T) + (\delta_{IJ} + Z_{IJ}(\Phi, T)) \vec{\nabla} \varphi_I \vec{\nabla} \varphi_J/2 + \dots]$. Using the inverse scalar propagator in the homogeneous background field Φ one gets on the one-loop level $Z_{IJ}(\Phi, T) = Z_\varphi(\varphi^2, T) P_{IJ}^\varphi + Z_\chi(\varphi^2, T) P_{IJ}^\chi$, where $Z_\varphi = T[\lambda \bar{m}^2(3/m_\varphi^3 + 1/m_\chi^3)/4 - 2g^2/(m_\chi + m_T) + g^2 m^2(1/m_L^3 + 10/m_T^3)/16]/(4\pi)$ and $Z_\chi = T[2\lambda \bar{m}^2/(m_\varphi + m_\chi)^3 - 2g^2/(m_\chi + m_T) - g^2/(m_\varphi + m_T)]/(6\pi)$ with $P_{IJ}^\varphi = \varphi_I \varphi_J/\varphi^2$, $P_{IJ}^\chi = \delta_{IJ} - \varphi_I \varphi_J/\varphi^2$, $\varphi^2 = \sum_{I=1}^4 \varphi_I \varphi_I$, $\bar{m}^2 = \lambda \varphi^2$ and $m^2 = g^2 \varphi^2/4$.

Figure 3: The one-loop wave function correction δ_Z as a function of m_H for different values of γ .

Note, that despite the divergence of Z_φ at $\varphi \sim 0$ the correction to the surface tension (σ) is finite; $\sigma = \int_0^{\varphi_c} d\varphi \sqrt{2(1 + Z_\varphi(\varphi^2, T_c)) V_{eff}(\varphi^2, T_c)}$. A measure for the size of the loop correction to the Z -factor is (Fig. 3) the ratio $\delta_Z = \int d^3x Z_\varphi(\vec{\nabla} \bar{\varphi})^2 / \int d^3x (\vec{\nabla} \bar{\varphi})^2$, where $\bar{\varphi}$ is the saddle point solution at the nucleation temperature calculated from (1). For $\gamma = 0$ the perturbative expansion becomes unreliable at $m_H \sim 80$ GeV. The magnetic mass as an infrared cutoff ($\gamma = 1, 2$) could improve the convergence.

The next step in the series is the calculation of the effective potential in higher orders. This has been done in order g^4, λ in [5], in order g^4, λ^2 in [3], and two-loop results of the three-dimensional theory is also known [13].

Here some results of the order g^4, λ^2 of [3] are collected, with a particular emphasis on the breakdown of the perturbative approach. The full result of order g^4, λ^2 predicts a stronger first order phase transition than the lower order results. The potential is plotted using different approximations for the $SU(2)$ Higgs-model

Figure 4: Different approximations of $V_{eff}(\varphi, T_c)$ for $m_H = 70$ GeV.

at their respective T_c (Fig. 4). The expectation value of the Higgs field does not change dramatically, but there is an order of magnitude difference between the heights of the barrier.

The surface tension, $\sigma = \int_0^{\varphi^+} d\varphi \sqrt{2V(\varphi, T_c)}$, may be seen as a measure of the strength of the phase transition (Fig. 5). Again, for large m_H the perturbative approach breaks down.

The full standard model calculation with zero temperature renormalization has also been done. The qualitative behaviour of the potential and the conclusions are essentially the same as for the $SU(2)$ -Higgs model.

In order to improve the behaviour of the perturbative results an interesting result is obtained by [12]. It applies the Clausius-Clapeyron equation to the electroweak phase transition, which gives a simple relation between latent heat and jump of the order parameter. In leading order

$$\Delta Q = -\frac{1}{2} m_H^2 \Delta \rho (1 + \mathcal{O}(g^2, \lambda)) \quad (2)$$

The same expression has been derived in the three-dimensional theory, too [13]. This important relation will be checked in the next section.

The usually used ring-summation has problems, particularly in the neighbourhood of the symmetric phase. An elegant treatment to this problem has been presented in [12]. One can use different resummation procedures in the symmetric phase and in the broken phase. A two-loop analysis has been given for a gauge invariant effective potential, which gives a qualitatively similar description as the above discussed two-loop analysis, however, the convergence of perturbation theory is improved.

The results of this section are from Ref. [3, 2, 12], where the details of the calculations can also be found.

Figure 5: The surface tension calculated from the different potentials as a function of m_H .

3 Lattice Monte-Carlo simulations

As we have seen in the previous section in the realistic Higgs mass range ($m_H > 63$ GeV) the perturbative approach predicts $\mathcal{O}(100\%)$ corrections [4, 3] for the relevant quantities (e.g. interface tension or latent heat). Some of the nonperturbative estimates (e.g. magnetic mass [4]) suggest a weaker, others (e.g. vacuum-condensate [14]) a stronger first order phase transition than the perturbative approach. A way to solve the problem seems to be the use of lattice simulations.

Since fermions always have nonzero Matsubara frequencies, the perturbative treatment of these, at high temperatures very massive, modes could be satisfactory. Thus, the starting point of the lattice analyses is the $SU(2)$ -Higgs model, which contains the essential features of the standard model of electroweak interactions.

The 4-dimensional $SU(2)$ -Higgs model at finite T is defined by the following action

$$S = \int_0^\beta d\tau \int d^3x \left[\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + (D_\mu \phi)^\dagger (D_\mu \phi) - \frac{1}{2} m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \right], \quad (3)$$

where D_μ and $F_{\mu\nu}^a$ are the usual covariant derivative and the Yang-Mills field strength, respectively. $\beta = 1/T$, and the τ integration is over periodic bosonic fields. This model has been studied on the lattice by ref [6, 7, 8, 9]

The origin of the perturbative infrared problems is the appearance of zero Matsubara modes in the bosonic sector. Therefore, similarly to the case of the fermion fields with nonzero Matsubara frequencies one can integrate out all the massive, non-static bosonic modes of (3) at the one-loop level. Since these modes are heavy in the high temperature limit, the perturbative treatment of them could be well-founded. This dimensional reduction gives an effective 3-dimensional gauge-Higgs

model, where in addition, also an isovector field (A_0^a), the fourth component of the gauge fields, is present

$$S = \int d^3x \left[\frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{2} (D_i A_0)^a (D_i A_0)^a + (D_i \phi)^\dagger (D_i \phi) + \frac{1}{2} m_D^2 A_0^a A_0^a + \frac{1}{4} \lambda_A (A_0^a A_0^a)^2 + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2 + \frac{1}{2} h_3 A_0^a A_0^a \phi^\dagger \phi \right]. \quad (4)$$

Here all the 3-dimensional couplings g_3^2 , λ_3 , λ_A and h_3 and masses m_D , m_3 can be expressed in terms of the 4-dimensional couplings and the temperature [13, 15]. This model has been studied on the lattice by ref [11].

The next step is the elimination of the gauge degrees of freedom. One obtains a 3-dimensional $\mathcal{O}(4)$ ferromagnet model with cubic and quartic terms in the action.

$$S = \int d^3x \left[(D_i \phi)^\dagger (D_i \phi) + \bar{m}_3^2 \phi^\dagger \phi + \bar{\lambda}_3 (\phi^\dagger \phi)^2 - 2\bar{g}_3^2 (\bar{m}_T^2 + \phi^\dagger \phi)^{3/2} - \bar{g}_3^2 (\bar{m}_E^2 + \phi^\dagger \phi)^{3/2} \right]. \quad (5)$$

The parameters of this action can be similarly expressed in terms of the 4-dimensional couplings and the temperature as in the previous case [10]. The infrared stability is ensured by including magnetic ($\bar{m}_T$) and electric ($\bar{m}_E$) screening mass terms. Since the first of them has a nonperturbative origin, the approximation contains an uncertainty connected with the magnetic mass. This model has been studied on the lattice by Ref. [10]. Their choice of $\bar{m}_T$ has been suggested by the solution of the one-loop gap-equations of the $SU(2)$ -Higgs theory at finite temperature [4].

The results obtained by the different groups are briefly summarized. First the results of the 3-dimensional $\mathcal{O}(4)$ ferromagnet model, then those of the 3-dimensional gauge-Higgs model are presented. Finally the analyses based on the 4-dimensional $SU(2)$ -Higgs model are considered.

After discretizing the action (5) and performing the mean field analysis of this effective scalar model Ref. [10] presented the results of their Monte-Carlo simulations. The typical lattice sizes and statistics for a given lattice size were $8^3 - 18^3$ and 10^6 sweeps, respectively. The parameters used in this work correspond to the physical values of the W mass and vacuum expectation value of the Higgs field ($m_W = 80$ GeV and $v = 246$ GeV), with a Higgs mass of ≈ 35 GeV.

The simulations have shown a considerably weaker first order phase transition than the mean-field analysis. For the critical temperature (T_c), jump in the order parameter at the critical temperature (φ_c), latent heat ($\Delta\epsilon$) and interface tension (σ) the simulations have given $T_c = 114.6(36)$, $\varphi_c/T_c = 0.68(4)$, $\Delta\epsilon/T_c^4 = 0.122(8)$ and $\sigma/T_c^3 \approx 6.4 \cdot 10^{-4}$, respectively. The corresponding mean field results are $T_c = 99.6$, $\varphi_c/T_c = 1.3$, $\Delta\epsilon/T_c^4 = 0.262$ and $\sigma/T_c^3 = 0.024$, respectively.

In [13] the one- and two-loop effective potential is constructed using the dimensional reduction idea for the three-dimensional gauge-Higgs model (cf. eq. 4). With the help of the renormalisation group leading logarithms have been summed. The lattice results have been presented in [11]. The used W mass and gauge coupling were $m_W = 80.6$ GeV and $g = 2/3$, respectively. The runs have been done on lattices of sizes $8^3 - 32^3$. For $m_H = 35$ GeV the critical temperatures obtained on the lattice and in the perturbative approach are $T_c = 85$ GeV and $T_c = 95$ GeV, respectively; for $m_H = 80$ GeV the values are $T_c = 162.1(26)$ GeV and $T_c = 173.3$ GeV, respectively. The jump in the order parameter is $\varphi_c/T_c = 0.73(4)$ obtained by the lattice simulation and $\varphi_c/T_c = 0.47$ in perturbation theory. The informations obtained from the above lattice studies are in qualitative agreement with the vacuum-condensate picture [14].

In [6] the typical lattice sizes were $2 \cdot 16^3 - 2 \cdot 36^3$ and approximately 7000 measurements have been done for each set. The simulations have been performed for $m_W = 80$ GeV, $m_H \approx m_W$ and $g^2 = 0.5$. The lattice results show a clear first order phase transition. The phase transition is stronger than the one obtained in the one-loop perturbative approach. However, the lattice was presumably too small to analyse the weak first order phase transition.

A 4-dimensional, finite T analysis has been done by [7, 8, 9] too. The largest lattice used was $3 \cdot 48^2 \cdot 384$ with $\mathcal{O}(10^5)$ updating. The simulations have been performed for three set of parameters ($m_W = 80$ GeV): $m_H \approx 18$ GeV, $g_{bare}^2 = 0.5$ (low); $m_H \approx 35$ GeV, $g_{bare}^2 = 0.5$ (middle); $m_H \approx 49$ GeV, $g_{bare}^2 = 0.5$ (high) point. A moderate renormalisation of the gauge-coupling, $g_R^{2,low} = 0.5476(90)$ and $g_R^{2,low} = 0.5781(95)$, has been found. The simulations of our middle point has not been finished yet, however, the result of the surface tension calculations are available [9].

The latent heat has been calculated from the discontinuity of the energy density ($\Delta\epsilon$). The necessary partial derivatives along the lines of constant physics have been determined by use of the one-loop renormalisation group equations, whereas the critical points have been given by the inspection of the gauge-invariant effective potential [12]. The obtained values are: $(\Delta\epsilon/T_c^4)^{low} = 1.68(17)$ and $(\Delta\epsilon/T_c^4)^{high} = 0.125(19)$. The interface tension has been determined using the two-coupling method. The obtained results are: $(\sigma/T_c^3)^{low} = 0.84(16)$ $(\sigma/T_c^3)^{middle} = 0.065(10)$ and $(\sigma/T_c^3)^{high} = 0.008(2)$. Another possibility [9] is to measure the interface tension by use of the finite volume tunneling between the symmetric and the broken phase. This phenomenon leads to the splitting of the ground state, which gives the interface tension by $E_0 = C \exp(-L_t L_{xy} \sigma)$. Measuring correlation lengths of $\mathcal{O}(250)$ the final result reads: $(\sigma/T_c^3)^{middle} = 0.053(5)$.

Figure 6: Comparison of the perturbative results and the lattice data for $m_H \approx 18$ GeV and $m_H \approx 49$ GeV.

The comparison for φ_c/T_c , $\Delta Q/T_c^4$, σ/T_c^3 and T_c/m_H is shown in Fig. 6 ($m_H \approx 18, 49$ GeV). The comparison for $m_H \approx 35$ GeV is not presented, since the $T = 0$ simulations, which gives the renormalized parameters, are not available yet. The dots with error bars represent the perturbative result at one-loop (g^3) and at two-loop (g^4) level. For each quantity the dashed lines show the region allowed by the statistical error, whereas the dotted lines include the systematics, too. The values of g and m_H/m_W are those of [8].

For $m_H \approx 18$ GeV (cf. Fig. 7) both the one-loop and the two-loop results are in good agreement with the lattice data. For $m_H \approx 49$ GeV the two-loop results agree definitely better with the Monte Carlo data, except for the surface tension. The two plots may be interpreted in the following way. For small Higgs boson masses the perturbative approach is in very good shape, already the one-loop approximation gives a reliable result. As m_H grows, the higher order contributions become more and more important, yet a two-loop calculation is still satisfactory for $m_H \approx 49$ GeV. In this range of parameters the non-perturbative features of the symmetric phase are not important enough to destroy the perturbative picture.

Another interesting possibility is to test the Clausius-Clapeyron equation, which relates the latent heat and the jump in the order parameter. The comparison by use of the lattice data [12] is presented in the table. The first line of it contains ΔQ obtained by the Clausius-Clapeyron equation and from the lattice results for $\Delta\rho$ and m_H^2 . The second line contains the latent heat determined by lattice simulations. The numbers in the parentheses denote the statistical errors. The values for $m_H \approx 49$ GeV agree within one standard deviation. For $m_H \approx 18$ GeV there is a small discrepancy.

4 Conclusions, outlook

Perturbative studies show, that the electroweak phase transition can not be described above $m_H = 70$ GeV by perturbative methods. One systematic way to solve the problem is to perform lattice simulations. There is an intensive work to do Monte-Carlo simulations, in both three and four dimensions.

The good quantitative agreement between the perturbative and lattice results found for Higgs mass values

	$m_H \approx 18$ GeV	$m_H \approx 49$ GeV
ΔQ from C-C eq.	.0236(14)	.00171(15)
lattice result	.0194(15)	.00151(12)

Table 1: Comparison of the latent heat in lattice units obtained by using the Clausius-Clapeyron equation and lattice simulations. The data are from ref. [8].

$m_H \approx 18$ GeV and $m_H \approx 49$ GeV is interpreted as evidence for the correctness of the present understanding of the electroweak phase transition. Non-perturbative effects present in the symmetric phase are neglected by perturbation theory, but they should contribute to the lattice results. These effects should be more pronounced for larger Higgs boson masses. Thus, the main problem for the future is to perform lattice simulations in the physically relevant parameter region ($m_H > 65$ GeV) and to understand the above mentioned non-perturbative effects. This understanding is essential for a clear answer concerning the observed baryon asymmetry of the universe.

References

- [1] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B155 (1985) 36.
- [2] D. Bödeker, W. Buchmüller, Z. Fodor and T. Helbig, Nucl. Phys. B423 (1994) 171.
- [3] Z. Fodor, A. Hebecker, Nucl. Phys. B432 (1994) 127.
- [4] W. Buchmüller, Z. Fodor, T. Helbig and D. Waliser, Ann. Phys. (NY) 234 (1994) 260.
- [5] P. Arnold and O. Espinosa, Phys. Rev. D47 (1993) 3546.
- [6] B. Bunk, E. M. Ilgenfritz, J. Kripfganz, A. Schiller, Nucl. Phys. B403 (1993) 453.
- [7] F. Csikor, Z. Fodor, J. Hein, K. Jansen, A. Jaster, I. Montvay, Phys. Lett. B334 (1994) 405.
- [8] Z. Fodor, J. Hein, K. Jansen, A. Jaster, I. Montvay, Nucl. Phys. B439 (1995) 147.
- [9] F. Csikor, Z. Fodor, J. Hein and J. Heitger, Phys. Lett. B357 (1995) 156.
- [10] F. Karsch, T. Neuhaus, A. Patkós Nucl. Phys. B441 (1995) 629.
- [11] K. Kajantie, K. Rummukainen, M.E. Shaposhnikov, Nucl. Phys. B407 (1993) 356; K. Farakos, K. Kajantie, K. Rummukainen, M.E. Shaposhnikov, Phys. Lett. B336 (1994) 494.
- [12] W. Buchmüller, Z. Fodor and A. Hebecker, Phys. Lett. B331 (1994) 131; Nucl. Phys. B447 (1995) 317.
- [13] K. Farakos, K. Kajantie, K. Rummukainen and M. Shaposhnikov, Nucl. Phys. B442 (1995) 317.
- [14] M. Shaposhnikov, Phys. Lett. B316 (1993) 112.
- [15] A. Jakovác, K. Kajantie, A. Patkós, Phys. Rev. D49 (1994) 6810.