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GENERALIZED VENEZIANO MODEL WITH ISOSPIN

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A B S T R A C T

A generalization of the Veneziano model to the N point function was proposed which was crossing symmetric and bootstrap consistent in treating all particles as bound states of others. However, the model was so far limited to only neutral mesons. Here we propose a very simple general method for incorporating isospin into the model which preserves all the desirable properties and gives no unwanted states of high isospin. Previously known results for the four and five point functions appear as special cases of our solution.

A generalisation of the Veneziano model to the N point function has been proposed ¹⁾⁻⁵⁾ which could conceivably form the basis for a new phenomenology of multiple production processes, and even perhaps of a new dynamical bootstrap model. The model is crossing symmetric and incorporates resonance poles on linear trajectories as well as Regge asymptotic behaviour. In addition, it is consistent with the bootstrap hypothesis in treating all particles as bound states of others. However, the model is so far limited to a hypothetical world with only normal parity, neutral mesons lying on trajectories with negative intercepts. Before applications are possible, one should extend it to include at least :

- A - isospin and strangeness,
- B - trajectories with positive intercepts, such as the ρ and ω ,
- C - abnormal parity trajectories.

In this note we address ourselves to the problem A. Until now, the isospin structure has been worked out in a few particular cases only ⁶⁾⁻⁸⁾. Here, we present a very simple general solution which preserves all the desirable property of the neutral meson model and give previously known results as special cases. We have in mind the obvious goal of writing down the amplitude for the realistic case where the external lines are pseudoscalars (π , K...) and the internal lines are such high lying trajectories as the ρ and ω . For this reason, we shall use the symbols π , K..., etc. to label the isospin, strangeness and G parity of the external lines, and ρ , ω etc. to label the quantum numbers of the internal trajectories. To attain our goal, however, would require a general solution also for both B and C which has not been achieved ⁹⁾.

The N point function in the generalized Veneziano model for spinless neutral particles take the form ²⁾

$$A_N = \sum B_N(1, 2, 3, \dots, N) \quad (1)$$

where each term in the sum corresponds to a distinct cyclic ordering of the particles $1, \dots, N$. We do not need here the detailed form of the functions B_N occurring in (1). Relevant for us are only two properties of these functions :

- (i) - Factorization property. For instance, for $N = 6$, the residue at the pole $\alpha_{123} = 0$ corresponding to the intermediate I is given by

$$\text{Res. } B_6(1, 2, 3, 4, 5, 6) = B_4(1, 2, 3, I) B_4(I, 4, 5, 6) \quad (2)$$

$\alpha_{123} = 0$

- (ii) - Cyclic symmetry. B_N is invariant under a cyclic permutation of the external lines.

We seek to introduce isospin in a way which will ensure :

- 1 - factorization so as to preserve bootstrap consistency ;
- 2 - cyclic symmetry for identical external particles ; this guarantees crossing symmetry although it is not demanded by it ;
- 3 - absence of poles with isospin larger than 1. This is dictated by present experimental evidence.

The solution takes the form of simple isospin factors multiplying the terms in the sum (1), with each term and hence each cyclic ordering of the external lines, corresponding to a different isospin factor. For convenience, we distinguish the following cases.

a - Pions ($I = 1$) only as external lines

Give each external particle i the isospin label a_i ($a_i = 1, 2, 3$) and let τ_a ($a = 1, 2, 3$) denote the 2×2 Pauli matrices. Then the desired isospin factor for the term corresponding to the ordering $(1, 2, \dots, N)$ is simply the trace

$$\frac{1}{2} \text{Tr} (\tau_{a_1} \tau_{a_2} \tau_{a_3} \dots \tau_{a_N}) \quad (3)$$

Property 2 is immediate from the properties of traces. Property 3 follows since the product of any number of 2×2 matrices is again a 2×2 matrix and hence can represent only a combination of isospin 0 and 1. To see Property 1, we need only note the following identity :

$$\begin{aligned} & \frac{1}{2} \text{Tr} (\tau_{a_1} \tau_{a_2} \cdots \tau_{a_N}) \\ &= \left[\frac{1}{2} \text{Tr} (\tau_{a_1} \tau_{a_2} \cdots \tau_{a_M}) \right] \left[\frac{1}{2} \text{Tr} (\tau_{a_{M+1}} \tau_{a_{M+2}} \cdots \tau_{a_N}) \right] \\ &+ \sum_{a_I} \left[\frac{1}{2} \text{Tr} (\tau_{a_1} \tau_{a_2} \cdots \tau_{a_M} \tau_{a_I}) \right] \left[\frac{1}{2} \text{Tr} (\tau_{a_I} \tau_{a_{M+1}} \cdots \tau_{a_N}) \right]. \end{aligned} \quad (4)$$

where τ_{a_I} are again Pauli matrices. The two terms in (4) correspond respectively to intermediate states of isospin 0 and 1.

For the special case $N = 4$, the above prescription reduces to the form given by Lovelace⁶⁾ for $\pi \pi$ scattering. For $N = 6$, it can be shown to be identical to a form obtained earlier by Bardakçi¹⁰⁾. In each case the solution was shown to be unique.

As an immediate consequence of the isospin factor (3), we obtain the generalized exchange-degeneracy between the isospin 0 and 1 trajectories coupled to two pions (the ρ - f_0 degeneracy). This follows simply from the identity

$$\tau_a \tau_b = \delta_{ab} + i \epsilon_{abx} \tau_x \quad (5)$$

which implies that any two-body pole of a B_N occurs in both isospin states. However, when the B_N 's are summed over, as in (1), the $I = 0$ and 1 poles will occur on opposite signature trajectories because of the opposite symmetry of the two terms in (5) under interchange of a and b . Similarly there is also exchange degeneracy between isospin 0 and 1 trajectories of opposite signatures coupled to three or more external pions (e.g., the $\omega - A_2$ degeneracy).

b - Pions ($I = 1$) and kaons ($I = \frac{1}{2}$) as external lines

The rules in this case are most conveniently stated by introducing the 3×3 λ matrices of $SU(3)$. We use these matrices, however, only as a device to ensure the correct isospin structure without making any $SU(3)$ assumptions of mass degeneracy or coupling constant relations. To each external line we associate a λ matrix as follows :

$$\begin{aligned} \pi_a &\sim \begin{pmatrix} \tau_a & 0 \\ 0 & 0 \end{pmatrix} & \kappa &\sim \begin{pmatrix} 0 & \kappa \\ 0 & 0 \end{pmatrix} \\ \bar{\kappa} &\sim \begin{pmatrix} 0 & 0 \\ \bar{\kappa} & 0 \end{pmatrix} \end{aligned} \quad (6)$$

Then the isospin factor corresponding to the ordering $(1, 2, 3, \dots, N)$ is simply

$$\text{Tr}(\lambda_{a_1} \lambda_{a_2} \dots \lambda_{a_N}) \quad (7)$$

Equation (7) obviously satisfies all the conditions 1, 2 and 3 and in addition forbids poles with strangeness greater than 1.

Consider the following specific examples.:

- (i) - $\bar{K}_1, \bar{K}_2, \pi_3, \pi_4$ as external lines⁷⁾. Our rule (7) gives isospin factors $\bar{K}_2 \tau_{a_3} \tau_{a_4} K_1$ and $\bar{K}_2 \tau_{a_4} \tau_{a_3} K_1$ for respectively the orderings $(2, 3, 4, 1)$ and $(2, 4, 3, 1)$ but zero for the ordering $(2, 4, 1, 3)$. The absence of a Veneziano term corresponding to the ordering $(2, 4, 1, 3)$ implies exchange degeneracy of $I = \frac{1}{2}$ trajectories, $[K^*(890) \text{ and } K^*(1400)]$.
- (ii) - $\bar{K}_1, \bar{K}_2, K_3, \bar{K}_4$ as external lines⁷⁾. Our rule gives isospin factors, $(\bar{K}_2 K_3)(\bar{K}_4 K_1)$ and $(\bar{K}_2 K_1)(\bar{K}_4 K_3)$ respectively for the orderings $(2, 3, 4, 1)$ and $(2, 1, 4, 3)$ but zero for the ordering $(2, 1, 3, 4)$. We note that in the term corresponding to

the ordering (2,3,4,1), particles 2 and 3 must be in an isospin 0 state, whereas particles 1 and 2 can be in an isospin 0 or 1. Thus in the variable α_{12} one has degenerate isospin $0(\omega)$ and $1(\rho)$ trajectories, but in α_{23} there are only isospin $0(\emptyset)$ trajectories. If one identifies the two isospin 0 trajectories with different particles, not necessarily degenerate, then the couplings correspond to those of a perfect nonet.

(iii) - $K_1, \bar{K}_2, \pi_3, \pi_4, \pi_5$ as external particles ⁸⁾. The only non-zero isospin factors are $\bar{K}_2 \tau_{a3} \tau_{a4} \tau_{a5} K_1$ and those obtained from this by permutations of 3, 4 and 5. The absence of other terms implies as in (i) and (ii) the exchange degeneracy of $K^*(890) - K^*(1400)$ and of $\rho - \omega$.

(iv) - $K_1, \bar{K}_2, K_3, \bar{K}_4, \pi_5$ as external particles ⁸⁾. The only non-zero isospin factors are $(\bar{K}_2 K_3)(\bar{K}_4 \tau_{a5} K_1)$ and those obtained from this by permutations of the identical particles. One notes as in (ii) that for the term corresponding to $(\bar{K}_2 K_3)(\bar{K}_4 \tau_{a5} K_1)$, α_{23} has poles only in $I = 0(\emptyset)$, whereas α_{12} has poles in both $I = 0(\omega)$ and $I = 1(\rho)$. The same exchange degeneracies as in (i), (ii) and (iii) are implied.

c - I = 0 particles as external lines

$I = 0$ external particles can be included in two ways by introducing in the trace of Eq. (7) respectively the matrices

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} ; \quad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (8)$$

Isoscalar particles of the second type, as in the quark model, are not coupled to states with only isovector particles. We wish, however, to leave open the question of identifying these particles with η and X_0 .

The similarity of our solution to the quark picture ^{4),11)} is interesting.

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model, suggests a similar solution for B and C in the form of
ghost killing factors. The result is crossing symmetric and
gives as a special case the known solution for the five-point
function. Unfortunately, it leads in general to parity-dou-
bling of internal trajectories which we regard as unphysical.
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