CM-P00057834

Ref. Th. 919

Hodgins

A METHOD FOR INCLUDING RESONANCE PRODUCTION
IN A REGGEIZED MULTIPERIPHERAL MODEL

E. Plahte ^{*)} and R.G. Roberts
CERN - Geneva

A B S T R A C T

The phenomenological model of Chan et al. is generalized to include the formation of resonances between particles of low invariant mass. As well as removing an inconsistency of the model, resonance production successfully accounts for the pronounced backward peak in baryon longitudinal momentum distributions. The improved model has been applied to the reactions $\pi^+p \rightarrow p+(n-1)\pi$ at 8 GeV and $K^-p \rightarrow \Lambda+(n-1)\pi$ at 6 GeV with n varying from 3 to 6. Good qualitative agreement with the experimental two-particle effective mass distributions is obtained.

^{*)} On leave from Agricultural College of Norway, Vollebekk.

INTRODUCTION

Recently a phenomenological model was proposed by Chan, Koskiewicz and Allison ¹⁾ (referred to hereafter as CLA) which was able to describe many features of multiple particle production at intermediate and high energies. This model, in spite of its apparent crudeness, proved to be successful in explaining qualitatively single particle distributions for final states over a wide range of beam momentum and multiplicity. There was, nevertheless, an inconsistency in this achievement, namely that the model took no account of resonance production and at the same time attempted to describe reactions at energies where the formation of resonances is well known to dominate the cross-section. The aim of this paper is to rectify this deficiency by including the production of resonances within the model in a natural way.

The main feature of the CLA model is that it interpolates smoothly between the two cases of high and low energy production. The low energy region is supposed to be described simply by the statistical model ²⁾ while high energy behaviour of the amplitude is governed by the multi-Regge model ³⁾. The modulus of the amplitude $A^{(n)}$ corresponding to the process $A + B \rightarrow 1 + 2 + \dots + n$, shown in Fig. 1, can be written as a product of $(n-1)$ partial amplitudes $A(s_i, t_i)$.

That is,

$$|A^{(n)}| = \prod_{i=1}^{n-1} A_i(s_i, t_i) \quad , \quad (1)$$

where

$$s_i = (p_i + p_{i+1})^2 - (m_i + m_{i+1})^2 \quad (2)$$

The parametrization of the partial amplitude is

$$A_i(s_i, t_i) \propto \left(\frac{g_i s_i + c_2 a}{s_i + a} \right) \left(\frac{s_i + a}{a} \right)^{\alpha_i} \left(\frac{s_i + b_i}{b_i} \right)^{t_i} \quad (3)$$

Here g_i corresponds essentially to the coupling constant of the vertices which occur with each exchange of a Regge pole, c_2 is a two-body clustering constant, a defines the boundary between the "low" and "high" energy regions, α_i , t_i define the intercept and slope of the relevant Regge trajectory, and b_i is related to the slope of the residue exponent associated with the pole exchanged between particles i and $i + 1$.

The CLA model did not describe some longitudinal momentum distributions successfully, in particular it failed to account for the pronounced backward peak in these distributions. It is clear that such a discrepancy can easily be explained by the neglect of resonance production which effectively reduces the multiplicity of the final state. For example, in the reaction $\pi^+p \rightarrow \pi^+\pi^0p$ the $p\rho^+$ approximation to the final state has the effect of producing a more peripheral proton.

In order to develop a procedure for including resonance production, the following points should be kept in mind.

- i) : The very satisfactory results for simple particle distributions already achieved should not be worsened.
- ii) : The spirit of the model should not be lost. The outstanding feature of the model is its simplicity and the necessity of only a few parameters. Any modification of the amplitude should involve as few extra parameters as possible and avoid considerable complication of the model.
- iii) : The previous discrepancies in single particle distributions should be removed and all resulting mass distributions should be in satisfactory agreement with the data.

In Section 2, we describe a method which introduces resonance formation in a way which is immediately able to be extended to any reaction and which, we believe, satisfies the above requirements. The final Section presents the results obtained and includes some remarks concerning the comparison with experimental data.

2. -

PARAMETRIZATION OF THE AMPLITUDE

In order to account for the production of two-body resonances within the framework of the model proposed by CIA we make two assumptions. These have the effect of reducing the number of new parameters to a minimum.

- i) : A resonance is allowed to be formed only by pairs of adjacent particles in each graph to be considered.
- ii) : The probability for the formation of a particular resonance, say the $\rho(760)$, from any pair of adjacent particles whose quantum numbers are compatible with those of the resonance, is a constant times a Breit-Wigner function of the relevant effective mass.

A natural way to incorporate these ideas into the model is to replace the clustering constant c_2 occurring in each A_i amplitude by a new constant c_2' together with a weighted sum of Breit-Wigner functions for the resonances formed by particles i and $i + 1$,

$$c_2^2 \rightarrow c_2'^2 + \sum_r \gamma_r^2 BW_r^2(s_i) \quad , \quad (4)$$

where the constant γ_r^2 is a measure of the probability for the formation of the resonance r . From assumption (2), γ_r does not depend on the circumstances under which the resonance r is produced. In particular, for a given reaction γ_r is taken to be independent of the multiplicity of the final state. As we show below, the solution (4) corresponds to an incoherent addition of the relevant graphs for the reaction.

In order to explain our procedure more clearly, we consider a particular example, namely a four-body final state where the particles are in a definite order. The eight graphs which we want to consider are shown in Fig. 2.

The fifth graph, for example, represents the situation where particles 1 and 2 form a two-body cluster (with the associated formation of one or more resonances as a final state interaction) ; also

particles 2 and 3 form a cluster in the same sense. This particular graph in fact can be considered as a quasi-two-body final state where the two "particles" are labelled (123) and 4. The exchanged Regge pole is coupled to the (123) blob rather than to any single member of that cluster. The energy variable for the final state is clearly $s_{(123)4}$ where

$$S_{(123)4} = [(p_1 + p_2 + p_3) + p_4]^2 - [\sqrt{(p_1 + p_2 + p_3)^2 + m_4^2}]^2.$$

We emphasize this point for the reason that in the CLA model the quantity s_3 [see Eq. (2)] would be the corresponding energy variable. It would prove extremely tedious to evaluate all these "clustering subenergies" in practice. For example, the graphs of Fig. 2 require 11 different such quantities depending on the kinematic configuration of the final four particles.

A more important reason for choosing a different procedure is the fact that there is no clear-cut and unambiguous distinction between the eight graphs in Fig. 2. Also since it will be meaningless to take into account for example graph 8 in the extreme high energy region, phase-space would have to be divided into regions corresponding to the decomposition of the graph in Fig. 1 into a sum of graphs as in Fig. 2.

The great advantage of the CLA model is precisely to avoid this complex situation by allowing the same amplitude to be valid for the whole physical region. We therefore propose an alternative procedure which defines a set of "approximate" clustering subenergies. These have the advantage of being few in number (only three are required for all graphs in Fig. 2) and also interpolate between the correct subenergies and approximate to them in the asymptotic cases of each $s_i \rightarrow 0$ or $s_i \rightarrow \infty$ according to the configuration of the final state.

First we define the following sequences of four-vectors and their corresponding "masses" :

$$\left. \begin{aligned} r_1 &= p_1, \\ r_2 &= p_2 + \frac{a}{s_1+a} r_1, \\ r_3 &= p_3 + \frac{a}{s_2+a} r_2, \\ &\vdots \\ r_n &= p_n + \frac{a}{s_{n-1}+a} r_{n-1}, \end{aligned} \quad \begin{aligned} v_1 &= m_1, \\ v_2 &= m_2 + \frac{a}{s_1+a} v_1, \\ v_3 &= m_3 + \frac{a}{s_2+a} v_2, \\ &\vdots \\ v_n &= m_n + \frac{a}{s_{n-1}+a} v_{n-1}, \end{aligned} \right\} \quad (5)$$

$$\left. \begin{aligned} q_n &= p_n, \\ q_{n-1} &= p_{n-1} + \frac{a}{s_{n-1}+a} q_n, \\ &\vdots \\ q_1 &= p_1 + \frac{a}{s_1+a} q_2, \end{aligned} \quad \begin{aligned} \mu_n &= m_n, \\ \mu_{n-1} &= m_{n-1} + \frac{a}{s_{n-1}+a} \mu_n, \\ &\vdots \\ \mu_1 &= m_1 + \frac{a}{s_1+a} \mu_2. \end{aligned} \right\} \quad (6)$$

The approximate clustering subenergy $s_i^{(c)}$ is then defined by

$$s_i^{(c)} = (r_i + q_{i+1})^2 - (v_i + \mu_{i+1})^2, \quad i=1, 2, \dots, n-1. \quad (7)$$

Let us consider the properties of the $s_i^{(c)}$ in the limiting case where each s_i is very large or else very small:

- 1) - When particles i and $i+1$ are both in the same cluster, $s_i^{(c)}$ tends to the invariant "kinetic mass" of the cluster. E.g., in the above example, when s_1 and s_2 are very small but s_3 is large, $s_1^{(c)} \approx s_2^{(c)} \approx (p_1 + p_2 + p_3)^2 - (m_1 + m_2 + m_3)^2$.

- 2) - When i and $i+1$ are in separate clusters, $s_i^{(c)}$ tends to the invariant "kinetic mass" of the two-cluster system, e.g., in the same example $s_3^{(c)} \approx s_{(123)4}$. The term "cluster" is here taken in the widest sense - meaning simply a single particle or a group of particles which are adjacent to each other on the graph of Fig. 1 and whose s_i are small.
- 3) - For intermediate values of the s_i , $s_i^{(c)}$ interpolates smoothly (in the sense of CLA) between the two limiting cases.

Although entirely phenomenological, we believe the $s_i^{(c)}$ give a good parametrization of that part of the amplitude which determines the energy behaviour, i.e., the two last factors in the right-hand side of Eq. (3). Certainly it represents an improvement upon the corresponding amplitude of CLA. Moreover, in the situation when the clusters are considered as individual entities with definite quantum numbers, i.e., resonances, we expect the role of the $s_i^{(c)}$ to be most important.

These features are then approximately accounted for by the following parametrization of the amplitude A . We write

$$|A|^2 = \prod_{i=1}^{n-1} \left[|A_i^{(c)}(s_i, s_i^{(c)}, t_i)|^2 + \sum_r \gamma_r^2 \left| \frac{\frac{1}{2} \Gamma_r}{(\sqrt{s_i} - m_r) - \frac{i}{2} \Gamma_r} \right|^2 \right], \quad (8)$$

where

$$A_i^{(c)} = \frac{g_i s_i + c_2 a}{s_i + a} \left(\frac{s_i^{(c)} + a}{a} \right)^{\alpha_i} \left(\frac{s_i^{(c)} + b_i}{b_i} \right)^{t_i}. \quad (9)$$

By expanding the product in Eq. (8) and noting that when $s_i^{(c)} \ll a$, $A_i^{(c)} \approx c_2$, it is seen that the parametrization above corresponds to a decomposition of the type shown in Fig. 2 and the substitution (4) in the expression for $A_i^{(c)}$.

ACKNOWLEDGEMENTS

We are very grateful to Dr. Chan Hong-Mo for advice, criticism and discussions. We are indebted to the Aachen-Berlin-CERN-Krakow-Warsaw and Birmingham-Rutherford H.E. Lab.-Glasgow-Imperial College-Oxford-Munich collaborations for kind permission to use their data, some of which are unpublished. In particular we would like to thank Drs. D.R.O. Morrison, K. Paler and S. Goldsack for the generous co-operation.

or $\Lambda\pi^-$ distributions. This is directly a result of the fact that Λ and π^+ are adjacent in the graph which has no baryon exchange.

The ordering effect works very much in our favour for another reason. Since our parametrization takes no account of the charges of the resonances or of their decay products as such, there is no isospin factor to weight the decays $\rho^+ \rightarrow \pi^+\pi^0$, $\rho^0 \rightarrow \pi^+\pi^-$, etc. In fact, if two pions with like charge were to be adjacent to each other on a graph, our parametrization would admit a ρ resonance to be formed. An important consequence of the ordering effect, however, is that only those pions which can form ρ resonances can come together. This is shown in Figs. 8 and 10 where the $\pi^+\pi^+$ and $\pi^-\pi^-$ distributions show no definite structure while the $\pi^+\pi^-$, $\pi^+\pi^0$, $\pi^-\pi^0$ all show the characteristic ρ enhancement.

To sum up, we believe that the CLA model has successfully been taken a step further. The method we have described for including resonance production allows the extended model to explain a large class of two-body distributions. Since the method is quite general it could, in principle, be extended to include three-body resonance effects and we are now considering ways of parametrizing the amplitude which will allow the formation of many body low-energy clusters.

Among other questions to be considered in future developments of the model is, for example, the evaluation of the actual contribution from resonances to the low energy cross sections. This, in turn, would involve overcoming the problem of "double counting" ⁶⁾ of the contribution from (i) the direct channel resonances and (ii) the low energy extrapolation of the Regge amplitude.

Thus our parametrization approximately accounts for the effective change in the multiplicity of the final state when clustering and resonance formation take place. The degree of success achieved by this new parametrization can be judged by the improved fits to the data discussed in the next Section.

3. - COMPARISON WITH EXPERIMENT

We have used the form of the amplitude developed in the previous Section to compare with experimental data for two types of reaction :

$$\begin{aligned} \text{a)} \quad & \pi^+ p \rightarrow p + (n-1)\pi \quad \text{at 8 GeV}^{4)} , \\ \text{b)} \quad & K^- p \rightarrow \Lambda + (n-1)\pi \quad \text{at 6 GeV}^{5)} . \end{aligned}$$

In reaction a) two resonances were inserted, the $\Delta(1238)$ and the $\rho(760)$. The corresponding values of the resonance parameters were found to be

$$\gamma_{\Delta}^2 = 7.0 \quad , \quad \gamma_{\rho}^2 = 12.0 \quad .$$

We have ignored the pion charges, as did CLA, in order to simplify the problem a good deal. Figures 4-6 illustrate the results for three reactions with $n = 3, 4, 5$. The mass distributions for $n = 3$ are very similar to the corresponding curves of Ref. ¹⁾. It is interesting to compare the resulting proton longitudinal momentum distributions in Fig. 4 with the corresponding curves of CLA (shown in dashed curves). The tendency for the final pions to cluster into resonances is well shown by the marked backward peak in the proton distribution.

The comparisons with the data for reactions a) illustrate the fact that only one set of values for the resonance parameters is sufficient to obtain reasonable agreement for several multiplicities. This method is of course far superior to any alternative procedure of "removing" resonances from the data and somehow fitting the remainder with multi-Regge behaviour. No one really believes that observed events can be split up into two such distinct parts and the result is that there exists no unambiguous method for removing resonance contribution.

The study of reactions of type b) proves to be more interesting as the dependence of the amplitude upon the quantum numbers of the final particles is displayed clearly. It was pointed out by CLA that the charged pions of the final state must be ordered on the multi-Regge graph. For example, consider the reaction $K^- p \rightarrow \Lambda \pi^+ \pi^-$ where the only allowed graphs are those of Fig. 3.

The great advantage is that now we can distinguish the pions immediately. The consequences of this ordering of the pions, as far as single particle distributions are concerned, were pursued by CLA. We can now study the same effects in two-body distributions. According to our parametrization for including resonances, only those particles which are adjacent to each other on the graphs actually form two-body resonances. Consider the above example : - the first graph is the most important since it involves only $B = 0$ exchange and the second is next in importance since the third requires double exchange of a $B = 1$ Regge pole. Consequently we would expect a $\Lambda \pi^+$ resonance to have a greater weight than the corresponding $\Lambda \pi^-$ resonance. For reactions b) we again insert just two resonances - $\Sigma(1385)$ and ρ . We found the best values of the parameters to be

$$\chi_{\Sigma}^2 = 0.5 \quad , \quad \chi_{\rho}^2 = 4.0 \quad .$$

Figures 7 and 9 illustrate the $\Lambda \pi$ mass distributions for multiplicities 4 and 5. It is noticed that the $\Sigma(1385)$ dominates the $\Lambda \pi^+$ distributions to a much greater degree than the $\Lambda \pi^0$

REFERENCES

- 1) Chan Hong-Mo, J. Łoskiewicz, W.W.M. Allison - CERN Preprint Th. 866 to be published in Nuovo Cimento.
- 2) E. Fermi - Progr.Theor.Phys. 5, 570 (1950) ;
P. Srivastava and E.C.G. Sudarshan - Phys.Rev. 110, 765 (1958).
- 3) There are numerous papers on the multi-Regge model, a comprehensive list of which is given in Ref. ¹⁾. To those may be added :
I.T. Drummond - Cambridge University Preprint (March 1958) ;
L. Michejda, J. Turnau and A. Bialas - Nuovo Cimento 56, 241 (1968).
J.C. Polkinghorne - Cambridge University Preprint (June 1968).
- 4) Aachen-Berlin-CERN-Krakow-Warsaw collaboration.
- 5) Birmingham-Rutherford H.E. Lab.-Glasgow-Imperial College-Oxford-Munich collaboration.
- 6) R. Dolen, D.Horn and C. Schmid - Phys.Rev.Letters 19, 402 (1967) ;
Phys.Rev. 166, 1768 (1968) ;
G.F. Chew and A. Pignotti - Phys.Rev.Letters 20, 1078 (1968).

FIGURE CAPTIONS

- Figure 1 : Graph corresponding to the reaction $A+B \rightarrow 1+2+\dots+n$.
- Figure 2 : The eight graphs that contribute to the reaction $A+B \rightarrow 1+2+3+4$. The internal lines represent Regge pole exchanges and the blobs low effective mass clusters.
- Figure 3 : The three allowed graphs for the reaction $K^-p \rightarrow \Lambda \pi^+ \pi^-$.
- Figure 4 : Percentage distribution of protons in the c.m. longitudinal momentum $p_{\perp}^*$ for the reaction $\pi^+p \rightarrow p+(n-1)\pi$ at 8 GeV for n ranging 3 to 5. The histograms are experimental, the solid curves are calculated with the amplitude of this paper, the dashed curves are the results of Ref. 1).
- Figure 5 : Percentage πp and $\pi\pi$ effective mass distributions for the reaction $\pi^+p \rightarrow p2\pi^+\pi^-$ at 8 GeV. The curves represent the results with $\Delta(1236)$ and ρ production.
- Figure 6 : Percentage πp and $\pi\pi$ effective mass distributions for the reaction $\pi^+p \rightarrow p2\pi^+\pi^-\pi^0$ at 8 GeV. Curves as for Fig. 5.
- Figure 7 : Percentage $\Lambda\pi$ effective mass distributions for the reaction $K^-p \rightarrow \Lambda\pi^+\pi^-\pi^0$ at 6 GeV. The curves represent the results with $\Sigma(1385)$ and ρ production.
- Figure 8 : Percentage $\pi\pi$ effective mass distributions for the reaction $K^-p \rightarrow \Lambda\pi^+\pi^-\pi^0$ at 6 GeV. Curves as in Fig. 7.
- Figure 9 : Percentage $\Lambda\pi$ effective mass distributions for the reaction $K^-p \rightarrow \Lambda 2\pi^+2\pi^-$ at 6 GeV. Curves as in Fig. 7.
- Figure 10 : Percentage $\pi\pi$ effective mass distributions for the reaction $K^-p \rightarrow \Lambda 2\pi^+2\pi^-$ at 6 GeV. Curves as in Fig. 7.

FIG.1

FIG.2

FIG.3

FIG.5

FIG.6

FIG.7

FIG.8

FIG. 9

FIG.10