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A PHENOMENOLOGICAL MULTIPARTICLE AMPLITUDE WITH
REGGE TRAJECTORIES IN s AND t CHANNELS

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A B S T R A C T

We suggest a modified form of the model of Chan et al. which exhibits Regge phases at high energies and resonances at low energies. The new amplitude - suggested by the Veneziano model - gives an excellent estimation of shadow elastic scattering.

The intense study of the reggeized multiperipheral model during the last year ¹⁾ has considerably advanced our understanding of processes yielding three or more final state particles. In particular, the model proposed by Chan et al. ²⁾ (CLA model) has proved successful in explaining the main features observed in multiparticle states over a wide range of multiplicity and beam momentum. In spite of its success this model has certain shortcomings :

- i) no account is taken of two-particle resonances and yet the model claims to describe reactions at energies where the formation of resonances dominates the cross-section;
- ii) no prediction is made concerning the phase of the amplitude.

Although these two defects are closely related, an attempt ³⁾ was made to rectify i) only, which allowed the CLA model to extend its region of success to a wider class of distributions. However, in this note we show how both troubles may be cured simultaneously by making a simple modification which is strongly suggested by Veneziano's ⁴⁾ amplitude for $\pi\pi \rightarrow \pi\omega$. We shall show how this modification enables the model to accurately describe the Van Hove overlap function ^{5),6)} and hence give a good estimate for the shadow elastic scattering.

First we recall the CLA model describing the reaction $a+b \rightarrow 1+2+\dots+n$. The total amplitude, T , is an incoherent sum of amplitudes $A^{(n)}$ each describing a multi-Regge graph and expressed as

$$A^{(n)} = \prod_{i=1}^{n-1} A_i(s_i, t_i) \quad (1)$$

where

$$A_i(s_i, t_i) = \left(\frac{g_i x_i + c}{x_i + 1} \right) (1+x_i)^{\alpha_i(0)} \left(1 + x_i \frac{a}{b_i} \right)^{t_i} \quad (2)$$

and

$$x_i = \left[s_i - (m_i + m_{i+1})^2 \right] / a$$

The variables s_i, t_i and parameters $g_i, c, \alpha_i(0), b_i$ are explained in Ref. 2). This form of A_i interpolates smoothly between high energy behaviour corresponding to Regge exchange and low energy behaviour given by phase space. If one wishes to replace the idea of the low energy region being determined by the statistical model by a more realistic view of resonance formation, one is faced by the problem of interpolating from this situation to the Regge exchange situation. In particular, the question of double counting will arise if the resonance amplitude is simply added to the Regge amplitude. In fact, there is evidence ⁷⁾ that it may be more realistic to assume the Regge amplitude alone to be valid, in a "locally averaged" sense, in the resonance region.

The recent work of Veneziano ⁴⁾ gives a prescription for adding direct channel poles and crossed channel poles in a way which, for a two-particle amplitude, presumably does not involve double counting. His amplitude (for $\pi\pi \rightarrow \pi\omega$) is

$$A(s,t) = \frac{\Gamma(\alpha_s + \alpha_t - 1)}{\Gamma(\alpha_s)} \Gamma(1 - \alpha_t) \left[\frac{1 - \cos \pi \alpha_s}{\sin \pi \alpha_s} \sin \pi \alpha_t + 1 - \cos \pi \alpha_t \right] \quad (3)$$

where $\alpha_s = \alpha(s)$, $\alpha_t = \alpha(t)$ are Regge trajectories. The terms before the brackets may be considered as a "shrinkage" term times a residue function. Since α_s is complex above threshold, then the final term in (3) has resonance poles together with a real "background". Since $\text{Im} \alpha_s$ is supposed to be an increasing function of s , then eventually $(1 - \cos \pi \alpha_s) / \sin \pi \alpha_s \rightarrow i$ and so we get the Regge phase factor $1 - e^{-i\pi \alpha_t}$. Since the CLA model already has terms analogous to the Γ functions in (3), the following new form for A_i is suggested :

$$A_i(s,t) = g (1+x)^{\alpha_t} \left(1 + x \frac{a}{b}\right)^{\alpha_t' t} \times \left[\frac{1 + \cos \pi(\alpha_s - \sigma_s)}{\sin \pi(\alpha_s - \sigma_s)} \sin \pi(\alpha_t - \sigma_t) + 1 + \cos \pi(\alpha_t - \sigma_t) \right] \quad (4)$$

where σ_s is the spin of the lowest resonance on the trajectory α_s .

To test the new amplitude, we apply it to the reactions $\pi^+ p \rightarrow (n-1)\pi N$ at 8 and 4 GeV. As in Ref. 2), we take P , A_2 and nucleon for the exchanged trajectories (together with the same values of a , b_i). The amplitude (4) implies that a resonance is allowed to be formed only by pairs of adjacent particles on each graph. We take ρ and Δ trajectories in the relevant s channels in this case. We have to guess at the form of the imaginary part of α_s but a convenient formula ⁸⁾ is :

$$\text{Im} [\pi \alpha_s] = A \log (1+x) \quad (5)$$

A can be chosen so as to reproduce the correct width for the lowest resonance of the trajectory - giving $A_\rho = 0.7$, $A_\Delta = 1.3$. Asymptotically, the widths of the recurrences increase or decrease for $A \gtrless \frac{1}{2}$. Our chosen values reproduce roughly the widths of the first recurrences on each trajectory. Also the higher recurrences are sufficiently damped so that we obtain qualitative agreement with the mass distributions. When nucleon is exchanged $g = g_B$; when P or A_2 is exchanged $g = g_M$, giving just one parameter g_B/g_M which we fixed at 0.5 to give the correct backward peak for the proton.

At least in the high energy region, the contributions from graphs corresponding to different permutations of the final particles do not overlap and consequently the neglect of phases is, in general, a reasonable approximation. However, to calculate the overlap function we require the product of two amplitudes related by a rotation of the initial state. This depends critically on the phase of each amplitude and the incoherent summation of contributions from each graph is not a valid approximation. Therefore a really crucial test of the amplitude (4) is its prediction for the shadow elastic scattering. The CLA model, as such, fails to achieve any consistency with the directly measured values of $d\sigma_{el}/dt$ at small angles and this can be attributed to the lack of phases ⁶⁾. The normalized overlap function for $\pi p \rightarrow (n-1)\pi N$ is ⁹⁾

$$F_n(t) = \text{Re} \left[\frac{\int d\tau_n \langle \pi p, \theta=0 | T | (n-1)\pi N \rangle^* \langle (n-1)\pi N | T | \pi p, \theta=\theta_t \rangle}{\int d\tau_n |\langle \pi p, \theta=0 | T | (n-1)\pi N \rangle|^2} \right] \quad (6)$$

and unitarity gives

$$\text{Im } T_{el}(t) / \text{Im } T_{el}(0) = \sum \left(\frac{\sigma}{\sigma_{tot}} \right) F_n(t) \quad (7)$$

where $\sigma_n = \sigma(\pi p \rightarrow (n-1)\pi N)$ and we suppose that only these reactions contribute to σ_{tot} .

Figures 1 and 2 show our results. The values of σ_n are obtained by multiplying the measured cross-sections for particular charge modes by the isospin statistical factors of Bartke and Czyzewski, Ref. ¹⁰⁾. The values of $\text{Im } T_{el}(t)$ are obtained from the differential cross-sections for π^+p at 8 ¹¹⁾ and 4 ¹²⁾ GeV. The overlap functions, $F_n(t)$, for $n=3$ to 7, as calculated from the amplitude (4), are plotted together with their sum weighted according to (7). For comparison, we show the corresponding result at 8 GeV for the CLA model. The improvement achieved over the unmodified model is obvious.

We believe that our insertion of the "phase factor" increases the power of the CLA model as a phenomenological basis for understanding the features of inelastic processes. While maintaining good agreement for single particle distributions, the model is now able to explain a range of two-particle distributions. More important, however, is the removal of defects i) and ii) which puts the model on a firmer basis.

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FIGURE CAPTIONS

Figure 1 : Imaginary part of elastic amplitude calculated as shadow of the π^+p inelastic channels at 8 GeV. Dotted lines indicate $F_n(t)$. Solid lines are weighted sums.

Figure 2 : Imaginary part of elastic amplitude calculated as shadow of the π^+p inelastic channels at 4 GeV. Solid line is the weighted sum.

FIG. 1

FIG. 2