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 K^+p BACKWARD SCATTERING FROM THE VENEZIANO MODEL

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A B S T R A C T

The residue functions of the leading $Y_0^*(\Lambda_\alpha, \Lambda_\gamma)$ and $Y_1^*(\Sigma_\delta, \Sigma_\beta)$ trajectories in the backward K^+p scattering are parametrized according to the Veneziano model, consistent with the absence of the particles which are not found in nature. We then obtain a best fit for K^+p backward scattering data from 2.0 to 7.0 GeV/c. The extrapolated residues agree with experimental Y_0^* and Y_1^* widths.

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The appealing feature of the Regge pole model in boson-baryon backward scattering is the conjectured connection between baryon resonances and baryon exchange amplitude as expressed by trajectory functions $\alpha(\sqrt{u})$ based on the Chew-Frautschi plot and residue functions $\gamma(\sqrt{u})$. This connection has not been fully established up to the present. Arbitrariness of residue functions gave difficulty in predicting reasonable extrapolated magnitude of baryon resonance widths at baryon poles in most data fitting in backward directions.

As was shown in a previous paper ¹⁾, the Veneziano type model for kaon-nucleon scattering gives a reasonable approximation to elastic widths of the Y^* resonances, forward K^-p charge exchange scattering and K^+p backward scattering at $u=0$ at high energies. The absence of prominent resonances in the KN elastic channel makes the Veneziano model for KN scattering particularly simple, since this implies the t channel trajectories are exchange degenerate and also the u channel trajectories.

In the t channel we took into account the (g, A_2) trajectories, which were required to be exchange degenerate and the (ω, P') trajectories which were also required to be exchange degenerate. Since the $\pi\pi$ Veneziano formula forced the g and P' trajectories to be exchange degenerate ²⁾, we assumed

$$\alpha_g(t) = \alpha_{A_2}(t) = \alpha_\omega(t) = \alpha_{P'}(t) \equiv \alpha(t).$$

In the u channel the $Y_0^*(\Lambda_\alpha, \Lambda_\gamma)$ and $Y_1^*(\Sigma_\delta, \Sigma_\beta)$ trajectories, each of which are exchange degenerate, were assumed to be dominant.

The model appears to predict parity doubling of all resonances on the Y^* trajectories. Since the particle with $J^P = \frac{1}{2}^-$ on the leading Y_1^* trajectory and also parity partners of $\Lambda(\frac{1}{2}^+, 1115)$, $\Lambda(\frac{3}{2}^-, 1520)$ and $\Sigma(\frac{3}{2}^+, 1385)$ are not found experimentally, we eliminate them.

Then, we obtain from Ref. 1) the following representations for the invariant amplitudes A and B, which satisfy duality ³⁾, crossing symmetry and Regge asymptotic behaviour:

$$\begin{aligned}
 B_{K^-p}(u, t, s) &= \sum_{I=0}^1 \frac{\beta_{I0}^{(2)}}{\pi} B(1-\alpha(t), \frac{1}{2} - \alpha_{Y_0}(u)) \\
 &+ \sum_{I=0}^1 \frac{\beta_{I1}^{(2)}}{\pi} B(1-\alpha(t), \frac{3}{2} - \alpha_{Y_1}(u)) \\
 &+ \frac{\gamma_1}{\pi} C(2-\alpha(t), \frac{3}{2} - \alpha_{Y_1}(u)) + \dots \\
 &= -B_{K^+p}(s, t, u)
 \end{aligned} \tag{1}$$

and

$$\begin{aligned}
 A_{K^-p}(u, t, s) &= \sum_{I,J=0}^1 \frac{\beta_{IJ}^{(1)}}{\pi} C(1-\alpha(t), \frac{3}{2} - \alpha_{Y_J}(u)) \\
 &+ \frac{\gamma_0}{\pi} B(1-\alpha(t), \frac{1}{2} - \alpha_{Y_0}(u)) + \dots \\
 &= A_{K^+p}(s, t, u)
 \end{aligned} \tag{2}$$

where $B(X, Y) = \Gamma(X)\Gamma(Y)/\Gamma(X+Y)$, $C(X, Y) = \Gamma(X)\Gamma(Y)/\Gamma(X+Y-1)$,

$$\begin{aligned}
 \gamma_0 &= -(m_\Lambda - m) \sum_{I=0}^1 \beta_{I0}^{(2)}, \\
 \sum_{I=0}^1 \beta_{I0}^{(1)} &= (1.520 - m_\Lambda) \sum_{I=0}^1 \beta_{I0}^{(2)}
 \end{aligned}$$

and

$$\sum_{I=0}^1 \beta_{I1}^{(1)} = (1.385 + m) \gamma_1.$$

These are the most economical numbers of Veneziano terms, which provide a definite parametrization for the elastic widths of the Y^* resonances, for the forward K^-p charge exchange scattering amplitude and for the K^+p backward elastic scattering amplitude.

The purpose of this paper is only to show that the connection between baryon resonances and baryon exchange amplitudes is correct in the case of K^+p backward elastic scattering using the Regge parametrization from the Veneziano model ^{1),2),4)}.

Substituting the asymptotic form of Eqs. (1) and (2) into the u channel amplitude, defined by

$$f_1(\sqrt{u}, s) = \frac{E_u + m}{8\pi\sqrt{u}} [A(u, s) + (\sqrt{u} - m)B(u, s)]$$

we obtain ¹⁾ the following amplitudes for K^+p backward scattering ^{*)}

$$f_1^{I_u=0}(\sqrt{u}, s) = \frac{E_u + m}{\sqrt{u}} \frac{\gamma_{Y_0}(\sqrt{u})}{\Gamma(\alpha_{Y_0}(\sqrt{u}) + \frac{1}{2})} \frac{2}{\cos \pi \alpha_{Y_0}(\sqrt{u})} S^{\alpha_{Y_0}(\sqrt{u}) - \frac{1}{2}} \quad (3)$$

with

$$\gamma_{Y_0}(\sqrt{u}) = -\frac{1}{8\pi} [\beta^{(1)}(\alpha_{Y_0}(\sqrt{u}) - \frac{1}{2}) - \beta^{(2)}(\sqrt{u} - m_\Lambda)] a^{\alpha_{Y_0}(\sqrt{u}) - \frac{1}{2}} \quad (4)$$

and

$$\beta^{(1)} = (1.520 - m_\Lambda) \beta^{(2)} \quad (5)$$

$$f_1^{I_u=1}(\sqrt{u}, s) = \frac{E_u + m}{\sqrt{u}} \frac{\gamma_{Y_1}(\sqrt{u})}{\Gamma(\alpha_{Y_1}(\sqrt{u}) + \frac{1}{2})} \frac{2}{\cos \pi \alpha_{Y_1}(\sqrt{u})} S^{\alpha_{Y_1}(\sqrt{u}) - \frac{1}{2}} \quad (6)$$

with

$$\gamma_{Y_1}(\sqrt{u}) = -\frac{\gamma_1}{8\pi} (\alpha_{Y_1}(\sqrt{u}) - \frac{1}{2}) (\sqrt{u} + 1.385) a^{\alpha_{Y_1}(\sqrt{u}) - \frac{1}{2}} \quad (7)$$

^{*)} We followed the convention of always dealing with the $\ell = J - \frac{1}{2}$ amplitude. With this convention, Λ has a pole at $\sqrt{u} = -m_\Lambda$.

Here m_Λ denotes Λ particle mass and

$$\sum_{I=0}^1 \beta_{I0}^{(1)}, \sum_{I=0}^1 \beta_{I0}^{(2)}$$

in Eqs. (1) and (2) are replaced by $\beta^{(1)}, \beta^{(2)}$ respectively.

The important and essential features of the above Eqs. (3)-(7) are:

- 1) the scaling factor s_0 is not arbitrary but fixed to be a^{-1} , where a is the slope of boson trajectory;
- 2) they satisfy the FESR at fixed u ⁵⁾;
- 3) in the above expressions (3)-(7), the residue functions vanish at the position of $Y_1^*(J^P = \frac{1}{2}^-)$ and parity partners of $\Lambda(\frac{1}{2}^+, 1115)$, $\Lambda(\frac{3}{2}^-, 1520)$, $\Sigma(\frac{3}{2}^+, 1385)$.

We fit the data for elastic K^+p backward scattering from 2.0 to 6.9 GeV/c ⁶⁾⁻⁹⁾ using the above parametrized residue functions of the leading $Y_0^*(\Lambda_\alpha, \Lambda_\gamma)$ and $Y_1^*(\Sigma_\delta, \Sigma_\beta)$ trajectories and the trajectory functions $\alpha_{Y_0} = -0.70 + 0.98u$ and $\alpha_{Y_1} = -0.33 + 0.93u$ obtained from baryon mass spectra. By the crossing symmetry relation, we calculate $f_1(\sqrt{s}, u)$ to be

$$\begin{aligned} f_1(\sqrt{s}, u) = \frac{E_s + m}{2\sqrt{s}\sqrt{u}} \sum_{i=Y_0^*, Y_1^*} \left\{ C_i \frac{\sqrt{u} - \sqrt{s} + 2m}{\Gamma(\alpha_i(\sqrt{u}) + \frac{1}{2})} \frac{2\gamma_i(\sqrt{u})}{\cos \pi \alpha_i(\sqrt{u})} S^{\alpha_i(\sqrt{u}) - \frac{1}{2}} \right. \\ \left. + C_i \frac{\sqrt{u} + \sqrt{s} - 2m}{\Gamma(\alpha_i(-\sqrt{u}) + \frac{1}{2})} \frac{2\gamma_i(-\sqrt{u})}{\cos \pi \alpha_i(-\sqrt{u})} S^{\alpha_i(-\sqrt{u}) - \frac{1}{2}} \right\} \end{aligned} \quad (8)$$

with the isospin crossing coefficient C_i .

The differential cross-section is given by ¹⁰⁾

$$\frac{d\sigma}{du} = \frac{\pi}{q^2} \left\{ |f_1(\sqrt{s}, u)|^2 + |f_1(-\sqrt{s}, u)|^2 - 2 \cos \theta \operatorname{Re} [f_1(\sqrt{s}, u) f_1^*(-\sqrt{s}, u)] \right\} \quad (9)$$

In our fit we have used only three adjustable parameters $\beta^{(1)}$, $\beta^{(2)}$ and γ_1 . The slope of boson trajectory was fixed to be $a = 0.95 \text{ GeV}^{-2}$. We then have obtained the best χ^2 values for the following sets:

$$\beta^{(1)} = 495.0 \text{ GeV}^{-1}, \quad (10)$$

$$\beta^{(2)} = 904.7 \text{ GeV}^{-2}, \quad (11)$$

$$\gamma_1 = -52.7 \text{ GeV}^{-2}. \quad (12)$$

In order to yield a positive sign for the baryon resonance widths we fixed the sign of $\beta^{(1)}$ and $\beta^{(2)}$ to be positive. Under this condition we obtained the best χ^2 for negative value of γ_1 as indicated above. As the results show, the Y_1^* contribution is one order of magnitude smaller than the Y_0^* contribution ^{1), 11)}. This has to be compared with the results from Ref. 1)

$$\begin{aligned} \beta^{(1)} &= 341 \begin{matrix} +101 \\ -76 \end{matrix} \text{ GeV}^{-1} \\ \beta^{(2)} &= 841 \begin{matrix} +249 \\ -189 \end{matrix} \text{ GeV}^{-2} \\ \gamma_1 &= -43.8 \begin{matrix} +28 \\ -39 \end{matrix} \text{ GeV}^{-2} \end{aligned}$$

obtained from Y_0^* and Y_1^* widths. The condition (5) was not required in our fit since the parity partner of $\Lambda(\frac{3}{2}^-, 1520)$ is far away from the physical region of backward scattering and its position is not precisely known ¹²⁾. However, we would like to point out that the above solution, Eqs. (10) and (11), approximately satisfy Eq. (5). A mass of the parity partner of $\Lambda(\frac{3}{2}^-)$, which was killed, corresponds to 1.66 GeV in our solution ¹³⁾.

In Fig. 1 we show the angular distributions and in Fig. 2 the s dependence of the K^+p backward scattering cross-sections as results of our fit compared with the experimental data.

We also extrapolate our residue functions $\gamma_{Y_0}(\sqrt{u})$ and $\gamma_{Y_1}(\sqrt{u})$ given in Eqs. (4) and (5) together with Eqs. (10)-(12) to baryon poles, using ¹⁾

$$2 \int_{l-}^{el} \frac{(\sqrt{s_{l-}} + m)^2 - m_K^2}{q_{l-}^{2l+1}} \frac{M(l) \Gamma(l)}{(2a)^{l-1}} = \frac{\beta^{(2)}}{\pi^2 a_0} \quad (13)$$

for the Y_0^* trajectory and

$$2 \int_{l+}^{el} \frac{(\sqrt{s_{l+}} - m)^2 - m_K^2}{q_{l+}^{2l+3}} \frac{M(l+1) \Gamma(l)}{(2a)^l} = - \frac{\gamma_1}{\pi^2 a_1} \quad (14)$$

for the Y_1^* trajectory ^{*)}. In the Table, we show the predicted elastic widths of baryon resonances as a result of our fit compared with experimental ones from Rosenfeld table ¹⁴⁾. Except for the $\sum (\frac{7}{2}^+, 2030)$ width which is not well established from experiments, we find very good agreement. Our solution gives a value for the Λ coupling constant, $g_\Lambda^2/4\pi = 23.4$. Finally, we would like to mention that the constants $\beta^{(1)}$, $\beta^{(2)}$, γ_1 in Eqs. (10), (11), (12) and Ref. 13) should be compared with the same constants obtained from forward K^-p charge exchange scattering ¹⁾:

$$\beta^{(1)} = 255 \text{ GeV}^{-1} \quad (15)$$

$$\beta^{(2)} = 553 \text{ GeV}^{-2} \quad (16)$$

$$\gamma_1 = -19.4 \text{ GeV}^{-2} \quad (17)$$

^{*)} The function $M(l)$ is defined by $P'_l(z) = M(l)z^{l-1} + \dots$ and a_0 , a_1 are the slopes of Y_0^* , Y_1^* trajectories.

which give qualitative agreement within a factor two ^{*)}.

In conclusion, we would like to emphasize the importance of the correct parametrization, which is consistent with duality and crossing symmetry in the analysis of boson-baryon scattering, since the fascinating feature of the Regge pole theory is the close connection between low energy resonances and their trajectory exchange in backward scattering. This kind of approach would also be powerful in the analysis of $p\bar{p} \rightarrow K\bar{K}$.

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) However, one must keep in mind that in forward K^-p CEX scattering, duality requires all Y^ trajectories to contribute in an average sense, while the agreement is only within a factor two. There is still freedom to add non-leading Veneziano terms which affect the elastic widths of many daughters and may therefore give a possible explanation for the above discrepancy.

TABLE

Y_8^0 trajectory				Y_7^0 trajectory		
ℓ	J^P	Elastic width for K^-p $\Gamma_{\ell^-}^{\text{rel}}$ experiment MeV	Elastic width for K^-p $\Gamma_{\ell^-}^{\text{rel}}$ calculated MeV	J^P	Elastic width for K^-p $\Gamma_{\ell^+}^{\text{rel}}$ experiment MeV	Elastic width for K^-p $\Gamma_{\ell^+}^{\text{rel}}$ calculated MeV
1	$\Lambda(1115) \frac{1}{2}^+$			$\Sigma(1385) \frac{3}{2}^+$		
2	$\Lambda(1520) \frac{3}{2}^-$	3.6 ± 0.8	2.14	$\Sigma(1765) \frac{5}{2}^-$	23.0 ± 3.5	20.45
3	$\Lambda(1815) \frac{5}{2}^+$	24.3 ± 3.5	19.1	$\Sigma(2030) \frac{7}{2}^+$	6.0	24.44
4	$\Lambda(2100) \frac{7}{2}^-$	21.0	20.4	$\Sigma(2250) ?$	?	21.85
5	$\Lambda(2350) ?$	?	16.4			

R E F E R E N C E S

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In an unpublished version of the CERN preprint TH. 993 (1969),
the term

$$\frac{\delta_1}{\pi} C \left(2 - \alpha_I(t), \frac{3}{2} - \alpha_{Y_1}(s) \right) + \dots$$

was missing in Eq. (1) and the term

$$\frac{2\delta_1}{\pi} C \left(2 - \alpha_I(t), \frac{3}{2} - \alpha_{Y_1}(s) \right) + \dots$$

was missing in Eq. (2).

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- 12) In the limit of vanishing $\text{Im } \alpha$ with linear trajectory, Eq. (5) must be strict. Since difficult questions with parity doubling are not solved yet, Eq. (5) was not required in fitting.
- 13) When we require condition (5) in our fit, we obtain a best χ^2 for the following two parameters

$$\beta^{(2)} = 597.1 \text{ GeV}^{-2}$$

$$\delta_1 = -52.4 \text{ GeV}^{-2}$$

This yields a coupling constant $g_\lambda^2 / 4\pi = 15.6$. Then the calculated Y_0^* widths are a factor two smaller than the experiment.

- 14) Rosenfeld table (Jan. 1969).

FIGURE CAPTIONS

Figure 1:

Backward K^+p elastic scattering angular distributions. The BNL-Rochester data are from Ref. 6), the Illinois data from Ref. 7), the CERN-Saclay data from Ref. 8), the CERN data from Ref. 9). The solid line represents the results of our fit to these data.

Figure 2:

Energy dependence of $d\sigma/du$ at $u=0$ for K^+p elastic scattering. The solid line represents the results of our fit.

FIG. 2