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CORRELATION EFFECTS BETWEEN PIONS IN THE REACTIONS $\bar{p}p \rightarrow n\pi$

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A B S T R A C T

Using Lovelace's $\pi\pi$ phase shifts, we construct a model based upon the Reggeized multiperipheral model which can explain many of the features observed in the processes $\bar{p}p \rightarrow n\pi$ above ~ 2.5 GeV/c. An improvement upon previous models is the good agreement achieved for the Goldhaber effect.

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I. - INTRODUCTION

Many experiments ¹⁾⁻¹⁷⁾ have been carried out on multipion production from annihilation of anti-protons in flight with the result there now exists a comprehensive set of data for several multiplicities for beam momenta up to 7 GeV/c. The interesting features observed in the multipion final states may be divided into two parts : (i) single particle distributions and (ii) two (or more) particle distributions, i.e., correlation effects.

A step forward in our understanding of many-body production processes was the phenomenological Reggeized multi-peripheral model introduced by Chan et al. ¹⁸⁾ (hereafter referred to as the CLA model). In particular the model was applied by Chen ¹⁹⁾ to the reactions $\bar{p}p \rightarrow n\pi$ and he obtained good agreement for both single particle distributions and cross-sections. However, the model failed to achieve any success in explaining the correlation effects which perhaps are the most puzzling and therefore the most interesting features of these reactions. Especially the so-called Goldhaber effect ²⁰⁾ has long been a source of intensive study ²¹⁾. The Goldhaber angle χ is the angle between a pair of pions measured in the over-all centre-of-mass frame. The observed distributions in $\cos\chi_U$ and $\cos\chi_L$ (where U and L refer to unlike and like pion pairs) show strong deviations from phase space. χ_U tends always to be larger than χ_L especially at low average pion momentum.

It is not surprising that the CLA model fails to produce this effect since for low dipion energies the two-particle amplitude is simply tending to a constant. An attempt ²²⁾ was made to insert some structure into the low-energy CLA amplitude in the form of resonance formation. The modified amplitude still produced a significant anti-Goldhaber effect (i.e., $\chi_L > \chi_U$), however.

In this paper we attempt to describe low energy $\pi\pi$ behaviour in a more realistic way. The hope is that by giving the $\pi\pi$ interaction the correct phase for each partial wave, there is

a greater chance of reproducing the observed correlation effects. As we shall see, this hope is borne out - at least qualitatively. In Section III we compare the predictions of our model with the data and find that for momenta above ~ 2.5 GeV/c

- (a) single particle distributions are very well predicted,
- (b) qualitative agreement with correlation effects,
- (c) failure to reproduce the magnitudes of the cross-sections.

The construction of our amplitude for $\bar{p}p \rightarrow n\pi$ requires a knowledge of the $\pi\pi$ phase shifts. Recently Lovelace²³⁾ has derived a set of such phase shifts from a unitarized form of the Veneziano amplitude. Furthermore, there are compelling reasons²⁴⁾ for believing in these phase shifts from their success in fitting $\pi\pi$ data. Our model may be regarded as being rather naïve but it has the appealing feature of fitting a wide range of data with no free parameters (apart from normalization).

II. - THE MODEL

In Chen's¹⁹⁾ application of the CLA model to the reaction $\bar{p}p \rightarrow n\pi$ the amplitude at high energies corresponds to the graph of Fig. 1, in which a Reggeized nucleon is exchanged at each stage. Experimentally only final states with either (a) no π^0 , or (b) one π^0 , are observed. In case (a) the ordering of the charged pions in Fig. 1 is fixed, while in case (b) several graphs contribute since the π^0 may occur at any stage. In this paper we shall calculate only the final states of type (a) since computation is faster when only one graph has to be evaluated. However, our discussion may easily be extended to case (b).

In the CLA model, the full amplitude (corresponding to the graph of Fig. 1) may be written

$$A = \prod_{i=1}^{n-1} A_i(s_i, t_i) \quad (1)$$

where $A_i(s_i, t_i)$ describes the two-particle final state of pions i and $i+1$. This multiplicative form of the amplitude given by Eq. (1) is the basis of our model but we shall take an expression for A_i different from that used in the CLA model. Instead of getting the high energy behaviour of A_i correct and making a simple approximation for the low energy interaction (as in the CLA model) we attempt to insert a realistic low energy behaviour and not worry too much about the high energy region. The motivation behind this approach is that the observed $\pi\pi$ mass distributions are concentrated in the low energy region and therefore it is reasonable to believe that the final state distributions are governed by the dynamics of the low energy $\pi\pi$ interaction.

Our simple-minded approach regards A_i as representing a scattering process between two Reggeons $\bar{N}_{i-1}$, with momentum q_{i-1} ($q_{i-1}^2 = t_{i-1}$), and N_{i+1} , with momentum q_{i+1} ($q_{i+1}^2 = t_{i+1}$), producing pions i and $i+1$. We expand A_i in partial waves using as our variable $\cos\theta_i$ which is the angle between pion i and $\bar{N}_{i-1}$ in the dipion centre-of-mass frame. From linear unitarity, we know that the phase of A_i , the partial wave amplitude, will be the corresponding $\pi\pi$ elastic phase shift δ_ℓ . In our model we go one step further and approximate A_i by the partial wave amplitude for elastic $\pi\pi$ scattering. We expect this to be a valid approximation at low energies since threshold behaviour is the same and the widths of the ρ , ϵ and f^0 should be similar. However, as $s_i (= (p_i + p_{i+1})^2)$ increases, the approximation will clearly become worse since the Regge behaviour of A_i is given by baryon exchange not meson exchange. So we write

$$A_i = \frac{2k}{\sqrt{s_i}} \sum_{\ell} e^{i\delta_\ell} \sin \delta_\ell P_\ell(\cos\theta_i) \quad (2)$$

Since we are concerned with values of $\sqrt{s_i}$ less than about 1.5 GeV, we truncate the sum in Eq. (2) at $\ell = 3$, i.e., we need to include only S, P, D and F waves. We take the phase shifts given by Lovelace's prescription ²³⁾ which is based upon a method of unitarising Veneziano's $\pi\pi$ amplitude. Thus our model has no free parameters, apart from normalization and is also very simple in form.

Before comparing with the data we expose an inherent fault of the model. From Eqs. (1) and (2) we see that a situation may arise where adjacent pairs of pions may simultaneously form resonances. If the model were an infinitely narrow width model then this would correspond to the occurrence of a double pole in the variables s_i and s_{i+1} , for example. From the extension of Veneziano's model to multi-particle amplitudes ²⁵⁾ it is known that such double poles do not occur - always there exists a zero such that one gets at most single poles. In our phenomenological model we have no compensating damping factors and as a result we overestimate the density of events lying in resonance overlap regions. Consequently we expect resonance formation to be too strong in general and this discrepancy will be most strongly felt at low energies.

III. - COMPARISON WITH EXPERIMENT

In this paper we discuss those reactions with only charged pions in the final state. We shall therefore refer to the reactions $\bar{p}p \rightarrow 2\pi^+2\pi^-$, $3\pi^+3\pi^-$, $4\pi^+4\pi^-$ as reactions (2), (3) and (4). The data with which we compare are taken from Refs. ¹⁾⁻¹⁷⁾.

A - Single pion distributions

Typical examples of such distributions are shown in Fig. 2 - in this case for reaction (2) at 5.7 GeV/c. The agreement appears to be quite satisfactory. The most interesting single pion distribution is the angular distribution of the π^- (π^+) measured with respect

to the direction of the $\bar{p}$ (p) in the over-all centre-of-mass system. To discuss the $\cos\theta_\pi$ distribution more fully, it is traditional to define the following parameters :

$$\text{Asymmetry} \quad A = F/B$$

$$\text{Collimation} \quad C = P/E$$

where

$$\begin{aligned} F &= \text{number of events for which } \cos\theta_\pi > 0 \\ B &= \text{ " " " " " } \cos\theta_\pi < 0 \\ P &= \text{ " " " " " } |\cos\theta_\pi| > \frac{1}{2} \\ E &= \text{ " " " " " } |\cos\theta_\pi| < \frac{1}{2} \end{aligned}$$

In Figures 3 and 4 we plot the variation in the parameters A and C with beam momentum and show also the corresponding curves predicted by the model. Below ~ 2.5 GeV/c there is no agreement with the values of A - this is connected with the remark at the end of Section II. As the momentum increases the agreement improves and at 5.7 GeV/c the fit is very good for each multiplicity. The behaviour of C appears to be correctly predicted at all energies.

B - Correlation effects

The best known of these is the Goldhaber effect. In Fig. 5, we show typical distributions in the Goldhaber angle χ where it can clearly be seen that the effect is strongest at low average pion momenta, i.e., either low beam momentum and/or high multiplicity. This variation with multiplicity and momentum is qualitatively well reproduced by our model although the fit with any one particular $\cos\chi$, on its own, is not outstanding. The ability of the model to explain the effect, we believe, arises from the inclusion of the strong P wave interaction at low energies. It is traditional ²¹⁾ to define asymmetry parameters $\delta^L = n_A^L/n_B^L$, $\delta^U = n_A^U/n_B^U$ where n_A , n_B are the number of pion pairs with $\chi_{\pi\pi} > \text{ or } < 90^\circ$ respectively. L refers to like pions, U to unlike. Also $n_A + n_B$ is normalized to unity. Furthermore

a correlation parameter C is given by

$$C = n_A^U - n_A^L = \frac{\delta^U - \delta^L}{(1+\delta^U)(1+\delta^L)}$$

i.e., $C = 0$ means no correlation.

Figure 6 illustrates the variation of δ^U, δ^L with beam energy and multiplicity. Again there is no agreement for δ^U below ~ 2.5 GeV/c and again this can be traced to the fault mentioned at the end of Section II. δ^L appears to be well reproduced at all beam energies, however. The net effect is shown in Fig. 7 where we plot the variation in the correlation parameter C . Qualitatively the behaviour given by the model is correct although the Goldhaber effect is, in general, too strong. Nevertheless this is an improvement upon previous models which predicted anti-Goldhaber effects.

A well-known two-particle correlation is the two-pion mass distribution. As might be expected the $\pi^+\pi^-$ experimental distributions display the known resonance peaks corresponding to ρ, f while $\pi^\pm\pi^\pm$ distributions are similar to those predicted by phase space. However, this is not the complete description and deviations from this simple picture have been noticed²⁶⁾ for many multipion final states. At low $\pi\pi$ energies there is a deficiency of events in the $\pi^+\pi^-$ spectrum when compared with phase space and, in contrast, a concentration of events in the $\pi^\pm\pi^\pm$ distribution. This correlation effect is closely connected with the Goldhaber effect. Consider two pions in the final state labelled by A and B - as shown in the non-Euclidean velocity diagram²⁷⁾ of Fig. 9. OA and OB represent the centre-of-mass velocities of the pions. χ is the Goldhaber angle. OQ is the velocity of the centre-of-mass of the dipion system. QB is the velocity of pion B in the dipion centre-of-mass system. At low, non-relativistic energies, the diagram lies in a plane. AB is thus a measure of the $\pi\pi$ energy $\sqrt{s_{\pi\pi}}$. When A, B are like pions, χ tends to be smaller than the mean

value determined from phase space alone. Thus $s_{\pi^{\pm}\pi^{\pm}}$ tends to favour smaller values - giving a distortion in the mass distribution towards the lower end. On the other hand, when A, B are unlike pions χ tends to be large and consequently there is a lack of events in the low energy end of the mass spectrum.

Therefore since our model gives a fairly good description of the Goldhaber effect it should also describe this correlation effect in mass distributions. This can be seen from Fig. 8. Because the model overestimates the events lying in the resonance overlap region, the $\pi^+\pi^-$ spectra are too pronounced at the ρ and f positions.

IV. - CONCLUSIONS

At low energies the interaction between unlike pions is dominated by one or two partial waves which exhibit resonance behaviour. In order to explain the correlation effects observed in multi-pion final states, it is necessary to construct a model which takes this into account. Our aim here has been to formulate a simple model which incorporates the realistic description of low energy $\pi\pi$ interaction. We have disregarded the correct high energy behaviour since most events lie in the region of phase space corresponding to low $\pi\pi$ energies. The rather naïve model described in Section II is able to reproduce the general features seen in the Goldhaber angular distributions with the aid of no adjustable parameters. Furthermore the model gives a very good fit to all the single particle distributions.

In the model we have essentially replaced Reggeized nucleon exchange by a meson Regge pole. We are able to get away with this as far as the distributions are concerned but in attempting to explain cross-sections it virtually amounts to criminal neglect. The fast decrease in all cross-sections for $\bar{p}p \rightarrow n\pi$ is a sure indication of the occurrence of baryon exchange. Clearly the next step will be

to construct a model which incorporates baryon-exchange but also contains the $\pi\pi$ low energy interaction we have inserted perhaps as a final state interaction.

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FIGURE CAPTIONS

- Figure_1 Diagram of multi-Regge amplitude for the reaction $\bar{p}p \rightarrow n\pi$ involving only charged pions.
- Figure_2 Percentage histograms for the reaction $\bar{p}p \rightarrow 2\pi^+2\pi^-$ at 5.7 GeV/c. The distributions are of p_L , longitudinal momentum of π^- (reflected π^+) ; $\cos\theta_\pi$, direction of π^- (reflected π^+) ; p_T , transverse momentum of pion.
- Figure_3 Asymmetry parameter A as a function of beam momentum for reactions (2), (3), (4).
- Figure_4 Collimation parameter C as a function of beam momentum for reactions (2), (3), (4).
- Figure_5 Distributions in the Goldhaber angle χ for reactions (2), (3), (4).
- Figure_6 Asymmetries χ^U and χ^L in the Goldhaber angle for reactions (2), (3), (4).
- Figure_7 Correlation parameter C of the Goldhaber effect as a function of beam momentum, for reactions (2), (3), (4).
- Figure_8 Mass distributions for $\bar{p}p \rightarrow 3\pi^+3\pi^-$ at 3.6 GeV/c. Dashed curves correspond to phase space.
- Figure_9 Velocity diagram for a pion pair.

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