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ELECTROMAGNETIC NUCLEON FORM FACTORS IN THE
SPACELIKE ASYMPTOTIC REGION

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A B S T R A C T

We apply $O(3,1)$ analysis of current matrix elements to the electromagnetic current between single-nucleon states. The asymptotic behaviour of the form factors is described in terms of singularities of two reduced vertex functions. We consider in particular poles and cuts.

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1. - INTRODUCTION

In two previous articles ^{1),2)} we developed the harmonic analysis of tensor operators of the inhomogeneous Lorentz group on the homogeneous subgroup. We showed that this formalism if applied to the current density operator between single-particle states, can be used to describe the asymptotic properties of the form factors involved in the vertex function. The most elementary example, which is nevertheless of great physical interest, is the electromagnetic current between single-nucleon states. Apart from the transformation properties of the current matrix element under the proper orthochronous Lorentz group, we have to take into account parity and time reversal invariance, isospin, current conservation, and the hermiticity of the current operator. In order to include isospin we need only apply our formalism to the isoscalar and isovector vertex functions separately. The known transformation properties of the vertex function under the discrete symmetries imply certain symmetry and reality properties of the Fourier transforms which are easily figured out. Current conservation implies a normalization of the charge form factors at $q^2 = 0$, which can be accounted for by elementary means, and vanishing of the third component of the vector matrix element in the brick wall frame, which follows also from parity and time reversal invariance and the hermiticity of the current operator.

In Section 2 we derive the Fourier transforms of the vertex functions on the homogeneous Lorentz group ("reduced vertex functions") for the four possible types of generalized Dirac matrices. Because of the discrete symmetries only two of these reduced vertex functions are essentially independent. In Section 3 we give the contributions of poles in these reduced vertex functions to the form factors. According to whether such pole appears in the first or in the second independent reduced vertex function, we shall call it a "class one pole" or a "class two pole". In Section 4 we generalize this result to the case that reduced vertex functions possess cut singularities.

Our notations of physical quantities are conventional. Dirac type form factors F_1 and F_2 are normalized, Sachs type form factors G_E and G_M are not. For further details on the mathematical framework and notations we refer to Refs. ^{1),2)}.

2. - REDUCED VERTEX FUNCTIONS

We consider vertex functions

$$\Gamma_Q^J(p_2, p_1)_{q_2 q_1}$$

defined as the appropriately normalized ²⁾ matrix elements of the electromagnetic current density operator $j_\mu(0)$ between single-nucleon states of momenta p_2, p_1 and Wigner's spin q_2, q_1 (no helicities). The nucleon is assumed to have the common mass M . J, Q denote the components of the four-vector in the "canonical basis". This basis is most convenient for our purposes. The relation between the canonical basis and the familiar vector basis was explained in Ref. ²⁾, Appendix 2. We refer always to that system of inertia where particle 1 is at rest,

$$\begin{aligned} p_1 &= (M, 0, 0, 0) \\ p_2 &= (M \cosh \eta, 0, 0, -M \sinh \eta), \quad \eta \geq 0 \end{aligned} \quad (1)$$

such that

$$q^2 = (p_1 - p_2)^2 = -4M^2 \sinh^2 \frac{1}{2} \eta \quad (2)$$

Correspondingly we write

$$\Gamma_Q^J(p_2, p_1)_{q_2 q_1} = \Gamma_Q^J(\eta)_{q_2 q_1}$$

These vertex functions are expressible by form factors of the Sachs type as

$$\begin{aligned}\Gamma_0^0(\eta)_{\pm\frac{1}{2}, \pm\frac{1}{2}} &= e \pi^{\frac{1}{2}} \operatorname{ch} \frac{1}{2} \eta G_E(q^2) \\ \Gamma_0^1(\eta)_{\pm\frac{1}{2}, \pm\frac{1}{2}} &= e \left(\frac{\pi}{3}\right)^{\frac{1}{2}} \operatorname{sh} \frac{1}{2} \eta G_E(q^2) \\ \Gamma_{\pm 1}^1(\eta)_{\mp\frac{1}{2}, \pm\frac{1}{2}} &= e \left(\frac{2\pi}{3}\right)^{\frac{1}{2}} \operatorname{sh} \frac{1}{2} \eta G_M(q^2)\end{aligned}\quad (3)$$

In (3), either the upper or the lower signs have to be taken, all other combinations vanish.

Following the definition of Ref. ²⁾, Eq. (45), we introduce reduced vertex functions by

$$\begin{aligned}M_{\nu\mu}(\chi) &= \\ &= 2^{-\frac{1}{2}} e^{-1} \sum_{j_2} (-1)^{j_2} (2j_2+1) \langle \bar{\chi}; j_2 \| T^{j_2(1-\mu, 1-\nu)} \| \bar{\chi}_{\nu\mu}; \frac{1}{2} \rangle \\ &\times \sum_{Q Q_2} \left(\frac{1}{2}, Q+Q_2 | j_2 Q_2, j_2 Q \right) \int_0^\infty d\eta \operatorname{sh} \eta d_{\frac{1}{2} j_2 Q_2}^{\bar{\chi}}(\eta) \Gamma_Q^{j_2}(\eta)_{Q_2, Q+Q_2}\end{aligned}\quad (4)$$

where ν, μ take the values 0 and 1 and label the four four-vector operators $T_Q^J(\nu, \mu)$ of $SL(2, \mathbb{C})$, the "generalized Dirac matrices". χ denotes a representation of $SL(2, \mathbb{C})$ and $d_{J_1 J_2 Q}^{\chi}(\eta)$ the corresponding representation function in the canonical basis as defined by Ref. ¹⁾, Eq. (1.37). The round bracket denotes a Clebsch-Gordan coefficient of $SU(2)$. For further details of the notations see Refs. ^{1), 2)}. Only those representations are admitted which possess states with spin $\frac{1}{2}$. Using the notation

$$\chi = (m, \lambda)$$

where λ is connected with Gelfand's variable g used in ^{1), 2)} by

$$\lambda = \frac{i}{2} \vartheta$$

and m is an integer, we have representations χ with

$$m = \pm 1, \quad \lambda \quad \text{arbitrary complex.}$$

We consider first the cases $\nu = 1, \mu = 0$ and $\nu = 0, \mu = 1$. With the form factors F_1 and F_2 of Dirac type, the evaluation of (4) yields

$$\begin{aligned} \pi^{-\frac{1}{2}} M_{10}(1, \lambda) &= \\ &= \int_0^\infty d\eta \, \sinh \eta \left\{ (2F_1 + \kappa F_2) \left[\frac{\sinh(\lambda + \frac{1}{2})\eta}{\lambda + \frac{1}{2}} + \frac{\sinh(\lambda - \frac{1}{2})\eta}{\lambda - \frac{1}{2}} \right] \right. \\ &\quad \left. - 2(F_1 + \kappa F_2) [\sinh(\lambda + \frac{1}{2})\eta - \sinh(\lambda - \frac{1}{2})\eta] \right\} \end{aligned} \quad (5)$$

In (5) and later in (8), we presuppose that the integral transformations converge absolutely on the imaginary λ axis, i.e., the principal series of $SL(2, \mathbb{C})$. We can later abandon this hypothesis and replace it by a weaker assumption as was shown in Ref. ²⁾, Section 5. We inspect from (5) that $M_{10}(1, \lambda)$ is a real function on the imaginary λ axis, that it is also real on those pieces of the real λ axis on which it possesses a unique analytic continuation, and that it is symmetric like

$$M_{10}(1, \lambda) = M_{10}(1, -\lambda) \quad (6)$$

From Ref. ²⁾, Eq. (49), we have in addition

$$M_{01}(-1, \lambda) = -M_{10}(1, \lambda) \quad (7)$$

Next we consider $\nu = \mu = 1$ and $\nu = \mu = 0$. In these cases we have four reduced vertex functions $M_{11}(\pm 1, \lambda)$ and $M_{00}(\pm 1, \lambda)$ connected by

$$M_{11}(1, \lambda) = M_{11}(-1, \lambda)$$

$$M_{00}(1, \lambda) = M_{00}(-1, \lambda)$$

Evaluating (4) as before we obtain

$$\begin{aligned} \pi^{-\frac{1}{2}} M_{11}(1, \lambda) &= \\ &= \int_0^{\infty} d\eta \sinh \eta \left\{ (2F_1 + \kappa F_2) \frac{\sinh(\lambda + \frac{1}{2})\eta}{\lambda + \frac{1}{2}} \right. \\ &\quad \left. - \frac{1}{2} \kappa F_2 \left[\frac{\sinh(\lambda + \frac{3}{2})\eta}{\lambda + \frac{3}{2}} + \frac{\sinh(\lambda - \frac{1}{2})\eta}{\lambda - \frac{1}{2}} \right] \right\} \end{aligned} \quad (8)$$

The function $M_{11}(1, \lambda)$ is real on the line

$$\lambda = -\frac{1}{2} + i\sigma, \quad \sigma \text{ real}$$

and the real λ axis provided it possesses there a unique analytic continuation off the imaginary λ axis. In addition we inspect the symmetry

$$M_{11}(1, \lambda) = M_{11}(1, -\lambda - 1) \quad (9)$$

From Ref. ²⁾, Eq. (49), we find

$$M_{00}(1, \lambda) = (\lambda + \frac{1}{2})(\lambda - \frac{3}{2}) M_{11}(1, -\lambda) \quad (10)$$

This function $M_{00}(1, \lambda)$ is real on the line

$$\lambda = +\frac{1}{2} + i\sigma, \quad \sigma \text{ real}$$

and the real λ axis, and exhibits the symmetry

$$M_{00}(1, \lambda) = M_{00}(1, -\lambda + 1) \quad (11)$$

The reflection symmetries (6), (9), (11) can be made more explicit if we define instead of (5)

$$\begin{aligned} \pi^{-\frac{1}{2}} M_{10}^{(+)}(1, \lambda) &= \\ &= \frac{1}{2} \int_0^\infty d\eta \sinh \eta \left\{ (2F_1 + \kappa F_2) \left[\frac{e^{(\lambda + \frac{1}{2})\eta}}{\lambda + \frac{1}{2}} + \frac{e^{(\lambda - \frac{1}{2})\eta}}{\lambda - \frac{1}{2}} \right] \right. \\ &\quad \left. - 2(F_1 + \kappa F_2) [e^{(\lambda + \frac{1}{2})\eta} - e^{(\lambda - \frac{1}{2})\eta}] \right\} \end{aligned} \quad (12)$$

such that

$$M_{10}(1, \lambda) = M_{10}^{(+)}(1, \lambda) + M_{10}^{(+)}(1, -\lambda) \quad (13)$$

If the integral (12) converges absolutely on the imaginary λ axis, it converges also in the half plane $\text{Re } \lambda < 0$. In this half plane it defines a holomorphic function. The singularities of $M_{10}(1, \lambda)$ in the right half plane are therefore entirely contained in $M_{10}^{(+)}(1, \lambda)$ and those in the left half plane in $M_{10}^{(+)}(1, -\lambda)$. We may proceed similarly in the case of the functions $M_{01}(-1, \lambda)$, $M_{11}(\pm 1, \lambda)$, and $M_{00}(\pm 1, \lambda)$.

3. - CONTRIBUTIONS OF SINGLE POLES TO THE FORM FACTORS

The vertex functions can be represented as [see Ref. 2), Eq. (51)]

$$\begin{aligned}
 \Gamma_Q^J(\eta)_{q_2 q_1} &= \\
 &= \frac{i}{4\pi} \sum_{\nu, \mu=0,1} \sum_{m=\pm 1} \int_{-i\infty}^{+i\infty} d\lambda M_{\nu\mu}(m, \lambda) \\
 &\times \sum_{J_2=\frac{1}{2}, \frac{3}{2}} \langle \chi_i J_2 q_2 | T_Q^J(\nu, \mu) | \chi_{\nu\mu} i \frac{1}{2} q_1 \rangle e_{\frac{1}{2} J_2 q_2}^{\chi}(\eta)
 \end{aligned} \tag{14}$$

where $\chi = (m, \lambda)$ and $e_{J_1 J_2 q}^{\chi}(\eta)$ is the corresponding representation function of the second kind defined in Ref. ¹⁾, Eq. (1.40), which falls off exponentially when $\text{Re } \lambda$ tends to $-\infty$. The simplest possible analytic structure of the reduced vertex functions is meromorphy, and we are therefore interested in the contribution of one first order pole to the form factors. If the functions $M_{\nu\mu}$ are in fact meromorphic, the asymptotic behaviour of the form factors at $q^2 \rightarrow -\infty$ ($\eta \rightarrow +\infty$) is determined by the poles which are closest to the imaginary axis.

For $\nu = 1, \mu = 0$ and $\nu = 0, \mu = 1$, we make the ansatz

$$\begin{aligned}
 \pi^{-\frac{1}{2}} M_{10}(1, \lambda) &\sim \frac{1}{2} \left[\frac{\beta_1}{\lambda - \lambda_1} - \frac{\beta_1}{\lambda + \lambda_1} + \frac{\bar{\beta}_1}{\lambda - \bar{\lambda}_1} - \frac{\bar{\beta}_1}{\lambda + \bar{\lambda}_1} \right] \\
 \pi^{-\frac{1}{2}} M_{01}(-1, \lambda) &\sim \frac{1}{2} \left[\frac{\beta_1}{\lambda - \lambda_1} + \frac{\beta_1}{\lambda + \lambda_1} - \frac{\bar{\beta}_1}{\lambda - \bar{\lambda}_1} + \frac{\bar{\beta}_1}{\lambda + \bar{\lambda}_1} \right]
 \end{aligned} \tag{15}$$

where the symbol $(\sim)$ denotes "contains the term" and λ_1 and β_1 are complex numbers. For the sake of definiteness we assume $\text{Re } \lambda_1 < 0$. We call the quartet of poles in (15) a "pole contribution of the first class". If λ_1 is real, this quartet reduces to a doublet. We shall see later that this is necessary to exclude oscillating behaviour of the form factors. The ansatz (15) satisfies all symmetry requirements that were inspected from (5). Consequently the single contribution (15) to

the vertex functions behaves correctly under the discrete symmetry operations and current conservation. Only the normalization of the charge form factors cannot be achieved with a term (15), since the assumption that (15) represents the complete reduced vertex functions contradicts the known analytic properties of the form factors.

The contribution to the form factors due to the terms (15) is

$$G_E \sim -\frac{1}{2} \operatorname{Re} \left\{ \beta_1 \frac{2 \sinh^2 \frac{1}{2} \eta}{\sinh^3 \eta} \left(\frac{e^{(\lambda_1 - \frac{1}{2}) \eta}}{\lambda_1 - \frac{1}{2}} + \frac{e^{(\lambda_1 + \frac{1}{2}) \eta}}{\lambda_1 + \frac{1}{2}} \right) \right.$$

$$G_M \sim -\frac{1}{2} \operatorname{Re} \left\{ \beta_1 \left[\frac{2 \cosh \frac{1}{2} \eta}{\sinh^2 \eta} e^{\lambda_1 \eta} - \frac{\cosh \eta}{\sinh^3 \eta} \left(\frac{e^{(\lambda_1 - \frac{1}{2}) \eta}}{\lambda_1 - \frac{1}{2}} + \frac{e^{(\lambda_1 + \frac{1}{2}) \eta}}{\lambda_1 + \frac{1}{2}} \right) \right] \right\} \quad (16)$$

If λ_1 is complex, $\lambda_1 = \lambda_1' + i\lambda_1''$, we get oscillating terms proportional to

$$\cos \left[\lambda_1'' \log \left(-\frac{q^2}{M^2} \right) + c \right]$$

Setting $\lambda_1'' = 0$ and assuming that (16) describes the form factors in the asymptotic region, we find

$$G_E \cong C_E^1 \left(-\frac{q^2}{M^2} \right)^{\lambda_1 - \frac{3}{2}}$$

$$G_M \cong C_M^1 \left(-\frac{q^2}{M^2} \right)^{\lambda_1 - \frac{3}{2}}$$

$$C_E^1 : C_M^1 = \left(\lambda_1 - \frac{1}{2} \right)^{-1} \quad (17)$$

A priori we would exclude $\lambda_1 = -\frac{1}{2}$ because of the pole in (16). We may, however, approach this value, if we let β_1 tend to zero correspondingly. The result (17) remains valid in this case. We may even allow for positive λ_1 . In such case the Fourier transforms of

the vertex functions do not exist in the proper sense (see the discussion in Ref. ²⁾, Section 5). We regularize the form factors first and remove the cut-off in (14). This forces us to deform the integration contour in order to evade the poles moving against it. The expression (16) remains valid and the asymptotic behaviour is still given by (17) as long as $\lambda_1 \neq \frac{1}{2}$. In the case that precisely $\lambda_1 = \frac{1}{2}$ the asymptotic properties of (16) can easily be obtained in the limit $\beta_1 \rightarrow 0$.

A pole contribution of the second class is defined by

$$\begin{aligned} \pi^{-\frac{3}{2}} M_{11}(\pm 1, \lambda) &\sim \frac{1}{2} \left[\frac{\beta_2}{\lambda - \lambda_2} - \frac{\beta_2}{\lambda + \lambda_2 + 1} + \frac{\bar{\beta}_2}{\lambda - \bar{\lambda}_2} - \frac{\bar{\beta}_2}{\lambda + \bar{\lambda}_2 + 1} \right] \\ \pi^{-\frac{3}{2}} M_{00}(\pm 1, \lambda) &\sim \frac{1}{2} (\lambda_2 - \frac{1}{2})(\lambda_2 + \frac{3}{2}) \left[-\frac{\beta_2}{\lambda + \lambda_2} + \frac{\beta_2}{\lambda - \lambda_2 - 1} \right] \\ &\quad + \frac{1}{2} (\bar{\lambda}_2 - \frac{1}{2})(\bar{\lambda}_2 + \frac{3}{2}) \left[-\frac{\bar{\beta}_2}{\lambda + \bar{\lambda}_2} + \frac{\bar{\beta}_2}{\lambda - \bar{\lambda}_2 - 1} \right] \end{aligned} \quad (18)$$

In general β_2 and λ_2 are complex numbers. From (14) and (18) we obtain

$$\begin{aligned} G_E &\sim -\frac{1}{2} \operatorname{Re} \left\{ \beta_2 (\lambda_2 - \frac{1}{2})(\lambda_2 + \frac{3}{2}) \frac{2 \gamma h^2 \frac{1}{2} \eta}{\gamma h^3 \eta} \right. \\ &\quad \times \left(\frac{e^{(\lambda_2 - \frac{1}{2})\eta}}{\lambda_2 - \frac{1}{2}} + 2 \frac{e^{(\lambda_2 + \frac{1}{2})\eta}}{\lambda_2 + \frac{1}{2}} + \frac{e^{(\lambda_2 + \frac{3}{2})\eta}}{\lambda_2 + \frac{3}{2}} \right) \left. \right\} \end{aligned}$$

$$\begin{aligned} G_M &\sim -\frac{1}{2} \operatorname{Re} \left\{ \beta_2 (\lambda_2 - \frac{1}{2})(\lambda_2 + \frac{3}{2}) \frac{1}{\gamma h^3 \eta} \right. \\ &\quad \times \left(\frac{e^{(\lambda_2 - \frac{1}{2})\eta}}{\lambda_2 - \frac{1}{2}} - 2 \gamma h \eta \frac{e^{(\lambda_2 + \frac{1}{2})\eta}}{\lambda_2 + \frac{1}{2}} + \frac{e^{(\lambda_2 + \frac{3}{2})\eta}}{\lambda_2 + \frac{3}{2}} \right) \left. \right\} \end{aligned} \quad (19)$$

A non-vanishing imaginary part of λ_2 causes oscillations just as in (16). If λ_2 is real and if the form factors are dominated by a pole term of the second class (19), we have

$$\begin{aligned} G_E &\approx C_E^2 \left(-\frac{q^2}{M^2}\right)^{\lambda_2 - \frac{1}{2}} \\ G_M &\approx C_M^2 \left(-\frac{q^2}{M^2}\right)^{\lambda_2 - \frac{3}{2}} \\ C_E^2 : C_M^2 &= -\frac{1}{2} \left(\lambda_2 + \frac{1}{2}\right) \end{aligned} \quad (20)$$

This result (20) can be extended to all values except $\lambda_2 = \pm 1$.

The fact that the magnetic form factor in (20) drops faster than the charge form factor makes it in principle possible that the nearest pole is of class two and determines the asymptotic charge form factor, whereas a closely following pole of class one dominates the magnetic form factor. Because of the negative sign of the ratio $c_E^1 : c_M^1$ in (17) when $\lambda_1 \approx -\frac{1}{2}$ such a complicated model seems to be necessary to maintain the scaling law of the proton form factors in the asymptotic region (of course there is no reason for such desire). For the sake of a tentative comparison with the observed form factors we prefer a simpler model. We assume first that the isovector part of proton and neutron form factors dominate in the asymptotic region. Next we assume that the isovector form factors are described in this region by a pole of the first class. This implies that the magnetic form factors of proton and neutron tend to zero in the same fashion though from opposite sides, i.e., $\lim G_{Mp} : G_{Mn} = -1$. The magnetic form factors need not have zeros. The proton charge form factor has to go through a zero, however, whereas the neutron charge form factor may be positive definite. The zero of the charge form factor of the proton should lie below the asymptotic region, i.e., somewhere in

$$-\frac{q^2}{M^2} \lesssim 10$$

4. - A GENERALIZATION

In Ref. ²⁾, we tried to make plausible that poles in the reduced vertex functions reflect a tower structure of the external particles. Such tower structure of the particles and resonances observed in production processes has often been suggested but never been verified convincingly, probably partly due to the lack of a unique concept for the description of these towers. Without further verification let us assume that poles in the reduced vertex functions are an appropriate tool for this task. Then the problem arises to allow for modifications in this picture which take into account the approximate nature of the infinite tower model. It may happen that the poles have to be smeared out, such that the reduced vertex functions exhibit cut singularities, whose discontinuities consist of a background with resonances superimposed.

A hint for the existence of such cuts comes also from attempts to determine the form factors in the asymptotic region by field theory, either with perturbative methods ³⁾, or by estimating solutions of a Bethe-Salpeter equation ⁴⁾. These authors found a dominating term of the type

$$\left(-\frac{q^2}{M^2}\right)^{\frac{1}{2}\alpha} \left[\log\left(-\frac{q^2}{M^2}\right)\right]^{\beta}$$

with integers α and β . The logarithmic factors can also be traced back to cuts. Though the methods of Ref. ⁴⁾ cannot exclude oscillating terms of the kind discussed in Section 3, it seems ^{3), 5)} that field theory excludes these terms and that the cuts of the reduced vertex functions have consequently to be placed on the real λ axis. We suggest therefore the following singularities

$$\begin{aligned} \text{disc } \{ \pi^{-\frac{1}{2}} M_{10}(1, \lambda) \} &= \\ &= \pi^{-\frac{1}{2}} M_{10}(1, \lambda + i\varepsilon) - \pi^{-\frac{1}{2}} M_{10}(1, \lambda - i\varepsilon) \\ &= 2\pi i \left[\beta_1(\lambda) \Theta(\lambda_1 - \lambda) - \beta_1(-\lambda) \Theta(\lambda_1 + \lambda) \right] \end{aligned} \quad (21)$$

with a real measure $\beta_1(\lambda)d\lambda$. We assume $\lambda_1 < 0$. This inequality eventually holds only after a regularization, which we may remove afterwards. The reflection symmetry (6) has been taken into account in (21). The corresponding cut structure of $M_{01}(-1, \lambda)$ can be obtained from (7) and (21). For $M_{11}(\pm 1, \lambda)$ we make the corresponding ansatz

$$\begin{aligned} \text{disc } \{ \pi^{-\frac{1}{2}} M_{11}(\pm 1, \lambda) \} = \\ = 2\pi i (\lambda - \frac{1}{2})^{-1} (\lambda + \frac{3}{2})^{-1} [\beta_2(\lambda) \Theta(\lambda_2 - \lambda) - \beta_2(-\lambda - 1) \Theta(\lambda_2 + \lambda + 1)] \end{aligned} \quad (22)$$

in which $\beta_2(\lambda)d\lambda$ is again a real measure. In addition we assume $\lambda_2 < -\frac{1}{2}$. The cut structure of $M_{00}(\pm 1, \lambda)$ follows from (10) and (22). Inserting (21), (22) into (14), and assuming that the functions $M_{\nu\mu}$ do not increase faster than

$$e^{\alpha|\lambda|^\gamma}, \quad \gamma \leq 1$$

parallel to the real λ axis (parallel to the imaginary axis these functions fall off like a Fourier transform of an infinitely differentiable function) we obtain

$$\begin{aligned} G_E &= G_E^1 + G_E^2 \\ G_M &= G_M^1 + G_M^2 \end{aligned} \quad (23)$$

$$\begin{aligned} G_E^1 &= \frac{1}{2} \int_{-\infty}^{\lambda_1} d\lambda \beta_1(\lambda) \frac{2\gamma h^{\frac{1}{2}} \eta}{\gamma h^3 \eta} \left[\frac{e^{(\lambda - \frac{1}{2})\eta}}{\lambda - \frac{1}{2}} + \frac{e^{(\lambda + \frac{1}{2})\eta}}{\lambda + \frac{1}{2}} \right] \\ G_M^1 &= \frac{1}{2} \int_{-\infty}^{\lambda_1} d\lambda \beta_1(\lambda) \left[\frac{2\gamma h^{\frac{1}{2}} \eta}{\gamma h^2 \eta} e^{\lambda \eta} - \frac{\gamma h \eta}{\gamma h^3 \eta} \left(\frac{e^{(\lambda - \frac{1}{2})\eta}}{\lambda - \frac{1}{2}} + \frac{e^{(\lambda + \frac{1}{2})\eta}}{\lambda + \frac{1}{2}} \right) \right] \end{aligned}$$

$$G_E^2 = \frac{1}{2} \int_{-\infty}^{\lambda_2} d\lambda \beta_2(\lambda) \frac{2\gamma h^2 \frac{1}{2}\eta}{\gamma h^3 \eta} \times \left[\frac{e^{(\lambda-\frac{1}{2})\eta}}{\lambda-\frac{1}{2}} + 2 \frac{e^{(\lambda+\frac{1}{2})\eta}}{\lambda+\frac{1}{2}} + \frac{e^{(\lambda+\frac{3}{2})\eta}}{\lambda+\frac{3}{2}} \right]$$

$$G_M^2 = \frac{1}{2} \int_{-\infty}^{\lambda_2} d\lambda \beta_2(\lambda) \frac{1}{\gamma h^3 \eta} \times \left[\frac{e^{(\lambda-\frac{1}{2})\eta}}{\lambda-\frac{1}{2}} - 2\gamma h \eta \frac{e^{(\lambda+\frac{1}{2})\eta}}{\lambda+\frac{1}{2}} + \frac{e^{(\lambda+\frac{3}{2})\eta}}{\lambda+\frac{3}{2}} \right] \quad (24)$$

If $\gamma = 1$ these representations (24) are valid only for $\eta > \alpha$. At the poles $\lambda = \pm\frac{1}{2}, -\frac{3}{2}$ the weight functions $\beta_{1,2}(\lambda)$ must have appropriate zeros. After a removal of the regularization we may permit any real numbers $\lambda_{1,2}$ in (24).

If the weight functions $\beta_{1,2}(\lambda)$ fall off sufficiently fast, the representations (24) can be continued analytically to $q^2 = 0$ ($\eta=0$). Analyticity of G_E and G_M at $q^2 = 0$ allows us to derive sum rules. In particular we find the normalization condition

$$G_E(0) = \frac{1}{2} \int_{-\infty}^{\lambda_1} d\lambda \beta_1(\lambda) + \int_{-\infty}^{\lambda_2} d\lambda \beta_2(\lambda) \quad (25)$$

It seems premature to attempt a global fit of the form factors by means of their representations (24). A very promising way to gain more theoretical information on the weight functions $\beta_{1,2}(\lambda)$ is to solve a Bethe-Salpeter equation with sufficient rigour. Such a solution could give us a hint on the connection between the structure of a tower of bound states, in particular its mass and spin spectrum, and the form of the weight functions, in particular the existence of resonances, their positions and widths. If the nearest resonance in a weight function is close to the threshold $\lambda_{1,2}$, it determines the form factors in a semi-asymptotic domain, in this domain the formulae for the poles given in

Section 3 are valid. In the true asymptotic domain only an infinitesimal neighbourhood of the threshold counts, and the fall-off of the form factors is necessarily slower than in the semi-asymptotic region.

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