EXCHANGE DEGENERACY FROM FESR AND THE $Y^*$ RESONANCES

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ABSTRACT

Exchange degeneracy takes different forms in FESR bootstrap models and in N/D models. We show that the more restrictive prediction of the FESR is verified experimentally: the sequences of $Y^*$'s, as observed in $K^-p$ elastic scattering, show exchange degeneracy for $\alpha(s)$ and $\beta(s)$, although there exist important coupled channels ($\Xi^-\Sigma^-, \Xi^-\Lambda$) which show a breaking of exchange degeneracy.

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We assume that the KN system does not contain resonances (at least in a first approximation) *). We further assume that forces are due to one-particle exchanges. (This corresponds to the assumption of resonance saturation in the FESR.) Under these assumptions the channel $\bar{K}N \rightarrow \bar{K}N$, considered by itself (without coupled channels), contains only "direct forces", i.e., meson exchanges, but no "exchange forces", i.e., no baryon exchanges, because there is no baryon with the quantum numbers of (KN).

If there are only direct forces, they generate resonances which lie on one trajectory for even and odd \( \ell \), e.g., \( 1^+, 3^-, 5^+ \), \( 1^-, 3^+, 5^- \), \( 2^+, 2^-, 2^+ \), \( 2^-, 2^+, 2^- \). The couplings of each sequence are given by one residue function.

But in our case there exist important other channels which couple to $\bar{K}N$, namely $\Lambda T$ and $\Sigma T$. In these other channels there are both types of forces, meson exchange and baryon exchange. The same is true for $\bar{K}N \rightarrow (\Lambda \text{or} \Sigma) T$.

In the coupled channel case the difference between various dynamical schemes becomes important. If we do an N/D bootstrap, we conclude that there is no reason to have any type of exchange degeneracy, because some of the channels have both types of forces. And because of unitarity this breaking of exchange degeneracy propagates into all coupled channels. On the other hand, in a FESR bootstrap scheme it is rigorously possible to consider each amplitude separately, $\bar{K}N$ elastic, $T \Sigma$ elastic, $\bar{K}N \rightarrow T \Sigma$, etc.; and it immediately follows (assuming resonance saturation) that an FESR bootstrap scheme predicts that the $Y^*$ resonances as observed in $\bar{K}N \rightarrow \bar{K}N$ are exchange degenerate (in $\mathcal{A}$ and $\mathcal{B}$, i.e., in mass and coupling strength).

*) The 1865 MeV $K^+p$ bump, even if it is a true resonance, is unimportant. For $t=0$, $\sigma_{\text{tot}}$, we note that \((J+\frac{1}{2})_X\) is only 0.35 compared to 1.90 for the $Y^*_0(1815)$. For $t>0$, where the resonance is used in FESR bootstrap calculations, it becomes relatively even less important, because it is only a P wave ($P_1$ or $P_3$), while the $Y^*_0(1815)$ is an $F_5$ wave.
Let us test this prediction experimentally. We consider the $Y^*_0$ in the Rosenfeld table. First we notice the beautiful exchange degenerate sequence which starts with the $\Lambda (1115): \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \frac{7}{2}^-, \frac{9}{2}^+$ (?). This is shown in Fig. 1a.

But there seem to be quite a few other $Y^*_0$ resonances which do not fit at all into the exchange degeneracy pattern. There are no exchange degenerate partners for the following resonances: $Y^*_0(1405, \frac{1}{2}^-)$ which dominates low energy $\bar{K}N$ scattering, $Y^*_0(1650, \frac{3}{2}^-)$, $Y^*_0(1830, 5/2^-)$, $Y^*_0(1660, 1^+)$. 

We must remember that the FESR do not predict that the $Y^*_0$ per se are exchange degenerate. Only when we select those $Y^*_0$'s which strongly couple to $\bar{K}N$ will we observe exchange degeneracy. Note, e.g., that the $Y^*_0(1815, 5/2^+)$ and the $Y^*_0(2100, 7/2^-)$ couple to $\bar{K}N$ seven times more strongly than to $\Sigma \bar{N}$, while the $Y^*_0(1830, 5/2^-)$ coupling to $\bar{K}N$ is five times weaker than to $\Sigma \bar{N}$, and the $Y^*_0(1690, \frac{3}{2}^-)$ coupling to $\bar{K}N$ is three times weaker than to $\Sigma \bar{N}$.

We further select those $Y^*_0$'s which are peripheral, $\ell \approx kR$, and neglect the ones in central partial waves, $\ell \ll kR$; in other words we neglect the S and P wave resonances at higher energy.

Both selections (large partial width to $\bar{K}N$, peripheral $\ell$) are most conveniently done in a quantitative way by considering the relative strength of the resonances in $\bar{K}N$ (s channel resonances) at $t = m_\rho^2$. The $P_\ell (z_s)$ in the resonance numerator will automatically enhance peripheral resonances. The resonance strength at $t = m_\rho^2 m_\omega^2$ or $t = m_{A_2}^2$ is more relevant in dynamical calculations, e.g., in a FESR bootstrap.

*) The couplings $g^2$ have different dimensions for different spin. In order to compare them numerically we must introduce a mass. A very convenient mass is $m_\rho^2$. 
Figure 1b shows quantitatively *) that those resonances which do not fit into the exchange degeneracy scheme couple very weakly to $\bar{K}N$ at $t=m_\rho^2$, either because of small partial width or because they occur in central partial waves. We have plotted $C[\text{Im}B]$ which is the contribution of one resonance to $\int \text{Im}Bd\theta$. We consider the amplitude $B(I_s=0)$, as opposed to $B(I_t=0,1)$.

Figure 1b also shows that not only the masses are exchange degenerate, but also the couplings **). If we draw a smooth curve through the couplings of $1^+,\frac{5^+}{2},\frac{9^+}{2}$ and another smooth curve with the same shape but different normalization through $\frac{3^-}{2},\frac{7^-}{2}$, we see that the normalization is about the same in both cases ***).

The situation for the $Y_1^*$'s is analogous, and it is shown in Figs 2a and 2b. Note that the $\Sigma(1193)$ and the $Y_1^*(1910,5/2^+)$ do not occur in Fig. 2a, because they both couple very weakly to $\bar{K}N$. This implies a special value of the $f/d$ ratio for both the $1^+$ and the $\frac{5^+}{2}$ octets: $\alpha = f/(f+d) \approx \frac{1}{3}$.

*) We used Kim 2) for the $\Lambda(1115)$ and $\Sigma(1193)$, Warnock and Frye 3) for the $Y_0^*(1405)$ and $Y_1^*(1385)$, the CERN-Heidelberg-Saclay 4) values for the mass region 1660-2000 MeV, and the Rosenfeld table 5) for the rest.

**) Collins and Squires 6) suggested that only the masses should be degenerate but not the residue functions. Their argument was based on the quark model.

***) Semi-local duality predicts the expected shape of the curves in Fig. 1b. Since $B_{\text{Regge}} \sim \sqrt{\alpha(t)-1}$ we expect $C[B] \sim \text{const}$ for those energies for which saturation of the amplitude by this one tower of resonances is good. At higher energies we expect the contributions of this tower to drop off "exponentially".
The SU$_3$ classification changes along the exchange degenerate $Y_1^*$ sequence. The $Y_1^*(3^+)$ and $Y_1^*(7/2^+)$ are decuplet partners of the $(3^+, 1238)$ and $(7/2^+, 1920)$. On the other hand the $Y_1^*(5/2^-)$ has no partner in the \(\Delta\) series \(^*)\); and the \(\Delta\) series shows no exchange degeneracy, because there is a strong baryon exchange force. Therefore the $Y_1^*(5/2^-)$ cannot belong to a decuplet. Rather it belongs into an octet with the $N(1680)$ and $\Lambda(1830)\). This situation is analogous to $\pi^+\pi^- \rightarrow \pi^+\pi^-$ scattering. The absence of a force corresponding to $\pi^+\pi^+$ exchange forces the $f_0$ and the $f$ to be exchange degenerate in this particular channel, although they have a different SU$_2$ classification.

The N/D and the FESR schemes do not give contradicting predictions about exchange degeneracy. Unitarity (i.e., the N/D scheme) does not say that the exchange degeneracy must be broken for the $Y^*$'s in the $\bar{K}N$ channel, it merely says that in general it will be broken, unless there is some "accidental" cancellation of the effects of baryon exchange forces. The FESR predict that this "accident" must necessarily happen. Experiment confirms this prediction of the FESR.

Summarizing we can say: for every missing particle in some channel, there is a corresponding exchange degeneracy of resonances in the two crossed channels.

Our discussion has dealt with resonances on their mass shells, i.e., with exchange degeneracy for $\alpha(t)$, $\beta(t)$ with $t>0$. Exchange degeneracy for total cross-sections, i.e., at $t=0$, is also well established. On the other hand the situation for $t<0$ (and large $s$) is far from clear \(^**\).

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\(^*)\) The $\Delta(5/2^-, 1950)$ found by Lovelace "requires some imagination" \(^5\). In any case, it would not be exchange degenerate with the $\Delta(3^+)$ and the $\Delta(7/2^+)$.  

\(^**\) If we assume that the $Y^*_o$'s and the $Y^*_1$'s are exchange degenerate not only for $u>0$ but also for $u<0$, and if we assume that cuts are not important for $u<0$ then we predict that there is no polarization in $K\bar{N}$ backward scattering at high energies because the amplitudes are purely real.
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REFERENCES


4) R. Armenteros et al., CERN preprint, Sept. 1968.

5) N. Barash-Schmidt et al., UCRL-8030, Aug. 1968.


7) R.D. Tripp, rapporteur talk at the Vienna Conference (1968).
FIGURE CAPTIONS

Figure 1: a) shows the exchange degenerate sequence \( \Lambda_\alpha - \Lambda_\gamma \); b) shows that those resonances which do not fit into the exchange degeneracy pattern give very small contributions where it counts dynamically, namely at \( t = m_\rho^2 \); b) also shows that the resonances included in a) have not only exchange degenerate masses, but also exchange degenerate couplings. The parameters are taken from Refs. 2), 3), 4) and 5).

Figure 2: Same as Fig. 1 but for the exchange degenerate sequence \( \Sigma_\delta - \Sigma_\rho \). Note that the \( \Sigma_\alpha \) and \( \Sigma_\gamma \) trajectories couple very weakly to \( \bar{K}N \).
FIG. 2a

FIG. 2b

C[ImB] (GeV⁻¹)

at \( t = m_p^2 \)
in \( \bar{K} N \)