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A METHOD FOR THE DETERMINATION OF THE SPIN AND PARITY
OF A RESONANCE THROUGH SPIN-ROTATION PARAMETER MEASUREMENTS

Nils A. Törnqvist ^{*})

CERN - Geneva

A B S T R A C T

A simple graphical method for studying polarization phenomena is presented. It provides a new method for the determination of the spin and parity of a resonance. This method of spin parity determination requires spin rotation parameter data. It is based on the observation that the spin and parity of the dominating partial wave is simply related to the number of times and the direction the final polarization vector rotates, when the scattering angle is varied from 0 to 180°. This spin parity determination would be independent of corresponding determinations using angular distributions. Apart from the spin-parity determination, the graphical technique is useful for a compact description of spin effects in terms of directly observable quantities. The method is applied and tested by plotting predictions of $\pi N \rightarrow \pi N$ phase shift analysis.

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^{*}) On leave from Institute of Nuclear Physics, University of Helsinki, Helsinki.

1. - INTRODUCTION

The only directly observable quantities in a reaction $AB \rightarrow CD$ are the positive-definite cross-sections as functions of energy, scattering angle and spin configuration of the particles $\sigma(E, \theta, \bar{P}_A, \bar{P}_B, \bar{P}_C, \bar{P}_D)$, where A, B, C and D refer to other quantum numbers than those of space and spin, and $\bar{P}$ is the polarization vector. By using basic quantum mechanics and by imposing symmetries, theory furnishes the relations to express these cross-sections in terms of complex amplitudes, not observed directly. In this paper a particularly simple graphical method is presented, which is very useful for studying the spin dependence in the particular case of spin $(0^-, \frac{1}{2}^+)$ scattering. With this method, the relations between the observable cross-sections and the underlying theory can easily be understood, and as an application it provides a new method for a direct and sensitive determination of the spin-parity of a resonance.

The method involves graphs, which are projections in different planes of a three-dimensional diagram. Such a graph can be interpreted in three different ways :

- (i) As a plot of the final polarization vector projected on a particular plane. A special case is a plot of the spin rotation parameters A and R against each other or a plot of one of these against the polarization parameter α .
- (ii) As a representation of the complex quantity $r = g/f$ (the ratio between the spin-flip and the non-spin-flip amplitude) on the complex sphere (Riemann sphere) in a particular projection.
- (iii) As graphs relating the observable quantities $\sigma(\bar{P}_B, \bar{P}_D)/\sigma_0$ where σ_0 is the differential cross-section for an unpolarized target.

By varying the energy or the scattering angle one obtains trajectories in the graphs. When the scattering angle is varied from 0 to 180° , the number of loops in the trajectory and the direction of these loops are uniquely related to the spin and parity of the dominating partial wave. This property can be used for a direct determination of the spin and parity of a resonance. This method is independent of corresponding determinations from angular distributions.

Although no spin rotation data have yet been published which could be used for this kind of analysis, they can to-day, due to the advances in the technique of polarized targets, be measured with a remarkable accuracy. A group at CERN expects to achieve an accuracy of 10% in π^+p scattering at high energies ($p > 5$ GeV/c) and low momentum transfers ¹⁾. At lower energies it would be possible to measure in the whole angular interval because the energy of the scattered nucleon is always low.

For other methods of spin and parity determination, see the review article in Ref. ²⁾. Predictions of spin rotation parameters from phase shift analysis or Regge pole theory have been studied by several authors ³⁾⁻⁶⁾. Many articles discussing polarized targets and their use can be found in Ref. ⁷⁾.

In the following Section, we briefly summarize the basic formulas for spin effects in spin $(0^-, \frac{1}{2}^+)$ scattering, in order to define the notation and the conventions. In the third Section, we describe the graphical method and discuss the spin parity determination. In Section 4, we discuss the possible applications and test our method by studying predictions of phase shift analysis in $\pi N \rightarrow \pi N$ scattering.

2. - SPIN EFFECTS IN PSEUDOSCALAR MESON AND SPIN $\frac{1}{2}^+$ BARYON
SCATTERING

The scattering matrix for spin $(0^-, \frac{1}{2}^+)$ scattering can be written in different ways

$$M = -A + iB \gamma_\mu q_\mu = f_1 + f_2 \frac{\bar{\sigma} \cdot \bar{k}^f \bar{\sigma} \cdot \bar{k}^i}{|\bar{k}^i|^2} = f + ig \bar{\sigma} \cdot \bar{n} \quad , \quad (1)$$

where A and B are the Lorentz invariant amplitudes, f_1 and f_2 the helicity amplitudes, and f and g the non-spin-flip and the spin-flip amplitude, respectively, q_μ is the four-momentum, $\bar{k}^i$ and $\bar{k}^f$ are the initial, resp., final c.m.s. momenta of the baryon, $\bar{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices and $\bar{n}$ is the normal to the scattering plane. Furthermore one can use the helicity non-flip and the helicity flip amplitudes

$$f_{++} = \cos \theta/2 f - \sin \theta/2 g \quad , \quad (2)$$

$$f_{+-} = \sin \theta/2 f - \cos \theta/2 g \quad , \quad (3)$$

where θ is the c.m.s. scattering angle.

A convenient way to discuss polarization effects is in terms of the spin space density matrix. If the target baryon has a polarization $\bar{P}^i$, its density matrix is

$$\rho^i = \frac{1}{2} (1 + \bar{P}^i \cdot \bar{\sigma}) \quad . \quad (4)$$

The density matrix for the final baryon is then

$$\rho^f = M \rho^i M^\dagger = \frac{1}{2} \sigma_0 \{ (1 + \alpha \bar{P}^i \cdot \bar{n}) 1 + \bar{P}^f \cdot \bar{\sigma} \} \quad , \quad (5)$$

where σ_0 is the differential cross-section for an unpolarized target, α is the ordinary polarization parameter, $\bar{n}$ is the normal to the scattering plane, and $\bar{p}^f$ is the polarization vector of the outgoing baryon

$$\sigma_0 = |f|^2 + |g|^2, \quad (6)$$

$$\alpha = 2 \operatorname{Im} fg^* / \sigma_0, \quad (7)$$

$$\bar{n} = \frac{\bar{k}^i \times \bar{k}^f}{|\bar{k}^i \times \bar{k}^f|}, \quad (8)$$

$$\begin{aligned} \bar{p}^f &= (\alpha + \bar{p}^i \cdot \bar{n}) \bar{n} + \beta \bar{p}^i \times \bar{n} - \gamma (\bar{p}^i \times \bar{n}) \times \bar{n} = \\ &= \{ \alpha + (1 - \gamma) \bar{p}^i \cdot \bar{n} \} \bar{n} + \beta \bar{p}^i \times \bar{n} + \gamma \bar{p}^i, \end{aligned} \quad (9)$$

where

$$\beta = 2 \operatorname{Re} fg^* / \sigma_0, \quad (10)$$

$$\gamma = \{ |f|^2 - |g|^2 \} / \sigma_0. \quad (11)$$

The polarization parameter α is the easiest one to measure. This can be done either by measuring the differential cross-section from a target polarized in the direction $\bar{n}$, or by observing the polarization of the final baryon in the same direction.

In order to measure the two other parameters β and γ the target polarization must be in the scattering plane. Then we have for the final polarization vector

$$\bar{p}^f = \alpha \bar{n} + \beta \bar{m} + \gamma \bar{l}, \quad (12)$$

where $\bar{n}$, $\bar{m}$ and $\bar{l}$ are orthogonal vectors, $\bar{m} = \bar{l} \times \bar{n}$ and $\bar{l} = \bar{P}^i$. Assuming for simplicity that the initial polarization is 100%, $\bar{l}$, $\bar{m}$ and $\bar{n}$ form an orthonormal basis (the helicity frame of the initial baryon).

The components of the polarization which are orthogonal to the momentum are the most readily measured quantities, while the parallel component would involve a spin rotating magnet. However, both β and γ can be measured as the orthogonal component if the experimental conditions are arranged so that the target polarization is parallel (β) or orthogonal (γ) to the momentum of the scattered baryon. Experimentally more convenient quantities are the Wolfenstein parameters ³⁾

$$A = -\beta \cos \theta - \gamma \sin \theta, \quad (13)$$

$$R = -\beta \sin \theta + \gamma \cos \theta, \quad (14)$$

which are equal to the orthogonal component (in the scattering plane) of the final polarization, when the target polarization is parallel (A), resp., orthogonal (R) to the beam momentum (see Fig. 1). In terms of the amplitudes f_{++} and f_{+-} [Eqs. (2) and (3)] we have

$$A = -2 \operatorname{Re} f_{++} f_{+-}^* / \sigma_0 \quad (13')$$

$$R = \{ |f_{++}|^2 - |f_{+-}|^2 \} / \sigma_0 \quad (14')$$

Thus in the helicity frame of the final baryon the components of the polarization vectors are α , A and R.

All the quantities above are defined in the c.m.s. They are in a non-relativistic approximation correct also in the laboratory if θ is replaced by the laboratory scattering angle θ_L . The relativistically correct expressions for A and R are obtained by replacing θ by $\theta + \bar{\theta}_L$ ⁸⁾ rather than by θ_L (the bar denotes

the complement angle $180^\circ - \theta = \bar{\theta}$). Thus relativistically there appears an additional spin rotating effect in the scattering plane by the angle $2\theta_L - \bar{\theta}$ ^{9),10)}. (This is related to the so-called Wick rotation.) The Wolfenstein parameters for the recoil baryon in the laboratory are thus ¹¹⁾

$$A_{rec} = -\beta \cos(\bar{\theta} - \bar{\theta}_L) + \gamma \sin(\bar{\theta} - \bar{\theta}_L), \quad (15)$$

$$R_{rec} = \beta \sin(\bar{\theta} - \bar{\theta}_L) + \gamma \cos(\bar{\theta} - \bar{\theta}_L). \quad (16)$$

Because of conventional reasons there is a change of sign in the definition of A_{rec} and R_{rec} compared to A and R .

Observe finally that α , β and γ are not independent but satisfy

$$\alpha^2 + \beta^2 + \gamma^2 = 1, \quad (17)$$

and similarly

$$\alpha^2 + A^2 + R^2 = 1, \quad (18)$$

$$\alpha^2 + A_{rec}^2 + R_{rec}^2 = 1. \quad (19)$$

Thus measurement of α and one of the spin rotation parameters determines the spin dependence completely up to a sign of A or R . A useful quantity is the spin rotation angle

$$\phi = \arctan \frac{\beta}{\gamma} = \arctan \frac{A}{R} - \theta. \quad (20)$$

If the spin dependence is completely known experimentally then f and g can be determined up to a common phase, which at least in principle would determine the phase shifts unambiguously.

3. - THE GRAPHS AND THEIR APPLICATION FOR SPIN AND PARITY DETERMINATION

In the following we shall assume that the initial polarization vector $\vec{P}^i$ is in the scattering plane, and we shall always be working in the c.m.s.

Let us represent the final polarization vector $\vec{P}^f$, (12), as the radius vector in the three-dimensional space spanned by the basis vectors $\vec{l}$, $\vec{m}$ and $\vec{n}$. Then due to (17), $\vec{P}^f$ lies on the unit sphere. Varying the scattering angle or the energy, $\vec{P}^f$ describes trajectories on this sphere. The components of $\vec{P}^f$ are α , β and γ . See Fig. 2. The projection of the three-dimensional trajectory on any plane is a curve which is confined to be inside the unit circle. The trajectories must form closed loops, since for $\theta = 0$ and 180° , the spin-flip amplitude vanishes. The curves thus begin and end at the point $(0, 0, 1)$. Exceptionally this need not be true, if also f accidentally vanishes in forward or backward scattering, i.e., there is a dip in the differential cross-section.

In terms of the amplitudes f and g , we can give another interesting interpretation of the three-dimensional graph. From the definition of α , β and γ ,

$$\alpha = -\frac{2 \operatorname{Im} r}{1 + |r|^2}, \quad \beta = \frac{2 \operatorname{Re} r}{1 + |r|^2}, \quad \gamma = \frac{1 - |r|^2}{1 + |r|^2}, \quad (21)$$

where $r = g/f$, it follows directly ¹²⁾ that we can interpret the three-dimensional diagram as the representation of the complex number r or the complex sphere (Riemann sphere). The origin $r = 0$ is represented by the "north pole" $(0, 0, 1)$, the unit circle $|r| = 1$ by the "equator" $(\alpha, \beta, 0)$, and the infinity $|r| = \infty$ by the "south pole" $(0, 0, -1)$.

It is also of some interest to note that a parallel projection of the three-dimensional graph on a plane can be interpreted as the complex plane for a quantity simply related to f and g . Thus for example, the projections on the co-ordinate planes $(\alpha, \beta, 0)$, $(\alpha, 0, \gamma)$ and $(0, \beta, \gamma)$ correspond to

$$\alpha + i\beta = 2ifg^*/\sigma_0, \quad (22)$$

$$\alpha + i\gamma = i(f+g)(f-g)^*/\sigma_0, \quad (23)$$

$$\beta + i\gamma = i(f-ig)(f+ig)^*/\sigma_0. \quad (24)$$

Note also that an SU_2 transformation of the vector $\begin{pmatrix} f \\ g \end{pmatrix}$ corresponds to a O_3 rotation of the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ and a set of homographic transformations on r ($r' = \frac{ar+br}{cr+dr}$). This just expresses the well-known correspondences between these groups of transformations.

Finally we note that the cross-section for a process where the initial polarization is in the scattering plane and the final polarization is analyzed in the direction $\bar{M}$ is

$$\begin{aligned} \sigma_{\bar{M}} &= |e^{-i\psi_M/2} \cos \theta_M/2 f + e^{i\psi_M/2} \sin \theta_M/2 g|^2 = \\ &= \frac{1}{2} \sigma_0 (1 + \alpha \sin \psi_M \sin \theta_M + \beta \cos \psi_M \sin \theta_M + \gamma \cos \theta_M) \end{aligned} \quad (25)$$

Here θ_M and ψ_M are the spherical angles of the direction $\bar{M}$ in a reference frame where $\bar{P}^i$ is the z axis and θ_M is the angle between $\bar{P}^i$ and $\bar{M}$, and ψ_M is the angle between $\bar{M}$ and the scattering plane. The quantities $y = 2\sigma_{\bar{M}}/\sigma_0$ are graphically represented as the distance from a tangent plane of the sphere to the point on the sphere given by f and g . The tangent plane is given by the point of intersection between the direction $-\bar{M}$ and the sphere. This shows clearly the close relationship between points in the graph and measurable

quantities, and how the continuum of observable cross-sections (one for every θ_M and φ_M) are related. For example if the final polarization is analyzed in the scattering plane, then in a $\beta - \gamma$ plot (see Fig. 2), the distance from the tangent to the point on the curve is the quantity y *).

Determination of spin and parity

We assume that one partial wave dominates and neglect the others at the moment. Then from the partial wave expansion for f and g

$$f(\theta) = \lambda \sum_l \{ (l+1) a_{l+} + l a_{l-} \} P_l(\cos \theta), \quad (26)$$

$$g(\theta) = \lambda \sum_l (a_{l+} - a_{l-}) P'_l(\cos \theta), \quad (27)$$

we see that the ratio $r(\theta) = g(\theta)/f(\theta)$ is

$$r_{l+}(\theta) = \frac{1}{l+1} \frac{P'_l(\cos \theta)}{P_l(\cos \theta)}, \quad \text{if } j = l + \frac{1}{2} \quad (28)$$

$$r_{l-}(\theta) = -\frac{1}{l} \frac{P'_l(\cos \theta)}{P_l(\cos \theta)}, \quad \text{if } j = l - \frac{1}{2}. \quad (29)$$

This is a real quantity from which it follows that the polarization parameter α is zero, while the spin rotation angle is

$$\phi_{l\pm}(\theta) = \arctan \frac{2r_{l\pm}(\theta)}{1 - (r_{l\pm}(\theta))^2} = 2 \arctan r_{l\pm}(\theta). \quad (30)$$

*) When the graphs are read in this way, one can see the relationship to the graphs for studying isospin invariance ¹³⁾, where branching ratios for different charge channels of a reaction are plotted against each other.

These functions are very smooth and monotonous functions of the scattering angle (they are in fact well approximated by the linear function $\pm 2\ell \theta$, see the example in Fig. 3, where ϕ_{3+} is displayed). They have the interesting property that they vary from 0 to $\pm 2\pi\ell$ when θ is varied from 0 to π . Thus if there is no background from other partial waves, the curves in the $\rho - \gamma$ plot form ℓ loops along the unit circle. In Fig. 4 to the left we indicate the behaviour of the curves for the lowest ℓ . The arrows indicate the direction of increasing θ . Let N be the number of loops and let the sign of N be positive if the rotation is anticlockwise for increasing θ 's and otherwise negative. Then the orbital angular momentum ℓ , the total angular momentum j and the parity P of the partial wave are

$$\ell = |N|, \quad (31)$$

$$j = |N + \frac{1}{2}|, \quad (32)$$

$$P = -(-1)^N. \quad (33)$$

Similar results are obtained in a plot of A against R (see Fig. 4 to the right). Then there is an additional rotation by π (and a reflection due to sign conventions), whereby $\ell = |N + \frac{1}{2}|$, $j = |N|$ and $P = -(-1)^{N + \frac{1}{2}}$.

When spin rotation data become available, these properties can be used for a direct determination of the spin and parity of a resonance. This determination is experimentally completely independent of spin determinations from angular distributions. Of course, generally, there is always a background from other partial waves. However, if the background is not too large, we can expect that it mainly has the effect of moving the loops from the boundary inside the circle and distorting their circular shape. In the following Section we shall find that this indeed is the case in $\pi N \rightarrow \pi N$. If the background is large, the picture will of course be obscured and one must use phase shift analysis to entangle the resonances. A good illustration of the importance of such interference effects is the following rather amusing effect.

We assume ideal parity doubling, i.e., that only two equally large partial waves of same j but opposite parity contribute. Then it follows from the properties of the Legendre polynomials that for any $j = l + \frac{1}{2}$

$$r(\theta) = \frac{1}{l+1} \frac{P_{l+1}'(\cos \theta) - P_l'(\cos \theta)}{P_{l+1}(\cos \theta) + P_l(\cos \theta)} = \cot \frac{\theta}{2} \quad (34)$$

and

$$\phi(\theta) = -\theta. \quad (35)$$

The curve in the $\beta - \gamma$ plot is the lower half-circle (see Fig. 4). It does not have to form a loop because in backward scattering the two partial waves cancel so that both f and g are zero. In an A-R plot, one would obtain just a point $R = 1$, $A = 0$. Thus for ideal parity doubling the final helicity is the same as the initial, i.e., $f_{+-} = 0$. This result could of course have been obtained directly, since f_{+-} is proportional to the difference between the two amplitudes of opposite parity.

Thus interference effects are important, but we see from the preceding discussion that spin rotation data are also very sensitive to the quantum numbers of an individual partial wave, while the polarization, α , only reflects interference effects.^{*)}

Finally we should point out that we have assumed the validity of the conventional formalism. If, for example, parity conservation would not be strictly valid, there would be four invariant amplitudes and the conclusions made above would not hold. Obviously spin rotation measurements are a very good test for the over-all correctness of the conventional formalism.

*) This is related to the fact that spin rotation measurements are very useful for resolving ambiguities in phase shift analyses^{4),5)}. In the $\beta - \gamma$ or R-A plots, different solutions give very different predictions to the curves.

4. - POSSIBLE APPLICATIONS. PREDICTIONS OF PHASE SHIFT
ANALYSIS IN $\pi N \rightarrow \pi N$ SCATTERING

The reactions for which our type of analysis is experimentally feasible are $\pi N \rightarrow \pi N$, $\pi N \rightarrow K \Lambda$, $\pi N \rightarrow K \Sigma$, $KN \rightarrow KN$, $KN \rightarrow \pi \Sigma$ and $KN \rightarrow \pi \Lambda$. The experiments must use a polarized target and the polarization of the outgoing baryon must be analyzed. In case the final baryon is a strange particle the polarization can be fairly easily analyzed by observing the decay asymmetry of the baryon. In the case of elastic scattering the final polarization must be analyzed by a rescattering. At present no such data have been published. But this type of experiments are being planned and performed, e.g., at CERN where spin rotation parameters are being measured in $\pi N \rightarrow \pi N$ by a group from Saclay. They hope to achieve an accuracy as high as 10% ¹⁾ which is more than sufficient for our type of analysis. This experiment is aimed at high energies (5-18 GeV/c) and low momentum transfers in order to test Regge pole theory predictions. At lower energies in the resonance region one could measure in the whole angular interval since the energy of the recoil nucleon is always low and the separation of inelastic events easier.

In the following we shall see what $\pi N \rightarrow \pi N$ phase shift analysis predicts in the energy regions of the lowest resonances. We use the phase shift analysis done by Lovelace et al. ¹⁴⁾ at CERN. In Fig. 5, the $\beta - \gamma$ and $\alpha - \gamma$ plots are shown at $E_{c.m.} = 1230$ MeV for $\pi^+ p \rightarrow \pi^+ p$. Here the $\Delta(1236)$ dominates and as expected the curve in the $\beta - \gamma$ plot describes one anticlockwise loop, which corresponds to a P wave and $j^P = \frac{3}{2}^+$. The background from other partial waves gives rise to a small α , which can be seen in the $\alpha - \gamma$ plot.

The next resonance $N(1470)$ is in the $T = \frac{1}{2}$ channel. We show the predicted $\beta - \gamma$ plots both for the pure $T = \frac{1}{2}$ and for the $\pi^- p \rightarrow \pi^- p$ channel. The $\pi^- p \rightarrow \pi^- p$ contains some ($\frac{1}{3}$) of the $T = \frac{3}{2}$ amplitude, but it is difficult to experimentally separate the $T = \frac{1}{2}$ process since it would require spin rotation measurements for

the charge exchange process as well as for the elastic scattering. We see, however, in Fig. 6, that in both the $T = \frac{1}{2}$ and in the $\pi^- p \rightarrow \pi^- p$, there is again one loop but in the opposite direction to that of the previous case. Thus we quite correctly get $\ell = 1$ and $j^P = \frac{1}{2}^-$.

The predictions in the following resonance region $N(1518)$ are shown in Fig. 7. Here there is already considerable background due to other partial waves. We can nevertheless still see the two clockwise loops, which should be expected from a $j^P = \frac{3}{2}^-$ resonance.

Around $E_{c.m.} = 1680$ MeV, there are two nearby $T = \frac{1}{2}$ resonances of same $j = \frac{5}{2}$, but opposite parity $N(1680)$ and $N(1688)$. Thus strong interference effects are expected. From the phase shift analysis predictions in Fig. 8, we see that in backward scattering (especially in the pure $t = \frac{1}{2}$ channel) the curves behave very much as should be expected from parity doubling (cf. Fig. 4f).

Figure 9 and Fig. 10 show the predictions in the regions of the $\Delta(1950)$ and the $N(2190)$. Due to the huge background - the latest CERN phase shift analysis predicts several new, although not well established, resonances in this region - and due to the large inelasticity of the resonances, the "signal-to-noise ratio" is very small. The curves can then behave in an unpredictable manner. The fact that the curve in Fig. 10 forms four clockwise loops (although the fourth is very small), which quite correctly correspond to the quantum numbers $j^P = \frac{7}{2}^-$ of the $N(2190)$, must therefore be considered partly as an accident. There is, however, much structure, which shows that there is a great amount of information to be obtained from spin rotation data.

5. -

CONCLUDING REMARKS

We have shown that spin rotation data can be used for a direct and independent determination of the spin and parity of a resonance if the background from other partial waves is not too large. The method is described using a simple graphical technique, by which spin and parity are related to loops in a plot of spin rotation parameters against each others. Apart from the spin and parity determination, the graphical method is a useful tool to study spin effects phenomenologically, since points in the diagram are simply related both to directly observable cross-sections and to the spin-flip and spin-non-flip amplitudes.

When the method is tested by using predictions from pion-nucleon phase shift analysis to spin rotation parameters, the curves behave as expected from the known spins and parities of the low mass resonances. In the case of the higher mass resonances, the loops are distorted by interference effects from other large partial waves. However, if the background is not very large, the expected loops are mainly distorted in their shape and position, while their number and their direction, which are the properties which determine the spin and parity, are left intact.

When spin rotation data become available, this method will be useful for studying the data.

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REFERENCES

- 1) L. Van Rossum - Private communication.
- 2) R.D. Tripp - Ann.Rev.Nucl.Science 15, 325 (1965).
- 3) L. Wolfenstein - Phys.Rev. 96, 1654 (1954) and Ann.Rev.Nucl. Science 6, 43 (1956).
- 4) Y.S. Kim - Phys.Rev. 129, 862 (1963).
- 5) L.D. Roper - Phys.Rev. 155, 1744 (1967).
- 6) W. Rarita, R.J. Riddel, C.B. Chiu and R.J.N. Phillips - UCRL-17523 (1967), unpublished.
- 7) Proceedings of the International Conference on Polarized Targets and Ion Sources, Saclay (1966).
- 8) H.P. Stapp - Phys.Rev. 103, 425 (1956).
- 9) G.C. Wick - Ann.Phys. 18, 65 (1962).
- 10) A. Bia~~l~~as and B.E.Y. Svensson - Nuovo Cimento 42, 908 (1965).
- 11) H. Steiner - UCRL-17903 (1967), unpublished.
- 12) P. du Val - "Homographies, Quaternions and Rotations", Oxford University Press (1964).
- 13) N. Törnqvist - Phys.Rev. 161, 1581 (1967).
- 14) C. Lovelace - Proceedings Heidelberg International Conference on Elementary Particles, 1967, North-Holland Publishing Company (1968) ;
A. Donnachie, R.G. Kirsopp and C. Lovelace - Phys.Letters 26B, 161 (1968).

FIGURE CAPTIONS

- Figure 1 Illustration of the kinematics and the notation in the c.m.s. and in the laboratory system.
- Figure 2 The final polarization vector visualized in a three-dimensional co-ordinate system and the projection in the $\beta - \gamma$ plane. The curve shows a possible trajectory obtained when varying the scattering angle from 0 to 180° .
- Figure 3 An example of the spin rotation angle as a function of the scattering angle, eq. (30), when dominance of one partial wave is assumed. The example is for $\ell = 3$, $j = \frac{7}{2}$.
- Figure 4 The expected behaviour of the curves in a $\beta - \gamma$ plot (to the left) and an A-R plot (to the right) when assuming dominance of one partial wave (4a-e). Figure 4f shows the behaviour in the case of ideal parity doubling.
- Figure 5 The $\beta - \gamma$ (a) and the $\alpha - \gamma$ (b) plot for $\pi^+p \rightarrow \pi^+p$ in the energy region of the $\Delta(1236)$, with varying scattering angle as predicted by the CERN phase shift analysis. The numbers along the curves are the cosine of the c.m.s. scattering angle.
- Figure 6 The $\beta - \gamma$ plots for the pure $T = \frac{1}{2}$ channel (a) and the $\pi^-p \rightarrow \pi^-p$ channel in the energy region of the $N(1470)$ with varying scattering angle as predicted by the CERN phase shift analysis.
- Figure 7 Same as in Fig. 6, but in the energy region of the $N(1518)$.

FIG.1

FIG. 2a

FIG. 2b

FIG. 3

FIG.4

FIG. 5a

FIG. 6a

FIG. 7a

FIG. 5b

FIG. 6b

FIG. 7b

FIG. 8a

FIG. 8b

FIG. 9

FIG. 10a

FIG. 10b