CM-P00057876

Archives

Ref. TH.962

FINAL STATE CLASSIFICATION AND NEW PHASE SPACE PLOT
FOR MANY-BODY HADRON COLLISIONS

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A B S T R A C T

The final states of hadron collisions at high energy are grouped in classes which generalize to many-body processes the distinction of two-body collisions into forward elastic, backward elastic and exchange processes. The use of longitudinal phase space plots is proposed for the phenomenological analysis of experiments and their comparison with dynamical models. The simple geometrical properties of these plots at high energy are pointed out.

1. Irrespective of multiplicity, high energy hadron collisions have the property that the transverse momentum distribution of outgoing particles is essentially independent of collision energy and is restricted to values $p_t \lesssim 0.4$ GeV/c. For two-body collisions $A + B \rightarrow A' + B'$ (A', B' = particles or resonances), this entails forward and backward peaking of the differential cross section and allows to classify the collisions according to the exchanged internal quantum numbers (baryon number, strangeness, isospin, G-parity for non-strange mesons). After sketching this classification, we show how it can be extended to many-body collisions and we propose a new type of phase space plot appropriate to its experimental study and to new tests of dynamical models.

2. Consider $A + B \rightarrow A' + B'$; let A, A' be forward and B, B' backward. Denoting the internal quantum numbers by IQN, we distinguish the following classes of two-body collisions.

D processes (D for diffractive) :

$$\text{IQN}(A) = \text{IQN}(A') \quad \text{and} \quad \text{IQN}(B) = \text{IQN}(B')$$

This class contains forward elastic scattering and forward diffractive excitation of resonances.

E processes (E for exchange) :

$$\text{IQN}(A) \neq \text{IQN}(A') \quad \text{and/or} \quad \text{IQN}(B) \neq \text{IQN}(B')$$

This class further subdivides into :

E_e processes (extrinsic exchange):

$$\text{IQN}(A) = \text{IQN}(B') \neq \text{IQN}(B) = \text{IQN}(A') \quad (1)$$

They contain backward elastic scattering and "backward diffractive excitation".

E_i processes (intrinsic exchange):

They contain the E processes not verifying (1).

In Regge pole language, the D processes allow exchange of the P (= Pomeranchuk) as well as of other Regge trajectories; it is dominated by the former. E processes require exchange of non-vacuum trajectories, i.e. trajectories carrying IQN different from the vacuum values. The distinction between E_e and E_i processes, which will be important for the generalization to many-body collisions, is based on the fact that E_e differs from D only by the momenta of the outgoing particles (D and E_e are the forward and backward parts of the same angular distribution), whereas E_i differs from $D + E_e$ by the IQN of the outgoing particles irrespective of their momenta.

3. We propose to extend the above classification to many-body collisions

$$A + B \rightarrow C^1 + \dots C^n \quad (2)$$

(C^i = particles or resonances, $n \geq 3$). The class E_i can be defined very simply on the basis of IQN alone.

E_i processes (intrinsic exchange): collision (2) is of type E_i when, for any permutation $i_1, \dots, i_n$ of $1, \dots, n$ and any $j \leq n$, one has

$$\text{IQN}(C^{i_1} + \dots C^{i_j}) \neq \text{IQN}(A) \text{ and/or } \text{IQN}(C^{i_{j+1}} + \dots C^{i_n}) \neq \text{IQN}(B)$$

Examples of many-body E_i processes are:

$$\bar{p} + p \rightarrow \text{mesons (baryon annihilation)}$$

$$K^- + p \rightarrow \text{hyperon + non-strange mesons (hypercharge annihilation)}$$

$$\pi^- + p \rightarrow \text{neutrals (charge annihilation)}$$

$$p + p, \quad K^+ + p \quad \text{and} \quad \pi^+ + p \rightarrow N^{*++} + \text{neutrals.}$$

It is less straightforward to define a subdivision of the reactions (2) which are not of E_i type into distinct classes analogous to D and E_e above. Only experiment can decide whether such a subdivision is realized in nature. Its most interesting realization would be through occurrence of two or more clearly separated maxima in the $(3n-4)$ -dimensional phase space distribution of the final states $C^1 + \dots C^n$, analogous to the occurrence of the forward and backward maxima in the angular distribution of two-body collisions. The following definitions could then be given:

D processes (diffractive processes): The maxima which, through their asymptotic independence of collision energy and through their localisation in phase space, can be interpreted in terms of P exchange (Fig. 1).

E_e processes (extrinsic exchange): The other maxima.

4. A process of type D (Fig. 1) can be regarded as diffraction dissociation of A and B into non-resonant sets of particles

$$A \rightarrow C^{i_1} + \dots C^{i_j}, \quad \text{IQN}(C^{i_1} + \dots C^{i_j}) = \text{IQN}(A)$$

$$B \rightarrow C^{i_{j+1}} + \dots C^{i_n}, \quad \text{IQN}(C^{i_{j+1}} + \dots C^{i_n}) = \text{IQN}(B)$$

Note that one or more subdivisions of $C^1 + \dots C^n$ into two sets with the above IQN exist for any process which is not E_i . The dissociated set of A should go mainly forward, the one of B backward. For fixed configurations of $C^{i_1} + \dots C^{i_j}$ in the rest system of A as well as fixed configurations of $C^{i_{j+1}} + \dots C^{i_n}$ in the rest system of B, the cross section should become constant at high energy and fixed momentum transfer. At intermediate energies, it could have an energy variation of the type obtained by replacing in Fig. 1 the P trajectory by others like P', ω, A_2, ρ , etc. Some D processes could correspond to double P exchange.

The E_e processes cannot be interpreted in terms of diffraction dissociation. In the multi-Regge model ¹⁾, they would correspond to diagrams (Fig. 2) where the permutation $i_1, \dots, i_n$ is such that none of the Regge trajectories R_{i_1} can be P. Other permutations exist where a R_{i_1} can be P; they provide contributions to D processes.

As examples of D and E_e processes we quote (f, b refer to forward, backward):

$$\begin{aligned} D &: (K^+)_f + (p)_b \quad (K^+ + \omega)_f + (p)_b \\ E_e &: (K^+)_f + (p)_b \quad (p)_f + (K^+ + \omega)_b \\ E_e &: (K^-)_f + (p)_b \quad (K^0 + \omega)_f + (\pi^- + p)_b \end{aligned}$$

5. The experimental separation and study of possible peaks in the phase space distribution would seem a priori to be a difficult task because of the high dimensionality of n-body phase space $(3n-4)$. We now indicate that this problem may be much simpler than usually thought because of the smallness of all transverse momenta. We recommend to tackle it by using longitudinal phase space plots which record each collision $A + B \rightarrow C^1 + \dots C^n$ as a point in the space $(q_1, \dots, q_n)$, where q_i is the c.m. longitudinal momentum of particle C^i . Such points obey the constraints

$$\sum_i q_i = 0, \quad \sum_i (q_i^2 + m_i'^2)^{1/2} = W \quad \text{with} \quad m_i'^2 = p_{ti}^2 + m_i^2, \quad (3)$$

where W is the c.m. energy of the collision and m_i , p_{ti} are the mass and transverse momentum of C^i . If one neglects the small spread due to the transverse momenta, which is of relative order p_{ti}/W for pions and smaller for heavier particles, the plot reduces to a $(n-2)$ -dimensional manifold of simple shape which will advantageously replace the full phase space.

Thus, for $n = 3$, one can plot every collision as a point Q in the plane of Fig. 3, where q_1, q_2, q_3 are the distances of Q to the lines OA, OB, OC with the sign convention indicated. Eqs.(3) define a curve (see solid curve in Fig. 3) which is close to a regular hexagon, the rounding off at the vertices being due to the m'_i . The distance AD is of order m'_1 whereas OA is of order W .

For $n = 4$, every collision is a point Q in the space of Fig. 4, where the q_i ($i = 1, \dots, 4$) are the distances to the planes OAB, OAE, OBD, OCE , and the sign convention is such that $q_1 > 0$, $q_{2,3,4} < 0$ for the point Q shown in the figure. With all m'_i neglected, eqs.(3) define the polyhedron of Fig. 4 which has all its sides equal. The finite values of m'_i round off the sides and vertices, again to relative order m'_i/W .

6. The volume element $dq_1 \dots dq_n$ of non-relativistic phase space turns out to take a very simple form on the manifolds of Figs. 3 and 4 when $W \gg m'_i$. For $N = 3$ (Fig. 3) it becomes $c dW dq d\ell$, with c constant, $q = \sum |q_i|$ and $d\ell$ the element of length on the hexagon. The volume element is $c dW dq d_2 \Sigma$ for $n = 4$ (Fig. 4), where $d_2 \Sigma$ is the area element on the polyhedron and c is constant on each face, the values on square and triangular faces having the ratio $c_s/c_t = 3^{1/2}/2$. Relativistic phase space follows when dividing by $\prod (q_i^2 + m_i'^2)^{1/2}$. It strongly depends on the m'_i near the sides and vertices.

The longitudinal phase space plot allows simultaneous analysis of collisions according to the relative order of $q_1, \dots, q_n$, to the sign of individual momenta q_i , as well as to the magnitude of the longitudinal parts of two-body sub-energies $s_{ij} = (p_{C^i} + p_{C^j})^2$ and various momentum transfers. This makes it a convenient tool for the study of many-body collisions, especially the classification proposed above and the multi-Regge model. It should permit to extract new information from many existing bubble chamber experiments.

The author is indebted to Drs. A. Bia~~l~~as and S. Pokorski for discussions and to Drs. A. Eskreys, Y. Goldschmidt-Clermont, W. Kittel and D. Morrison for a first examination of existing data.

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Fig. 1

Fig. 2

Fig. 3

Fig. 4