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SEARCH FOR GHOSTS IN VENEZIANO'S MODEL

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ABSTRACT

The absence of ghost states in Veneziano's crossing symmetric Regge formula is shown to imply universal slopes. For certain trajectories it has been verified numerically that there are no ghosts among all the daughters up to the 50th recurrence. The asymptotic behaviour in the impact parameter was also investigated.

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I. In any application of the Veneziano model ¹⁾ the scattering amplitude is written as a sum of terms of the following type

$$A = \frac{\Gamma(\ell - \alpha(s)) \cdot \Gamma(\ell - \alpha(t))}{\Gamma(k - \alpha(s) - \alpha(t))} \quad \begin{matrix} \ell = 0, 1, \dots \\ k \leq 2\ell \end{matrix} \quad (1)$$

Since we want to apply this model mainly to $\pi\pi$ and $K\pi$ scattering, we choose $\ell = k = 1$ accordingly ²⁾. The general case can be treated along the same lines.

For integer $\alpha(s) - 1$ a pole occurs in (1). The residue is a polynomial of degree $\alpha(s)$ in $\alpha(t)$. Near the pole $\alpha(s) = n$, and we get from (1)

$$A_n = R_n \cdot \frac{n}{n - \alpha(s)} \quad (2)$$

with

$$R_n = \binom{1 - \alpha(t)}{n} \quad (3)$$

We assume linear trajectories

$$\begin{aligned} \alpha(t) &= \alpha_t + \beta \cdot \alpha' \cdot t \\ \alpha(s) &= \alpha_s + \alpha' \cdot s \end{aligned} \quad (4)$$

In Ref. 2) these values are

$$\begin{aligned} \alpha_s = \alpha_t &= .483 \\ \alpha' &= .886 \end{aligned} \quad (5)$$

Now we look for restrictions on the coefficient in (4) in order to have positive partial wave projections of R_n

$$R_n^\ell = \frac{1}{2} \int_{-1}^{+1} d\cos\theta P_\ell(\cos\theta) \cdot R_n > 0 \quad (6)$$

Since $\alpha(s) = n$, $\alpha(t)$ is a function of n . We can write

$$\alpha(t) = a_n + b_n \cdot \omega \theta \quad (7)$$

In the $\pi^+ \pi^-$ case of Ref. 2), we get

$$a_n = \alpha_t - b_n, \quad b_n = \frac{\beta}{2} (n - \alpha_s - 4\alpha' \mu^2) \quad (8)$$

and in $\pi^- K^+$

$$a_n = \alpha_t - b_n, \quad b_n = \frac{\beta}{2} (n - \alpha_s - 2(m^2 + \mu^2)\alpha' + \frac{\alpha'^2}{n - \alpha_s} (m^2 - \mu^2)^2) \quad (8')$$

where μ is the pion mass and m the kaon mass.

In the Appendix we derive explicitly R_n^l for $l = n, n-1, n-2, n-3$, by the generating function for the binomial coefficients R_n of (3). We get

$$l = n \quad R_n^n = r_n(b_n) \quad (10)$$

$$l = n-1 \quad R_n^{n-1} = r_{n-1}(b_n) \cdot (a_n + \frac{n-1}{2}) \quad (11)$$

$$l = n-2 \quad R_n^{n-2} = r_{n-2}(b_n) \cdot \frac{1}{2} \cdot \left(\frac{b_n^2}{2n-1} + a_n^2 + a_n(n-1) + \frac{(3n-1)(n-2)}{12} \right) \quad (12)$$

$$l = n-3 \quad R_n^{n-3} = r_{n-3}(b_n) \cdot \frac{1}{2} \cdot \left(\frac{b_n^2}{2n-3} (a_n + \frac{n-1}{2}) + \frac{1}{3} \left(a_n^3 + \frac{3}{2} a_n^2 (n-1) + \frac{(3n-1)(n-2)}{4} a_n + \frac{n}{8} (n-1)(n-3) \right) \right) \quad (13)$$

with

$$r_n(x) = \frac{n!}{(2n+1)!} (2x)^n$$

Let us first discuss the conditions in the $\pi\pi$ case. From (10) we get :

$$\alpha_s + 4\alpha'\mu^2 < 1 \quad (14)$$

(11) can be used for the proof that $\beta = 1$. Inserting (8) and (9) into (11) we find

$$n(1-\beta) + \beta(\alpha_s + 4\alpha'\mu^2) + 2\alpha_t - 1 > 0 \quad (15)$$

$\beta \leq 1$ follows from Eq. (15). Using crossing, we get in the t channel $\beta \geq 1$. Therefore (15) implies

$$\beta = 1 \quad (16)$$

From now on we assume $\beta = 1$. Equations (15) and (14) can be combined under one condition

$$1 - 2\alpha_t < \alpha_s + 4\alpha'\mu^2 < 1 \quad (A)$$

In Ref. 2) $\alpha_s = \alpha_t$ and $4\alpha'\mu^2 = 0.0687$. Then (A) reads

$$.310 < \alpha_s < .931$$

(12) is most restrictive for small n . Therefore we use (12) for $n=2$ and can write (12) as

$$\left(3\alpha_t - \frac{5}{2} + 2\alpha_s + 8\alpha'\mu^2\right)^2 > 3\left(1 - \left(\frac{1}{2} + \alpha_t\right)^2\right) \quad (B)$$

(B) is non-trivial only if $\alpha_t + \frac{1}{2} < 1$. The values (5) fulfil (A) but not (B). Therefore a ghost occurs, the s wave daughter of the f meson. A slight change in the parameter α_s would give a non-negative value of this residuum ($\alpha_s \geq 0.496$ instead of $\alpha_s = 0.483$).

Condition (13) gives no new information. We can write it in the following form

$$\left(\frac{b_n^2}{2n-3} + \frac{a_n^2}{3}\right) \left(a_n + \frac{n-1}{2}\right) + \frac{1}{24} \left[8a_n^2(n-1) + (6n-2)(n-2)a_n + n(n-1)(n-3)\right] \geq 0$$

If (A) is valid, the first term is positive and the term $\left[\right]$ is positive for $n \geq 3$, provided that

$$2\alpha_t + \alpha_s + 4\alpha'\mu^2 < 5$$

which is again a consequence of (A).

In the πK case the conditions (A) and (B) are more complicated. Only half of the condition (16), $\beta \leq 1$, follows from (A) since the crossing argument leading to $\beta \geq 1$ cannot be used since $\pi\pi \rightarrow K\bar{K}$ is an inelastic process. The universal slope can only be proved for trajectories with the same charge quantum numbers.

The corresponding conditions (A) and (E) are :

$$n > \beta \left(\alpha_s + 2\alpha' (m^2 + \mu^2) - \frac{\alpha'^2}{n - \alpha_s} (m^2 - \mu^2)^2 \right) > 1 - 2\alpha_t - (1 - \beta)n \quad (A')$$

$$n \geq 1$$

$$\frac{2n}{\lambda_{n-1}} \left(6 - \frac{2n-1}{2n} \left(\alpha_t + \frac{n-1}{2} \right) \right)^2 > \frac{n+1}{12} - \frac{1}{2n} \left(\alpha_t + \frac{n-1}{2} \right)^2 \quad (B')$$

$$n \geq 2$$

(B') is only non-trivial if

$$\left(\alpha_t + \frac{n-1}{2} \right)^2 < \frac{n(n+1)}{6}$$

(13) is independent of (A') and (B') for an arbitrary choice of α_s and m^2, μ^2 . For π, K it is again a consequence of (A') and (B').

II. We could not prove that the necessary conditions (A) and (B) are sufficient. The partial wave decomposition of R_n of Eq. (3) can be easily done on the computer, since R_n is a polynomial in $\cos\theta$. Up to $N < 50$ we could not find any values of $\alpha_t, \bar{\alpha}_s = \alpha_s + 4\alpha'\mu^2$ in the regions $0 \leq \alpha_t \leq 1$ and $|\bar{\alpha}_s| \leq 1$, which fulfil (A) and (B), and have ghosts in one of the lower daughters. [Due to (A) and crossing, it makes no sense to enlarge this region.] In the $\alpha_t, \bar{\alpha}_s$ plane we mark the regions whenever

the conditions (A) or (B) are not satisfied (see Fig. 1). For higher values on n only (A) is important. The ghost-free region is separated then by the line $1-2\alpha_t = \bar{\alpha}_s$ from the ghost region.

This shows that, for practical purposes, conditions (A) and (B) are indeed sufficient.

A final remark concerns the magnitude of the residua. For given n we can plot the residua as function of the impact parameter $b = \frac{1}{q}(\ell + \frac{1}{2})$. The residua are small at the lower and upper end of the curves (Fig. 2). The value of the impact parameter, at which the maximum occurs, is different for even and odd ℓ values and independent of n , especially for

$$\begin{array}{ll} n+\ell & \text{even} \\ n+\ell & \text{odd} \end{array} \quad b_m = \begin{array}{l} 3.5 \\ 4.2 \end{array} \quad (17)$$

One would expect the independency of the maximum of n from the optical potential model. A possible logarithmic dependence of b_m on n cannot be seen in the interval from 10 up to 50. The different behaviour of odd and even n can be understood quite easily from the binomial coefficient

$$R_n = \frac{1}{n!} \alpha(t) \cdot (\alpha(t)+1) \cdots (\alpha(t) + n - 1) \quad (18)$$

For large n , R_n is near $\cos\theta=1$ ($\alpha(t) \sim \alpha_t$) of the order of 1, but in the backward direction one has $(-)^{n+1}$ [since $\alpha(t) \sim -n + (\alpha_t/2)$]. If one goes, in Eq. (18), from an even n to the next even n , one has the same rough behaviour of R_n , apart from two extra zeros which affect only the interference between higher and lower partial waves. Therefore one does not expect any drastic change. Going to the next (odd) n , the curve is changed as a whole, since it reaches now -1 at $\cos\theta=-1$. This explains the difference between even and odd n .

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A P P E N D I X

PARTIAL WAVE PROJECTION OF BINOMIAL COEFFICIENTS

Let us introduce the generating function

$$\begin{aligned} R(y, \alpha(t)) &= \sum_n R_n(\alpha(t)) \cdot y^n \\ &= e^{-\alpha(t) \cdot \log(1-y)} \end{aligned}$$

The partial wave expansion of $R(y, \alpha)$ is given by

$$R^\ell(y) = \frac{1}{2} \int_{-1}^{+1} P_\ell(\cos\theta) R(y, \alpha(t)) d\cos\theta$$

with $\alpha(t) = a + b \cos\theta$, we find

$$R^\ell(y) = \sqrt{\frac{\pi}{2b}} \frac{(1-y)^{-a}}{-\log(1-y)} i^{\ell+\frac{1}{2}} J_{\ell+\frac{1}{2}}(ib \log(1-y))$$

With the known expansion of the Bessel function we find :

$$R^\ell(y) = 2^\ell (1-y)^{-a} \sum_{\mu=0}^{\infty} \frac{(\mu+\ell)!}{\mu! (2\mu+2\ell+1)!} [-b \cdot \log(1-y)]^{\ell+2\mu}$$

To get the partial wave projection for R_n we have to pick up the coefficient of y^n in this expansion. This cannot be done for arbitrary $k=n-\ell$, but for $k=0,1,2,3$ leading to the formulae (10)-(13).

REFERENCES

- 1) C. Veneziano, Nuovo Cimento 57, 190 (1968).
- 2) C. Lovelace, CERN preprint TH.950 (1968).

FIGURE CAPTIONS

Figure 1 : Position of ghosts in the $(\bar{\alpha}_s, \alpha_t)$ plane for $\pi\pi$.
(A) and (B) are violated in the shaded regions.

Figure 2 : a) Residua for $n=30$ as a function of the impact
parameter. We have chosen the trajectories of
Eq. (5).
b) The same for $n=31$.

FIG. 1