WHERE SHOULD A MAGNETIC FIELD BE MEASURED?

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ABSTRACT

We argue that the answer to the above question is: "On the boundary of the volume in which the field is to be known." We assume that in this volume the field is divergence- and rotational-free, that the measurements have an error which is random and uncorrelated, and that only these errors limit the accurate description of the field.

In a practical case, it is shown that observations inside the volume can indeed be predicted accurately from measurements on the boundary.

The time and effort involved in the measurement can thus be substantially reduced.

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C. Measuring Techniques

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Suppose we want to know a component of a magnetic field inside a certain volume V , where the field is divergence- and rotational-free, such that at each point inside V we are able to predict the field to within a specified accuracy. Then the problem arises: "Where are we going to measure the field; or rather the field component?" We will discuss only the case of one component. But once we have an explicit expression for one component, the other components follow immediately.

One possibility is to measure at points so close together that linear or quadratic interpolation allows determination of the field value with the necessary accuracy. For a spectrometer magnet, with a large acceptance angle, this means measuring the field in a quarter million points or so.

We shall show that under certain assumptions, which will be discussed, measuring at the boundary is not only sufficient, but is in fact the optimum choice for the position of the points where the field is to be measured.

We first recall a few fundamental laws:

If the Laplacian of ψ is zero inside a volume V , then ψ is fully determined by its boundary values, and ψ , as well as its derivatives, cannot have an extremum inside V . Because of Maxwell's equations, this holds also for a field component. From this we draw the important conclusion that a field component is smoother inside V than on the boundary. Fig. 1 gives an example of this. Just as one would rather see a picture in focus than out of focus, it would seem logical to measure the field in the region where its variations are most distinct.

However, this is not always the case.

Suppose we were to do only one measurement; so that we are led to assume that the field is homogeneous. Then the best estimate of the average field would be obtained by measuring right in the middle of the volume, and not on the boundary.

What is the difference between these two cases? It is usually easier to see something in focus than out of focus, but this is not the case if one only makes observations at a few points. In fact, then, the opposite can be true, as is illustrated in Fig. 2.

To put this quantitatively: The criterion is the number of observations as compared to the number of coefficients necessary for an accurate description of the field, when this is expressed in an appropriate series.

So let us assume that the final error is due, not so much to the finiteness of the expansion, but to the errors in the individual coefficients.

Or, in other words, that we have made so many observations that the final error is determined by the accuracy of each measurement, rather than by the fact that we have not gathered enough information to make the expansion complete.

This is quite a reasonable assumption: a field may be measured at 100 000 points, whilst a few hundred, or one thousand coefficients at most, could be more than enough to describe the field completely.

Having made this first assumption, let us for a moment consider a particular case in which the number of observations would be just equal to the number of coefficients necessary, which we will call N .

If there were no errors in the measurements, all coefficients could be determined, even if the points were chosen at random. But let us now suppose that there are errors of measurement, and make the second assumption, that these errors are random, homogeneous and uncorrelated.

This is in practice not necessarily the case, but this need not alter the conclusion. We found, e.g. that the errors are also due to the uncertainty in the location of the probe, so that the error depends on the field gradient. Although the absolute error is consequently higher at the boundary, one does not gain information by measuring further inwards instead. If, however, the error is dominantly due to the finite probe size, the conclusion might conceivably be different.

But if we do assume random errors in the measurements, we expect random errors in the coefficients, and hence in the model. Now if the

N points were chosen at the boundary, then we could guarantee that if the maximum deviation is sufficiently small at the boundary, then it is good throughout the volume. The reason is that if not only the real field, but also the assumed field has zero Laplacian, then the deviation from the real field also has zero Laplacian, and this deviation must therefore have its maximum at the boundary.

But if, on the other hand, the N points were not all on the boundary but on some surface inside V, then the maximum deviation would be higher on the boundary.

So for the first N points, the best choice would be on the boundary.

If we now take 2N measurements, again on the boundary, we can expect the r.m.s. of the deviation to be $\sqrt{2}$ times smaller, as compared with the case of N points on the boundary.

For simplicity we now assume that a higher maximum deviation implies a higher r.m.s. of the deviation.

Then, if we choose N points on the boundary and N points inside the volume, the expected r.m.s. of the deviation would be higher: namely $\sqrt{\epsilon_1^2 + \epsilon_2^2} / 2$ as compared with $\sqrt{2}\epsilon_1/2$ (ϵ_1 and ϵ_2 are the expected r.m.s. of the deviations when taking N points on the boundary, and N points inside, respectively).

It is therefore also advantageous to choose the second N points on the boundary, and accordingly the third N, and so on.

The above theorem is of course, only useful if one can in fact fit a model obeying the constraints and evaluate, say, a thousand coefficients using, for instance, 20 000 observations.

This we can indeed do, and one example of such a fit is shown in Fig. 3. Here are shown the observations along one particular line at the boundary (I), the fit obtained, taking into account all boundary observations, when evaluated along the same line (II), and evaluated along a line further inwards (III). The latter coincides to within 0.1% with the observed values (IV). So we did not really have to measure there, since the computed values are as accurate. In fact we have good reason to believe that the computed values are more accurate than the observed ones.

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Figure captions

Fig. 1 : A magnetic field component is smoother further away from the boundary.

Fig. 2 : When observations are made at a few points only, it may be advantageous to look at the smoothed curve.

Fig. 3 :
I. Observations along one line at the boundary.
II. Result of fit evaluated along the same line.
III. Observations further inwards.
IV. Result of fit along this line. Agreement is better than 0.1%.

Fig. 1

Fig. 2

Fig. 3