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THE COUPLING OF π AND A_1 TRAJECTORIES BY DUALITYP.R. Auvil ^{*)} and F. Halzen ^{**)}

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A B S T R A C T

Duality sum rules implied by the absence of resonances in the scattering of pseudoscalar-pseudoscalar, pseudoscalar-vector and vector-vector meson states in channels with exotic quantum numbers are studied. It is shown that the assumption of exact duality and $SU(3)$ symmetry for the vertices leads inevitably to a decoupling of the π and the A_1 Regge trajectories. Next the view is taken that the experimental exchange degeneracy of the π and A_1 trajectories is not accidental and an alternative solution is proposed. It incorporates the normal degeneracy pattern for the leading vector and tensor trajectories but has a coupled triplet of exchange degenerate trajectories ($J^{PC} = 0^{-+}$, 1^{++} and 1^{+-}) at the secondary level. The implications of this solution for 1) meson spectroscopy, 2) meson-meson-reggeon coupling, and 3) the breaking of duality are investigated.

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1. INTRODUCTION

We investigate the implications of the absence of resonances in amplitudes involving exotic PP, PV and VV states, where P and V denote the pseudoscalar and vector meson octets, respectively.

Assuming duality between direct channel resonances and Regge exchanges, exchange degeneracy leads to a natural explanation of this phenomenon ¹⁾⁻⁴⁾. This is achieved by a cancellation of the imaginary parts of the exchange degenerate Regge poles which build up the exotic amplitudes. These considerations give rise to a definite degeneracy pattern for the leading trajectories (vector and tensor nonets). This has been realized by many authors, on the basis of PP scattering ⁵⁾. In the limit of SU(3) symmetry, a solution encompassing scattering in all PP, PV and VV states has been found ²⁾. It involves a quartet of secondary trajectories ^{*)} (nonets of 0^{-+} , 1^{++} , 1^{+-} and 2^{--} trajectories). This pattern is in reasonable agreement with the experimental situation (π , A_1 and B nonets) although no member of the postulated 2^{--} nonet has been observed in the expected mass region.

In this solution, however, the requirements of absence of resonant scattering in channels with exotic quantum numbers is achieved by a pairwise cancellation of exchange degenerate trajectories :

0^{-+} with 1^{+-}

1^{++} with 2^{--}

The experimental fact that π and A_1 seem to lie on one unsignatured trajectory, which is approximately parallel to the leading natural parity trajectories, is not explained by this solution, as π and A_1 are completely decoupled.

We look for a solution of the constraints where this degeneracy is not accidental. In contrast with the SU(3) symmetric approach, we proceed in a more phenomenological way.

^{*)} We use the notation J^{PC} not J^{PG} to refer to multiplets.

By phenomenological we mean that we consider the experimental degeneracies illustrated in Figs. 1, 2 and 3. These patterns are based on assumptions that Regge trajectories are linear and approximately parallel with a slope around 0.9. Figures 1 and 2 represent the leading and the lower lying unnatural parity trajectories. The strange trajectories are plotted separately in Fig. 3. We have included only the mesons whose existence is well confirmed and lower trajectories, falling at the right of η and K_A on the Chew-Frautschi plot for non-strange and strange patterns, respectively, are ignored. We also leave unresolved problems such as the A_2 splitting, the ambiguities in the A_1 region and the possibility that the Q band may not separate into K_A, K_B structures.

In contrast with an $SU(3)$ symmetric approach, we study equations from particular channels with definite particle and charge content in connection with the experimental degeneracy pattern. The degeneracy among $\Lambda - \Sigma$ baryon trajectories was studied by Capps⁴⁾ in an analogous way. Also in this approach we find that consideration of the total set of restrictions in PP, PV and VV channels leads to a minimal solution of the same basic form given by $SU(3)$, where pairwise cancellation occurs between a quartet of trajectories with π and A_1 decoupled.

Physically, it is however possible, and even plausible, that the consideration of all these restrictions goes beyond the basic principles with which we started. The duality arguments, as they emerged from the framework of finite energy sum rules, can only be trusted in so far as trajectories are leading and their contribution is not masked by trajectories with higher intercept and non-negligible coupling.

Furthermore, we expect cut contributions and daughter trajectories to become important at the level of secondary trajectories. Besides this, Mandula, Weyers and Zweig⁶⁾ pointed out that duality sum rules are likely to be valid only in a finite region of s and t . This leads to the conclusion that constraints from channels with a high threshold value may be unreliable. In this way they distinguish the meson-baryon $u(PB \rightarrow PB)$ and $t(PP \rightarrow B\bar{B})$ channel constraints and obtain a baryon spectrum in agreement with experiment. It is also consistent with Capps' approach to the $\Lambda - \Sigma$ problem on the basis of u channel constraints alone.

Our confidence in this threshold criterion in connection with mesons is further based on the fact, observed by Mandula et al., that $\pi - \eta'$ exchange degeneracy is only required by $VV \rightarrow VV$ scattering. Therefore we leave open the two following possibilities in our phenomenological "channel-by-channel" approach :

- i) duality sum rules are more restrictive to leading trajectories than lower lying trajectories;
- ii) duality sum rules for non-exotic processes in low threshold channels should be more reliable than those with higher threshold.

We find an alternative solution to the meson problem consistent with i) and ii). The secondary trajectories appear as a triplet of exchange degenerate trajectories (nonets of $0^{-+}, 1^{++}, 1^{+-}$) cancelling among themselves. This solution provides a natural explanation of the $\pi - A_1$ exchange degeneracy and does not require a 2^{--} nonet. It may also be relevant to observe that this solution naturally arises from considering the totality of channels where a π is scattered off any target ⁷⁾.

$SU(3)$ symmetry among the helicity residues is assumed aside from singlet octet mixing and $K_A - K_B$ mixing in the $(1^+) K^*$ systems. Explicit results are found for mixing angles and meson-meson-reggeon couplings which seem to be in agreement with experiment although only a few tests are available.

The equations are derived in Section 2 and their solutions are discussed in Section 3, where a discussion of the threshold criterion and its relation to $\pi - A_1$ and $\eta - B$ degeneracy is given. A discussion of the nature of the solution and its possible extension and comparison with experiment is given in Section 4.

2. DEGENERACY AND DUALITY EQUATIONS

Before constructing the duality sum rules we list below the Regge exchanges that will be considered. We group the trajectories tentatively as nonets ⁸⁾

$$\begin{array}{llllll}
 P \ (0^{-+}) : & \pi(140) & K(490) & \eta(550) & \eta'(960) & \\
 V \ (1^{--}) : & \rho(760) & K^*(890) & \phi(1020) & \omega(780) & \\
 T \ (2^{++}) : & A_2(1310) & K_N(1420) & f'(1514) & f(1264) & (1) \\
 A \ (1^{++}) : & A_1(1070) & K_\alpha(1243) & D(1288) & ?D' & \\
 B \ (1^{+-}) : & B(1235) & K_\beta(1320) & ?H' & H(1000) &
 \end{array}$$

The $I=Y=0$ members of these nonets are expressed as a superposition of $SU(3)$ symmetric singlet and octet states

$$\begin{aligned}
 \begin{bmatrix} \omega_1 \\ \omega_8 \end{bmatrix} &= \begin{bmatrix} \cos \vartheta_V & -\sin \vartheta_V \\ \sin \vartheta_V & \cos \vartheta_V \end{bmatrix} \begin{bmatrix} \omega \\ \phi \end{bmatrix} \\
 (f_1, f_8) &\xrightarrow{\vartheta_T} (f, f') \\
 (H_1, H_8) &\xrightarrow{\vartheta_B} (H, H') \\
 (D_1, D_8) &\xrightarrow{\vartheta_A} (D', D)
 \end{aligned} \tag{2}$$

Also K_α, K_β are in general mixtures of pure $Y=\pm 1, (1^{++})$ and (1^{+-}) states, such that

$$\begin{bmatrix} K_{+-} \\ K_{++} \end{bmatrix} = \begin{bmatrix} \cos \delta & -\sin \delta \\ \sin \delta & \cos \delta \end{bmatrix} \begin{bmatrix} K_\beta \\ K_\alpha \end{bmatrix} \tag{3}$$

Since $SU(3)$ symmetric couplings are assumed throughout the calculation, the following notation is convenient for the Regge helicity residues

$$[A B C]$$

Here A, B, C stand for $P_1, P_8, V_1, V_8, \dots$ etc. (AB) and C refer to the two external and the reggeized internal line, respectively.

Furthermore, the convention is adopted that the s channel is exotic. The equations therefore involve t and u channel Regge exchanges. The energy factor is denoted by R , e.g., $R_p = (s)^{\alpha_p(t)}$. Equations among residues on exchange degenerate trajectories should be looked at as relations among the residues at equal $\underline{l}$ values.

Since we assume that exotic channels cancel in all helicity states, we shall not explicitly do spin crossing. The equations are then valid for each helicity amplitude separately and the helicity indices on A and B can unambiguously be suppressed. Note, however, that

$$[ABC] = \tau_c (-1)^{s_A + s_B} [BAC]$$

where τ_c is the signature factor of Regge pole C .

We list the duality sum rules, referring to the exotic channel they come from. Exotic channels are considered in the order of magnitude of their thresholds. Reactions like $K^+ K^+ \rightarrow K^+ K^+$ with double hypercharge lead to constraints on $I=0$ and $I=1$ contributions separately from both t and u exchanges ⁵⁾.

$\pi\pi \rightarrow \pi\pi$

$$\begin{aligned} \underline{u} \quad & \frac{1}{3} \tau_v R_p [P_8 P_8 V_8]^2 + \frac{1}{5} \tau_T R_f \left([P_8 P_8 T_2] \sin \vartheta_\pi - \frac{\sqrt{10}}{4} [P_8 P_8 T_1] \cos \vartheta_\pi \right)^2 \\ & + \frac{1}{5} \tau_T R_{f'} \left([P_8 P_8 T_2] \cos \vartheta_\pi + \frac{\sqrt{10}}{4} [P_8 P_8 T_1] \sin \vartheta_\pi \right)^2 = 0 \quad (4) \end{aligned}$$

$\underline{t}$ same as $\pi\pi$ ($\underline{u}$) constraints.

$\pi K \rightarrow \pi K$

$$\underline{u} \quad \frac{1}{6} \tau_v R_{K^*} [P_8 P_8 V_8]^2 + \frac{3}{10} \tau_T R_{K_N} [P_8 P_8 T_2]^2 = 0 \quad (5)$$

$\underline{t}$ same as $\pi\pi$ ($\underline{u}$) constraints.

KK $\rightarrow$ KK

u ($I=0$)

$$\begin{aligned} & \frac{1}{4} \tau_V R_\varphi [P_8 P_8 V_8]^2 \cos^2 \vartheta_V + \frac{1}{4} \tau_V R_\omega [P_8 P_8 V_8]^2 \sin^2 \vartheta_V \\ & + \frac{1}{20} \tau_\pi R_{f'} ([P_8 P_8 T_8] \cos \vartheta_\pi - \frac{\sqrt{10}}{2} [P_8 P_8 T_1] \sin \vartheta_\pi)^2 \\ & + \frac{1}{20} \tau_\pi R_f ([P_8 P_8 T_8] \sin \vartheta_\pi + \frac{\sqrt{10}}{2} [P_8 P_8 T_1] \cos \vartheta_\pi)^2 = 0 \end{aligned} \quad (6)$$

u ($I=1$)

$$\frac{1}{12} \tau_V R_\rho [P_8 P_8 V_8]^2 + \frac{3}{20} \tau_\pi R_{A_2} [P_8 P_8 T_8]^2 = 0 \quad (7)$$

t same as KK(u) constraints.

$\pi\rho \rightarrow \pi\rho$

$$\begin{aligned} & -\frac{1}{3} R_\rho [P_8 P_8 V_8] [V_8 V_8 V_8] + \frac{1}{5} R_{f'} ([P_8 P_8 T_8] \cos \vartheta_\pi + \frac{\sqrt{10}}{4} [P_8 P_8 T_1] \sin \vartheta_\pi) \\ & \times ([V_8 V_8 T_8] \cos \vartheta_\pi + \frac{\sqrt{10}}{4} [V_8 V_8 T_1] \sin \vartheta_\pi) \\ & + \frac{1}{5} R_f ([P_8 P_8 T_8] \sin \vartheta_\pi - \frac{\sqrt{10}}{4} [P_8 P_8 T_1] \cos \vartheta_\pi) ([V_8 V_8 T_8] \sin \vartheta_\pi \\ & - \frac{\sqrt{10}}{4} [V_8 V_8 T_1] \cos \vartheta_\pi) = 0 \end{aligned} \quad (8)$$

$$\begin{aligned} & \frac{1}{3} \tau_\rho R_\pi [P_8 V_8 P_8]^2 + \frac{1}{3} \tau_\pi R_{A_2} [P_8 V_8 T_8]^2 + \frac{1}{3} \tau_A R_{A_1} [P_8 V_8 A_8]^2 \\ & + \frac{1}{5} \tau_V R_\varphi ([P_8 V_8 V_8] \cos \vartheta_V + \frac{\sqrt{10}}{4} [P_8 V_8 V_1] \sin \vartheta_V)^2 \\ & + \frac{1}{5} \tau_V R_\omega ([P_8 V_8 V_8] \sin \vartheta_V - \frac{\sqrt{10}}{4} [P_8 V_8 V_1] \cos \vartheta_V)^2 \\ & + \frac{1}{5} \tau_B R_{H'} ([P_8 V_8 B_8] \cos \vartheta_B + \frac{\sqrt{10}}{4} [P_8 V_8 B_1] \sin \vartheta_B)^2 \\ & + \frac{1}{5} \tau_B R_H ([P_8 V_8 B_8] \sin \vartheta_B - \frac{\sqrt{10}}{4} [P_8 V_8 B_1] \cos \vartheta_B)^2 = 0 \end{aligned} \quad (9)$$

$\pi K^* \rightarrow \pi K^*$

$\underline{t}$ same as $\pi \rho$ ($\underline{t}$) constraints

$\underline{u}$

$$\begin{aligned} & \frac{1}{6} \tau_P R_K [P_8 V_8 P_8]^2 + \frac{3}{10} \tau_V R_{K^*} [P_8 V_8 V_8]^2 + \frac{1}{6} \tau_T R_{K_N} [P_8 V_8 T_8]^2 \\ & + \frac{1}{6} \tau_A R_{K_\alpha} ([P_8 V_8 A_8] \cos \delta - \frac{3}{\sqrt{5}} [P_8 V_8 B_8] \sin \delta)^2 \\ & + \frac{1}{6} \tau_B R_{K_\beta} ([P_8 V_8 A_8] \sin \delta + \frac{3}{\sqrt{5}} [P_8 V_8 B_8] \cos \delta)^2 = 0 \end{aligned} \quad (10)$$

$K \rho \rightarrow K \rho$

$\underline{t}$ same as $\pi \rho$ ($\underline{t}$) constraint

$\underline{u}$ same as $\pi K^*(\underline{u})$ constraint.

$KK^* \rightarrow KK^*$

$\underline{t}$ ($I=1$)

$$-\frac{1}{12} R_\rho [P_8 P_8 V_8][V_8 V_8 V_8] + \frac{3}{20} R_{A_2} [P_8 P_8 T_8][V_8 V_8 T_8] = 0 \quad (11)$$

$\underline{t}$ ($I=0$) same constraint as $KK^*(\underline{t}(I=1))$.

$\underline{u}$ ($I=1$)

$$\begin{aligned} & \frac{1}{12} \tau_P R_\pi [P_8 V_8 P_8]^2 + \frac{1}{12} \tau_A R_{A_1} [P_8 V_8 A_8]^2 \\ & + \frac{3}{20} \tau_B R_B [P_8 V_8 B_8]^2 = 0 \end{aligned} \quad (12)$$

We do not explicitly write out the $\underline{u}(I=0)$ equation, and further sum rules indexed by $PP \rightarrow VV$ and $VV \rightarrow VV$ scattering are also omitted as we do not need their explicit form for the discussion of Section 3.

3. SOLUTION TO THE EQUATIONS

We first consider the $PP \rightarrow PP$ equations alone whose solution has been previously discussed⁹⁾. If we assume that $\underline{p}$ and $\underline{f}$ lie on one trajectory which is not the same as that for $\underline{f}'$ (see Fig. 1), then Eq. (4) is satisfied by $\underline{p} - \underline{f}$ cancellation with $\underline{f}'$ decoupling from the equation. Equations (5) and (7) imply $K^* - K_N$ and $\underline{p} - A_2$ exchange degeneracy, respectively. In Eq. (6) we note that no terms can reasonably be expected to vanish. Since we assume $\underline{f}$ and $\underline{f}'$ to be on separate trajectories, we must have pairwise cancellation. The natural choice is $\underline{f} - \underline{\omega}$ and $\underline{f}' - \underline{\phi}$ exchange degeneracy. These remarks lead uniquely to the following quantitative results :

$$\frac{[P_8 P_8 T_1]}{[P_8 P_8 T_8]} = \frac{4}{\sqrt{5}} S_2 \quad (13)$$

$$\frac{[P_8 P_8 T_8]}{[P_8 P_8 V_8]} = \frac{\sqrt{5}}{3} S_1$$

where S_1 and S_2 are arbitrary sign factors and

$$\begin{aligned} \tan \theta_T &= -\frac{1}{\sqrt{2}} S_2 \\ \tan^2 \theta_V &= \frac{1}{2} \end{aligned} \quad (14)$$

Passing now to $PV \rightarrow PV$ constraints we see that with no new assumptions, the $\pi \underline{p}$ $\pm$ channel yields

$$\frac{1}{5} \frac{[V_8 V_8 T_8]^2}{[V_8 V_8 V_8]^2} \left(1 + \frac{\sqrt{5}}{2} \frac{[V_8 V_8 T_1]}{[V_8 V_8 T_8]} S_2 \right)^2 = 1 \quad (15)$$

However, we must now make additional assumptions about exchange degeneracies to analyze the $\underline{u}$ channel constraint, Eq. (9). It is clear from our previous remarks that we should have $A_2 - \underline{\omega}$ cancellation and $\underline{\phi}$ decoupling. Thus, we obtain

$$\frac{[P_8 V_8 T_8]^2}{[P_8 V_8 V_8]^2} = \frac{9}{5} \quad (16)$$

and

$$\frac{[P_8 V_8 V_1]^2}{[P_8 V_8 V_8]^2} = \frac{16}{5} \quad (17)$$

respectively. There remain contributions from π , A_1 , H , and H' . If we assume that in this region trajectories are basically parallel straight lines, then a plausible assumption is that the π , A_1 , and H are exchange degenerate and the H' decouples. Equation (9) then implies two additional results

$$[P_8 V_8 P_8]^2 = [P_8 V_8 A_8]^2 + \frac{3[P_8 V_8 B_8]^2}{5 \sin^2 \theta_B} \quad (18)$$

$$\frac{\sqrt{10}}{4} \frac{[P_8 V_8 B_1]}{[P_8 V_8 B_8]} \tan \theta_B = -1 \quad (19)$$

It is interesting to observe that the $SU(3)$ solution introduces another trajectory (2^{--}) into Eq. (9) and then requires : H' decoupling, π - H cancellation, and A_1 -(2^{--}) cancellation. [We shall discuss later why this is necessary in the $SU(3)$ case.] Also, if the H were not present to participate in the π - A_1 degeneracy, then π - A_1 effects would cancel in all (not just exotic) channels in $\pi\pi$ scattering.

In Eq. (10), the K^* and K_N contributions cancel by using Eq. (16). The new information concerns the K - K_α - K_ρ system (see Fig.3). Because of the difficulty in interpreting the Q band data, no obvious cancellation scheme exists. We shall assume that two distinct particles exist in this band as we have shown them in Fig. 3. In this case the assumption that K_ρ decouples with K - K_α exchange degenerate yields a beautiful K - K_α trajectory precisely parallel to the K^* - K_N trajectory. In addition we obtain

$$\frac{\sqrt{5}}{3} \frac{[P_8 V_8 A_8]}{[P_8 V_8 B_8]} \tan \delta = -1 \quad (20)$$

and

$$\frac{[P_8 V_8 A_8]^2}{[P_8 V_8 P_8]^2} = \cos^2 \delta \quad (21)$$

Combining Eqs. (20), (21), (18) and (19) we find

$$\tan^2 \theta_B = \frac{1}{2} \quad (22)$$

$$\frac{[P_8 V_8 B_1]^2}{[P_8 V_8 B_8]^2} = \frac{16}{5} \quad (23)$$

$$\frac{[P_8 V_8 B_8]^2}{[P_8 V_8 P_8]^2} = \frac{5}{9} \sin^2 \delta \quad (24)$$

It may appear from Eq. (2) that we have assigned $\underline{H}$ to be basically a singlet. This is independent of our choice of notation for the $H-H'$ system, but is a prediction of the sum rules. The equations imply that the particle which is exchange degenerate with the $\underline{\pi}$ and A_1 is predominantly the singlet member of the $\underline{B}$ nonet.

Using $\rho-A_2$ exchange degeneracy, we find from the $K-K^*$ $\underline{u}$ channel equation [Eq. (11)]

$$\frac{[V_8 V_8 T_8]}{[V_8 V_8 V_8]} = \frac{\sqrt{5}}{3} S_1 \quad (25)$$

Combining this with Eq. (15)

$$\frac{[V_8 V_8 T_1]}{[V_8 V_8 T_8]} = \frac{4}{\sqrt{5}} S_2 \quad (26)$$

By use of Eqs. (21) and (24) we see that Eq. (12) (the KK^* $\underline{u}$ channel constraint) is formally satisfied, provided $R_{\underline{\pi}} = R_{A_1} = R_{\underline{B}}$. This is badly violated since it would imply a real degeneracy of $\underline{A}_1$ and $\underline{B}$.

In the $SU(3)$ solution we would have $\pi - B$ and $A_1 - (2^{--})$ cancellation instead, but from our point of view this is just as badly broken since $\pi - H$ degeneracy was required even in the $SU(3)$ solution.

The fundamental reason for introducing a (2^{--}) nonet into the $SU(3)$ solution can be found in the $PP \rightarrow VV$ constraints. In particular, if we look at the exotic process $\pi^+ \pi^+ \rightarrow \rho^+ \rho^+$, we find that the contribution from π and A_1 , in the crossed channel, is

$$\frac{1}{3} \tau_P R_\pi [P_8 V_8 P_8]^2 - \frac{1}{3} \tau_{A_1} R_{A_1} [P_8 V_8 A_8]^2 \quad (27)$$

Comparing this to Eq. (9), we see that $\pi - A_1$ cancellation cannot be maintained in both (even if H is included). Thus we are forced to decouple the $\pi - A_1$ trajectories and introduce a new (2^{--}) trajectory. (That this new trajectory must be 2^{--} can be seen by considering the $\pi \rho$ and KK^* channels simultaneously.)

4. DISCUSSION

Through a discussion of the scattering of PP and PV states we obtained the normal degeneracy pattern for the leading trajectories and found a solution that involves three nonets for the secondary trajectories. We realize, however, that this solution cannot face all restrictions (coming from KK^* and VV scattering). As we pointed out in the Introduction, this should not be looked upon as a disease, but may be completely within the logic of the underlying physical principles. So we investigate now if the calculational approach in the preceding Section is consistent with the two criteria for the breakdown of the sum rules anticipated in the Introduction.

Consider the progression of threshold masses of the channels leading to the degeneracy pattern of the secondary trajectories in the neighbourhood of the breakdown of this solution. We have the sequence

$$KK(990), \pi\rho(900), \pi K^*(1030), KK^*(1380), \rho\rho(1520)$$

It is plausible that the breakdown of exchange degeneracy (but not of the couplings) at the KK^* level, and then in the couplings as well at the $\rho\rho$ level is a direct reflection of the two large gaps between πK^* and KK^* . These gaps are indeed evident in the above sequence.

An alternative interpretation would be to assume $PP \rightarrow VV$ sum rules should not be used because these are for some reason analogous to t channel meson baryon constraints $PP \rightarrow B\bar{B}$. This brings us to our first criterion. Indeed, from the latter point of view $\rho\rho \rightarrow \rho\rho$ constraints should not be disregarded. These sum rules make sense if we allow for breaking of exchange degeneracy (but not couplings) of about the same size as that in KK^* . The sum rules imply η -B-D degeneracy which is not too badly broken. Eliminating $PP \rightarrow VV$ constraints we seem to get a uniform breaking of exchange degeneracy at the level of the B trajectory consistent with the first criterion. [It is straightforward to write down the constraints on the leading trajectories in $VV \rightarrow VV$ processes, but too many new couplings enter the equations to derive useful information.]

Besides these general conclusions, numerous quantitative relations have been derived in Section 3. It is difficult to find tests of these predictions. We have previously discussed a test of Eq. (13)⁹⁾ for the vector tensor nonets which is based on a comparison to total cross-section Regge fits¹⁰⁾. These involve the Regge residues at $t=0$ where we expect the relation to be well satisfied and where ambiguities in continuation of the residue function in momentum transfer do not enter. Equation (13) was indeed satisfied within uncertainty in fitting. Away from $t=0$ we do not know these residues at the same t values without reference to specific models like Veneziano's. However, if two exchange degenerate trajectories also have real particle degeneracies, then we can compare the residues at the particle poles directly. In the leading trajectories, f - A_2 degeneracy allows us therefore to compare $[P_8 P_8 T_1]$ and $[P_8 P_8 T_8]$ at the $J=2$ pole. These determine the decay coupling constants. It predicts a branching fraction $f' \rightarrow K\bar{K}$ of 2-3%. This is consistent with the data but it is not a good test since this rate is very sensitive to the value of the mixing angle and the choice of phase space.

A similar test is provided by the approximate $H-A_1$ degeneracy on the secondary trajectories. At this pole the coupling of the H to $\rho^+\pi^-$ is $\frac{9}{5}[\mathbb{P}_8 V_8 B_8]^2$ and that of A_1^0 to $\rho^+\pi^-$ is $[\mathbb{P}_8 V_8 A_8]^2$. The ratio of these two is $\tan^2 \delta$ [Eq. (20)] for both helicity states. Neglecting the $H-A_1$ mass difference, we get

$$\tan^2 \delta = \frac{\Gamma(H \rightarrow \rho^+\pi^-)}{\Gamma(A_1^0 \rightarrow \rho^+\pi^-)} \quad (28)$$

Evaluating Eq. (28) as it stands implies $\tan^2 \delta \sim \frac{1}{2}$ (8). However, a more accurate fit should take into account the mass difference and thus the model dependent S and D wave phase space factors which would enter the calculation.

Let us finally concentrate on the implication for meson spectroscopy. Equation (14) predicts a 35° mixing angle for both vector and tensor nonets¹¹⁾, this is known to be correct to an accuracy of 15%. We also find the same mixing angle for the B nonet [Eq. (22)]. If we assume the K_β and the H to be the partners of the B , the Gell-Mann-Okubo formula with mixing predicts the H' in the mass region 1340-1500 MeV. More specifically, if there is no $K_\alpha-K_\beta$ mixing, $m_{H'} = 1500$ MeV, if the $K_\alpha-K_\beta$ mixing is 35° , $m_{H'} \simeq 1450$ MeV. It is not clear how to estimate $K_\alpha-K_\beta$ mixing experimentally. With zero mixing, the $A_1-K_\alpha-D$ satisfy the pure octet mass formula quite well. However, a small D' (expected to be heavier than the D) admixture can easily be invoked to compensate the contribution of the heavier K_β , when mixed with K_α . We also note that decoupling the K_β from the πK^* channel also decouples it from the $K\rho$ channel. If it is heavy enough ($m \geq 1275$ MeV) it can still appear as a two-body $K\omega$ resonance. In the Q band we would expect a rather narrow $(K\pi\pi)$ enhancement in conjunction with a broader $(K^*\pi) K_\alpha$ resonance. Since the K^* itself is rather broad, it would be difficult to see an effect as this in the data.

As a review, we show in Fig. 4 the exchange degeneracy pattern of the vector and tensor nonets on the leading trajectories and compare it with the three nonet pattern on the secondary trajectories. Particles labelled with similar figures are exchange degenerate and are coupled by the equations of Section 2.

In the vector tensor case, we also have $\rho - A_2$ and $\omega - f$ exchange degeneracy which is not shown in the Fig. In the symmetry limit we would expect the $\pi - A_1 - B$ trajectories to merge. As we pointed out in Section 3, the necessary coupling relation is already satisfied. Since none have been definitely identified, we have not included the η' , D' and H' in the degeneracy pattern although it is certainly plausible that they should be exchange degenerate. In the degeneracy pattern, it is interesting to observe that we have large singlet-octet mixing in the B nonet and the $\pi - A_1$ are exchange degenerate with the predominantly singlet member. This is analogous to the $\rho - f$ and $A_2 - \omega$ pattern. However, there is some indication that mixing is small in the π and A_1 nonets and here it is the "octet" isoscalars which are degenerate with the isovector, B .

This scheme is certainly most attractive in the absence of a (2^{--}) trajectory approximately degenerate with the A_1 trajectory. Without this (2^{--}) trajectory, we must have $\pi - A_1$ degeneracy to maintain duality (i.e., cancellation of exotic channels) on the secondary trajectory level. If such a (2^{--}) state should be observed, the mechanism for $\pi - A_1$ coupling described here is not necessarily wrong but is much less compelling. When the experimental analysis of the meson spectrum above 1300 MeV is improved, conclusive checks of the other experimental consequences (H mixing, $K_\alpha - K_\beta$ mixing, and K_β decoupling) can be made.

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FIGURE CAPTIONS

- Figure 1 Exchange degeneracy pattern of vector and tensor trajectories.
- Figure 2 Exchange degeneracy pattern of unnatural parity trajectories.
- Figure 3 Exchange degeneracy pattern of strange trajectories.
- Figure 4 $SU(3)$ degeneracy patterns.

FIG. 1

FIG.4a

FIG.4b