CM-P00058223

Ref.TH.1106-CERN

Archives

TEST OF DUALITY SUM RULES FOR MESONS

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A B S T R A C T

Implications of duality and exchange degeneracy in pseudoscalar-pseudoscalar scattering are considered. Tests of these results at $t=0$, which are independent of detailed dynamical assumptions for the Regge residue functions, are proposed and found to be in agreement with experiment.

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We consider the implications of duality and exchange degeneracy, Refs. ^{1),2),3)}, in PP scattering. (P and V denote the pseudoscalar and vector octets, respectively.) We point out that the relations obtained may be tested directly by comparing with Regge fits to total cross-section data. Since we use our equations at $t=0$, this test avoids the ambiguity inherent in continuing relations among Regge residues to particle poles. One of the relations obtained has been previously noted ⁴⁾ but can be tested only by making the further hypothesis of duality sum rules following from pseudoscalar meson-baryon scattering in the $\underline{t}$ channel. We find a second relation which provides a direct test of meson duality and which agrees with experiment.

Because we study only relations which follow from PP scattering, our results involve only the leading reggeized meson exchanges. At this level we avoid any ambiguities arising from PV and VV scattering, which involve non-leading trajectories. In this way we encompass those relations which must be basic to any meson solution to the general duality sum rules.

We assume factorization of the Regge residues which we denote, e.g., $\beta_{\pi\pi}^{f^0}(t) \equiv [f^0\pi\pi]$. At this moment we do not assume that the residues are related by SU_3 , but later we point out why this may be interesting. From the absence of resonances in the exotic channels; $\pi^+\pi^+ \rightarrow \pi^+\pi^+$, $\pi^+K^+ \rightarrow \pi^+K^+$, $K^+K^+ \rightarrow K^+K^+$, and $K^+K^0 \rightarrow K^+K^0$, we find

$$\tau_{f'} [f'\pi^+\pi^-]^2 R_{f'} + \tau_{f^0} [f^0\pi^+\pi^-]^2 R_{f^0} + \tau_{\rho} [\rho^0\pi^+\pi^-]^2 R_{\rho} = 0 \quad (1)$$

$$\begin{aligned} \tau_{f'} [f'K^+K^-] [f'\pi^+\pi^-] R_{f'} + \tau_{f^0} [f^0K^+K^-] [f^0\pi^+\pi^-] R_{f^0} \\ + \tau_{\rho} [\rho^0K^+K^-] [\rho^0\pi^+\pi^-] R_{\rho} = 0 \end{aligned} \quad (2)$$

$$\begin{aligned} \tau_{f'} [f'K^+K^-]^2 R_{f'} + \tau_{f^0} [f^0K^+K^-]^2 R_{f^0} + \tau_{\omega} [\omega K^+K^-]^2 R_{\omega} \\ + \tau_{\varphi} [\varphi K^+K^-]^2 R_{\varphi} = 0 \end{aligned} \quad (3)$$

$$\tau_\rho [\rho^0 K^+ K^-]^2 R_\rho + \tau_{A_2} [A_2^0 K^+ K^-]^2 R_{A_2} = 0 \quad (4)$$

where τ_i is the trajectory signature factor and R_i is the energy factor, $(s) \alpha_i(t)$. The relations include both $\underline{t}$ and $\underline{u}$ channel constraints. The empirical fact that $(\rho - \omega - A_2 - f^0)$ and $(\varphi - f')$ seem to form two separated sets of exchange degenerate trajectories ⁵⁾ yields

$$[f^0 \pi^+ \pi^-]^2 = [\rho^0 \pi^+ \pi^-]^2 \quad (5)$$

$$[f^0 K^+ K^-]^2 = [\rho^0 K^+ K^-]^2 \quad (6)$$

$$[f^0 K^+ K^-]^2 = [\omega K^+ K^-]^2 \quad (7)$$

$$[\rho^0 K^+ K^-]^2 = [A_2^0 K^+ K^-]^2 \quad (8)$$

$$[f' K^+ K^-]^2 = [\varphi K^+ K^-]^2 \quad (9)$$

$$[f' \pi^+ \pi^-] = 0 \quad (10)$$

Although Eqs. (5)-(10) are expected to be valid in the region $t \gtrsim 0$, they are not easy to verify directly. We need a dynamical model to compare residues at the particle poles ⁶⁾, e.g., the left-hand side of Eq. (5) is related to $\underline{f}^0$ decay at $t = m_{f^0}^2$ whereas the right-hand side is related to ρ decay at $t = m_\rho^2$ so that we need a model of the $\underline{t}$ dependence of the residues such as the Veneziano model ⁷⁾ in order to compare with experiment.

However, at $\underline{t} = 0$, we can extract values for $[f \pi \pi]$, $[f \bar{K} K]$, $[\omega \bar{K} K]$, $[\rho \bar{K} K] / [\rho \pi \pi]$, $[\rho \bar{K} K] \times [\rho \bar{N} N]$, $[A_2 \bar{K} K] \times [A_2 \bar{N} N]$ from the total cross-section analysis of Barger et al. ⁴⁾. Thus we can test the ratio of Eqs. (5) and (6), Eq. (7), and possibly Eq. (8). Of these the latter is only a test of meson duality if $[\rho \bar{N} N] \simeq [A_2 \bar{N} N]$. It is easy to see that the first would also be correct in any scheme which is SU_3 invariant, decouples $\underline{f}'$ from the $(\pi \pi)$ channel, and has $\tan \theta_T = 1/\sqrt{2}$,

[see Eq. (14)]. As was shown by Chiu and Finkelstein ⁸⁾ and as we discuss later, in the SU_3 limit, duality implies $\tan\theta_T = 1/\sqrt{2}$. Thus only Eq. (7) is a direct test of meson duality. From Barger et al., we find

$$\frac{[f^0 K^+ K^-]}{[\omega K^+ K^-]} = .73 \pm .28 \quad (11)$$

and for completeness we include [these two relations have already been checked by Barger et al, ⁴⁾]:

$$\frac{[f^0 K^+ K^-]}{[f^0 \pi^+ \pi^-]} = .88 \pm .19, \quad \frac{[\rho^0 K^+ K^-]}{[\rho^0 \pi^+ \pi^-]} = 1.10 \pm .15 \quad (12)$$

$$\frac{[A_2^0 K^+ K^-][A_2^0 \bar{N} N]}{[\rho^0 K^+ K^-][\rho^0 \bar{N} N]} = 1.42 \pm .31 \quad (13)$$

The duality check, Eq. (11), and the SU_3 relations, Eq. (12), are satisfied to within the fitting errors, but Eq. (13), which involves the $\underline{t}$ channel meson-baryon constraint, may indicate some symmetry or duality breaking if the discrepancy is sustained by improved statistics,

This point of view is supported by the results of Refs. ^{3), 9)} which analyze meson-baryon scattering using only $\underline{u}$ channel constraints. In addition, Jacob and Weyers ¹⁰⁾ have pointed out that duality breaking is indicated by the polarization data in (pp) scattering.

Since Eq. (12) indicates that the residues appear to satisfy SU_3 symmetry, it is of interest to review the solution in this limit ⁸⁾. This will also yield an improved version of Eq. (7) because $[f^0 \pi^+ \pi^-]$ is more accurately known than $[f^0 K^+ K^-]$. We use the standard mixing conventions

$$\begin{bmatrix} f_1 \\ f_8 \end{bmatrix} = \begin{bmatrix} \cos \theta_\pi & -\sin \theta_\pi \\ \sin \theta_\pi & \cos \theta_\pi \end{bmatrix} \begin{bmatrix} f^0 \\ f' \end{bmatrix} \quad (14)$$

and

$$\begin{bmatrix} \omega_1 \\ \omega_8 \end{bmatrix} = \begin{bmatrix} \cos \theta_\eta & -\sin \theta_\eta \\ \sin \theta_\eta & \cos \theta_\eta \end{bmatrix} \begin{bmatrix} \omega \\ \varphi \end{bmatrix}$$

and the SU_3 normalization of de Swart ¹¹⁾. In terms of the residue ratios

$$\alpha \equiv \frac{\sqrt{5}}{2\sqrt{2}} \frac{[T_1 PP]}{[T_8 PP]}$$

and

$$\gamma \equiv \frac{2}{\sqrt{5}} \frac{[T_8 PP]}{[V_8 PP]}$$

we obtain (considering the exotic channels, $\pi^+ \pi^+ \rightarrow \pi^+ \pi^+$, and the $I=0$ contribution to $K^+ K^+ \rightarrow K^+ K^+$)

$$\begin{aligned} \alpha \tan \vartheta_T &= -1 \\ \gamma^2 [2 \cos \vartheta_T - \sin \vartheta_T]^2 &= 4/3 \\ \gamma^2 [2 \alpha \sin \vartheta_T - \cos \vartheta_T]^2 &= 4 \cos^2 \vartheta_V \\ \gamma^2 [2 \alpha \cos \vartheta_T + \sin \vartheta_T]^2 &= 4 \sin^2 \vartheta_V \end{aligned}$$

where T_1 and T_8 denote the tensor singlet and octet, respectively. The equations have the unique solution : $\alpha^2 = 2$, $\gamma^2 = 4/9$, $\tan^2 \vartheta_T = \tan^2 \vartheta_V = \frac{1}{2}$. All other exotic channel constraints are automatically satisfied including $K^* - K^{**}$ cancellation in $\pi^+ K^+ \rightarrow \pi^+ K^+$.

Besides the prediction of equal mixing between $I=Y=0$ partners in the 2^{++} and 1^{--} nonets ⁸⁾, with a value equal to that predicted by the quark model or the $\{35\}$ classification of SU_6 , the values of α and γ are tested experimentally ^{*)} by Eqs. (11) and (13). Also, in the $SU(3)$ limit, duality predicts :

*) The relation between the notation usually employed in Regge fits and that used here is

$$\begin{aligned} |\gamma_{MV}| &= \frac{1}{2\sqrt{3}} [V_8 PP] \\ |\gamma_{MT}| &= \frac{\sqrt{3}}{2\sqrt{5}} [T_8 PP] \\ |(1 + \frac{3}{2}\epsilon)\gamma_{MT}| &= \frac{\sqrt{3}}{8} [T_1 PP] \end{aligned}$$

Duality implies $\gamma_{MV} = \gamma_{MT}$ and $\epsilon = 0$ which are in agreement with a previous fit of Barger and Olsson ¹²⁾.

$$\frac{1}{2} \frac{[f^0 \pi^+ \pi^-]}{[\omega K^+ K^-]} = 1$$

and from Barger et al., we obtain

$$\frac{1}{2} \frac{[f^0 \pi^+ \pi^-]}{[\omega K^+ K^-]} = .82 \pm .21 \quad (15)$$

with somewhat smaller errors than in Eq. (11).

In conclusion we may say that we have looked at a test of duality which is independent of any dynamical assumptions about the t dependence of the residues. The results presented here seem to indicate that duality provides a consistent understanding of the lack of exotic resonances and exchange degeneracy in PP scattering. Although the errors are large, Eq. (13) is evidence for about a 10% breaking of t channel meson-baryon constraints.

This analysis can be extended to include lower lying trajectories by considering PV and VV scattering ^{13),14)}. However, it is clear that any extension must incorporate the results presented here.

ACKNOWLEDGEMENTS

It is a pleasure to thank Drs. M. Jacob and J. Weyers for helpful discussions. We are grateful to Drs. J. Prentki and W. Thirring for the hospitality extended to us in the Theoretical Study Division.

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