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THE SU_3 STRUCTURE OF THE INELASTIC SHADOW IN A
VENEZIANO MODEL FOR A NONET OF STABLE PARTICLES

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A B S T R A C T

In the framework of a model for the Veneziano n point function in a world with an SU_3 nonet of stable particles proposed by Paton and Chan we investigate the SU_3 structure of the inelastic shadow in the ladder approximation. The inelastic shadow is found to transform asymptotically as an SU_3 singlet with degenerate couplings to the octet and singlet particles; the asymptotic limit is well approximated already at low intermediate particle multiplicities. We also investigate the SU_3 structure in the case of symmetry breaking and find that the inelastic shadow acquires an octet component independently of the structure of the input symmetry breaking.

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1. INTRODUCTION

Paton and Chan ¹⁾ have proposed a formulation of the Veneziano n point function ²⁾ for the scattering of particles carrying isospin and SU_3 quantum numbers. The particularly simple form of the model makes it possible to study the isospin and SU_3 structure of multi-particle phenomena independently of the much more involved evaluation of their momentum space behaviour. We investigate here the SU_3 structure of the inelastic shadow in a world with an SU_3 nonet of stable particles, restricting ourselves to the ladder approximation. We find that the inelastic shadow consists asymptotically of an SU_3 singlet part with degenerate couplings to the octet and singlet particles which can be identified with a Pomernanchuk term and a part with the structure of the four-point function which can be identified with the mass renormalization of the direct channel resonances and will smooth the original Veneziano term. The separation into these two parts is well defined in contrast to a model investigated previously ³⁾ and we think this to be a satisfactory feature of the nonet model. The asymptotic behaviour is already well approximated at low intermediate state multiplicities.

The model does not account so far for the observation that the actual Pomeron couples differently to the octet particles (as indicated by the constant difference between pion and kaon nucleon total cross-sections). We have therefore introduced an SU_3 symmetry breaking which could account for dependences of the intermediate particle phase space or of the amplitudes themselves on the quantum numbers of the intermediate particles. It is found that for an arbitrary structure of the input symmetry breaking the inelastic shadow acquires an octet type component, to first order in the symmetry breaking. On the other hand the degeneracy between the couplings of the singlet component to the octet and the singlet particles remains.

In the next section we formulate the problem. In Section 3 we give some properties of the traces occurring in the model of Paton and Chan which will be useful for the evaluation of the intermediate

particle summation outlined in Section 3 for the SU_3 symmetric case. In Section 5 we give the results for the symmetric case. The results for the case of broken symmetry are discussed in Section 6, the derivation is given in Appendix I.

2. THE INELASTIC SHADOW IN THE LADDER APPROXIMATION

The formulation proposed by Paton and Chan ¹⁾ for the n point amplitude in a world with an SU_3 nonet of stable particles is given by

$$\tilde{V}_n(M_1, \dots, M_n) = \sum_P (\text{tr}(\underline{M}_{P_1} \dots \underline{M}_{P_n}) + \text{tr}(\underline{M}_{P_n} \dots \underline{M}_{P_1})) \times \\ \times V_n(k_{P_1}, \dots, k_{P_n}) \quad (2.1)$$

where V_n is the n point function for neutral particles ²⁾ corresponding to a certain ordering of the external particles indicated by the labels of their momenta k_i , M_i is the SU_3 λ matrix associated to the external meson M_i and the sum is over all permutations $P = \begin{pmatrix} P \\ 1 \dots n \end{pmatrix}$ of the external particles not related cyclically or anticyclically. The inelastic shadow contributing to elastic $M_1 M_2$ scattering is defined as

$$J_M T^{M_1 M_2}(p_1, p_2, p'_1, p'_2) = \sum_r \int d\varphi_r T_{M_1 M_2}(\underline{p}_1, \underline{p}_2) T_{M_1 M_2}^*(\underline{p}'_1, \underline{p}'_2) \quad (2.2)$$

The integration is over the intermediate particle phase space, the summation over all different intermediate states r . Inserting the $\tilde{V}_n$ of Eq. (2.1) as a lowest order approximation to the $T_{M_1 M_2 \rightarrow r}$

we obtain for fixed intermediate particle multiplicity and a fixed "global" charge configuration (defined by the numbers of each kind of particles present in the intermediate state) a sum of terms each corresponding to a pair of permutations P and P' occurring in the expressions for the $\tilde{V}_n$ (2.1), inserted for $T_{M_1 M_2 \rightarrow r}(p_1, p_2)$ and $T_{M_1 M_2 \rightarrow r}^*(p'_1, p'_2)$ respectively. Each term contains a product of trace factors multiplied by a unitarity integral over two neutral particle n point functions V_n corresponding to the permutations P and P' respectively. The restriction to the ladder approximation means now the restriction to the terms with $P=P'$ up to cyclic or anticyclic permutations. Using the invariance of the n point functions under cyclic and anticyclic permutations and the invariance of the phase space integration under an arbitrary permutation of the internal particle momenta all the ladder type integrals can be reduced to one of the following type

$$I_{ee'}(p_1, p_2, p'_1, p'_2) = \int (d\text{phase space}) V_n(p_1, k_1, \dots, k_e, p_2, \bar{k}_1, \dots, \bar{k}_e) \times V_n(p'_1, k_1, \dots, k_e, p'_2, \bar{k}_1, \dots, \bar{k}_e) \quad (2.3)$$

represented in Fig. 1. $l + l'$ is the intermediate state multiplicity and $n = l + l' + 2$. Collecting the trace factors of all the integrals equivalent to the integral (2.3) and summing over the different global charge configurations (in the sense defined above) we obtain for the coefficients $\tilde{S}^{M_1 M_2}(l, l')$ associated to the integral $I_{ll'}$

$$\tilde{S}^{M_1 M_2}(e, e') = S_{ee'}^{M_1 M_2} + 2 T_{ee'}^{M_1 M_2} + S_{e'e}^{M_1 M_2} \quad (2.4)$$

with

$$S_{ee'}^{M_1 M_2} = \sum_{\lambda, \lambda'} \left(\text{tr} \left(M_1 \prod_{k=1}^e N_{\lambda}^k M_2 \prod_{k'=1}^{e'} N_{\lambda'}^{k'} \right) \right)^2 \quad (2.5)$$

and *)

$$T_{ee'}^{M_1 M_2} = \sum_{\lambda, \lambda'} \text{tr} \left(\prod_{k=e'}^1 N_k^{\lambda} M_2 \prod_{k=e}^1 N_k^{\lambda} M_1 \right) \times \\ \text{tr} \left(M_1 \prod_{k=1}^e N_k^{\lambda} M_2 \prod_{k=1}^{e'} N_k^{\lambda'} \right) \quad (2.6)$$

The sums are now to be extended over all possible intermediate state charge configurations (λ, λ') specifying the quantum numbers of the mesons N_i^{λ} and $N_j^{\lambda'}$ not related by interchange of identical particles. In terms of the $I_{\ell\ell'}$ and $\tilde{S}^{M_1 M_2}(\ell, \ell')$ the ladder approximation to the inelastic shadow reads now

$$\text{Im } T^{M_1 M_2}(p_1, p_2, p_1', p_2') = \sum_{e, e'} \tilde{S}^{M_1 M_2}(e, e') T_{ee'}(p_1, p_2, p_1', p_2') \quad (2.7)$$

The $\tilde{S}^{M_1 M_2}(\ell, \ell')$ are now the quantities we are interested in because they determine the dependence of the inelastic shadow on the external particles M_1 and M_2 which defines its SU_3 structure.

Before we tackle the evaluation of the sums (2.5) and (2.6) we give in the next Section some details on the traces occurring in this summation which will greatly simplify our problem.

3. ALLOWED ORDERINGS AND THE EVALUATION OF THEIR TRACES

We call an allowed ordering an ordering of the different particles such that the corresponding trace is different from zero.

*) The terms $T_{\ell\ell'}^{M_1 M_2}$ were not mentioned in Ref. 3) because in the pion model they happen to be just $(-1)^n S_{\ell\ell'}^{M_1 M_2}$ with n the intermediate particle multiplicity.

Using the multiplication rules given in Table I one finds the following rules for allowed orderings:

- i) an allowed ordering is of the form

$$\dots \bar{K} \circ \circ \circ \circ \circ K \text{ --- } \bar{K} \circ \circ \circ \circ \circ K \text{ --- } \bar{K} \circ \circ \circ K \dots \quad (\text{cycl.}) \quad (3.1)$$

where $\circ \circ \circ \circ \circ$ is an arbitrary ordering of X and η mesons and --- is an arbitrary ordering of all non-strange mesons with the only restriction that charged pions occur in alternating order;

- ii) there are four different configurations of type --- ,

$$\overset{+}{\text{---}} \overset{+}{\text{---}} \quad \overset{+}{\text{---}} \text{---} \quad \text{---} \overset{+}{\text{---}} \quad \text{---} \text{---}$$

where the $\pm$ signs indicate the charges of the first and the last charged pion. Between any two of these configurations just one $\bar{K}K$ pair (possibly separated by a $\circ \circ \circ \circ$ ordering) is allowed. These are

$$\overset{+}{\text{---}} \bar{K} K \overset{+}{\text{---}} \quad \overset{+}{\text{---}} \bar{K} K \text{---} \quad \text{---} \bar{K} K \overset{+}{\text{---}} \quad \text{---} \bar{K} K \text{---}$$

(3.2)

Once a trace is allowed it is easily found that independently of its position (except for η) each particle contributes just a factor to the absolute value of the trace. These factors are given in Table III. This property is rather important for the summation procedure outlined in the next section because it allows to treat the sum over the squared traces of products of matrices as a sum over the products of the factors associated to these matrices.

For the evaluation of the $T_{ll'}^{ij}$, we note that the reversed ordering of an allowed ordering is allowed if and only if

- i) it does not contain both pions and kaons and
- ii) it does not contain both charged and uncharged kaons.

We can again attribute factors to the individual matrices if both of these conditions are fulfilled. They are also listed in Table III.

4. THE INTERMEDIATE PARTICLE SUMMATION IN THE LADDER APPROXIMATION

As formulated in Section 2 we have to evaluate, for $M_i M_j$ scattering, the sums

$$S_{ee'}^{M_i M_j} = \sum_{\lambda, \lambda'} (\text{tr } M_i C_{\lambda}^{(\mu)}(e) M_j C_{\lambda'}^{(\mu')}(e'))^2 \quad (4.1)$$

where, following Section 3, $C_{\lambda}^{(\mu)}(\ell)$ are configurations of one of the following types

$$\begin{aligned} C^{(1)}: & \text{---} \bar{K} \circ \circ \circ K \text{---} \bar{K} \circ \circ \circ K \text{---} \bar{K} \circ \circ K \text{---} \\ C^{(2)}: & \circ \circ \circ K \text{---} \bar{K} \circ \circ \circ \circ K \text{---} \bar{K} \circ \circ K \text{---} \bar{K} \circ \circ \circ \\ C^{(3)}: & \text{---} \bar{K} \circ \circ \circ K \text{---} \bar{K} \circ \circ \circ K \text{---} \bar{K} \circ \circ \circ \circ \\ C^{(4)}: & \circ \circ \circ K \text{---} \bar{K} \circ \circ \circ K \text{---} \bar{K} \circ \circ \circ K \text{---} \end{aligned} \quad (4.2)$$

ℓ specifies the length and λ one particular configuration of type μ . μ and μ' are determined by the external mesons M_i and M_j , as given in Table II. [There is more than one pair (μ, μ') if one of the particles is an η or X .] Using the fact that for allowed orderings the traces factorize into individual particle contributions we can rewrite (4.1) in the form

$$S_{ee'}^{M_i M_j} = \sum_{(\mu, \mu')} F(M_i) F(M_j) S_e^{(\mu)} S_{e'}^{(\mu')} \quad (4.3)$$

where

$$S_e^{(\mu)} = \sum_{\lambda} F(C_{\lambda}^{(\mu)}) \quad (4.4)$$

and $F(C)$ is the product of the factors contributed by the configuration C . The $F(M_i)$ are listed in Table III. For the evaluation of the $S_{\ell}^{(\mu)}$ we make the convention that the charge of the first and last charged particles ($\pi^{\pm}$, K , $\bar{K}$) is fixed by M_i or M_j . If this is not true (for M_i or M_j neutral), the factor 2 corresponding to the additional freedom is put into $F(M_i)$ or $F(M_j)$.

As an example we now give the evaluation of $S_{\ell}^{(1)}$ the other sums are done in a similar way. Consider first all configurations with a fixed number L of $\bar{K}K$ pairs and fixed positions of K 's and $\bar{K}$'s. Each of the ——— or ooooo type subconfigurations is then realizable independently of the others. So, the whole sum factors into a product of sums over all realizations of these subconfigurations.

Now, consider a subconfiguration of type oooooo of length m . At fixed numbers n_{η} and n_X of etas and X mesons ($n_{\eta} + n_X = m$) it contributes a factor $(\frac{4}{3})^{n_{\eta}} (\frac{2}{3})^{n_X}$ and is realizable in $\binom{m}{n_{\eta}}$ ways. So the sum associated to this subconfiguration is 2^m and for the total oooo type part we obtain a factor 2^M , M being its total length specified by the positions of the K 's and $\bar{K}$'s. Likewise, in a ——— type configuration of length k with fixed numbers n_c , n_X and n_{η} , n_c being the number of charged pions, contributes a factor $2^{n_c} (\frac{2}{3})^{n_X} (\frac{1}{3})^{n_{\eta}}$ and is realizable in $\binom{k}{n_c} \binom{n-n_c}{n_X} \binom{n-n_c-n_X}{n_{\eta}}$ ways. n_c is restricted to even (odd) numbers for different (equal) end charges, so the sum associated to this subconfiguration gives $\frac{1}{2} (4 \binom{k}{+} \delta_{k0})$. If only one end charge is restricted, one obtains 4^k and an additional factor 2 for the unrestricted case. For $L-1$ of the ——— subconfigurations the latter case is given if we sum over all charge realizations of the kaons, the first and last configurations are restricted at one end. So, for the whole of the ——— type configurations we obtain

5. THE SYMMETRIC CASE

Apart from the "twisting corrections" T^{ij} , Eq. (2.6), given in Appendix II our results for the case of unbroken symmetry discussed exclusively up to now can be given in a very simple form

$$\begin{aligned} \tilde{S}^{M_1 M_2}(e, e') = & \frac{8}{9} 6^{e+e'} (1-\delta_{e0})(1-\delta_{e'0}) + \frac{2}{3} (6^e \delta_{e'0} + 6^{e'} \delta_{e0}) \times \\ & \times (\text{tr } M_1 M_2 \bar{M}_1 \bar{M}_2 + \text{tr } (M_2 M_1 \bar{M}_1 \bar{M}_2)) + \\ & + T_{ee'}^{M_1 M_2} \end{aligned} \quad (5.1)$$

Here the additional δ_{l0} contributions mentioned below (4.6) have been taken into account. The twisting corrections amount to at most $8.2^{l+l'}$ and are, in general, negligible. For a multiplicity as low as $l+l'=3$ they amount to at most 1/6 and vanish for most of the scattering processes (especially all those between two charged particles) At $l+l'=4$ their contribution has already decreased to $\sim 10\%$. At higher multiplicities the inelastic shadow is a pure SU_3 singlet with degenerate couplings to the octet and the singlet particles, with the exception of the δ_{l0} contributions. The latter have the same SU_3 structure as the corresponding four point function and are easily understood as contributions to the mass renormalization of the direct channel resonances (especially providing them with a finite width). They should anyway not be included in the inelastic shadow if the latter is to be identified with the Pomeranchuk contribution. The structure of (5.1) is represented in Fig. 2). The decomposition (5.1) of the coefficients $\tilde{S}^{M_1 M_2}(l, l')$ determining the structure of the inelastic shadow into two parts of obvious physical interpretation seems to us a rather satisfactory feature of the nonet model. In a model with only an isospin triplet (pions) of stable particles investigated previously ³⁾ the terms analogous to the δ_{l0} contributions in (5.1) were present also for $l \neq 0$ (significant for $l \leq 3$) and in that case these terms spoiling the isospin independence of the inelastic shadow expected for a Pomeranchuk type contribution have no simple physical interpretation. The fact that the "twisting" corrections

which, because of their dependence on isospin and SU_3 quantum numbers, cannot be considered as a part of the Pomeron contribution are small even at low intermediate state multiplicities is likewise a satisfactory feature of the model in view of the fact that the Pomeron term observed in nature takes on its asymptotic form already at low multiplicities.

The structure of the inelastic shadow does not contain so far an SU_3 symmetry breaking part which could explain the difference in the couplings of the Pomeron to pions and kaons as indicated in nature by the constant difference of total cross-sections for πp and Kp scattering at high energies. Such a symmetry breaking need not be present in the lowest order of the theory. We will discuss in the next section a simple modification of the model including a small symmetry breaking effect.

6. A SIMPLE MODEL OF SYMMETRY BREAKING

As a simple model for a breaking of the nonet symmetry we assume a dependence of the unitarity integrals $I_{\ell\ell'}$ [Eq. (2.3)] on the quantum numbers of the intermediate particles in a rather simple form. We set

$$I_{\ell\ell'}(M_1, \dots, M_{\ell+\ell'}) = I_{\ell\ell'}^{(0)} \times \prod_{k=1}^{\ell+\ell'} (1 + \Delta_{M_k}) \quad (6.1)$$

where the M_k are the intermediate particles.

The summation becomes then somewhat more involved than in the symmetric case and we refer to Appendix I for the derivation. The resulting expressions for the $S^{M_1 M_2}(\ell, \ell')$ do not have the simple form (5.1) any more because we obtain, instead of the terms $\delta_{\ell\ell'}$, expressions of the type ϵ^{ℓ} where ϵ is of the order of the symmetry breaking Δ 's. For small ϵ these are, however, still almost zero for $\ell \neq 0$, and equal to 1 for $\ell = 0$ so that the form (5.1) represents the dominant features, also for the broken

symmetry results. We are now mainly interested in the change of structure of the asymptotically dominant term (the SU_3 singlet part in the symmetric case). It turns out that this term couples to the external particles in a factorizing form in the way expected for a t channel object. Up to common factors we obtain (denoting the exchanged object by P)

$$\begin{aligned}\langle \pi \bar{\pi} | P \rangle &= \frac{1}{2} (2 + \Delta_1 + \sqrt{(2 + \Delta_1)^2 + 32(1 + \Delta_K)^2}) \\ \langle \bar{K} K | P \rangle &= 4(1 + \Delta_K) \\ \langle \eta \eta | P \rangle &= \frac{1}{3} (5\sqrt{(2 + \Delta_1)^2 + 32(1 + \Delta_K)^2} - 3(2 + \Delta_1)) \\ \langle X X | P \rangle &= \frac{2}{3} \sqrt{(2 + \Delta_1)^2 + 32(1 + \Delta_K)^2}\end{aligned}$$

(6.2)

where $\Delta_1 = 3\Delta_K - \Delta_\eta$.

Expanding to first order in the symmetry breaking we obtain

$$\begin{pmatrix} \langle \pi \bar{\pi} | P \rangle \\ \langle K \bar{K} | P \rangle \\ \langle \eta \eta | P \rangle \\ \langle X X | P \rangle \end{pmatrix} = (4 + \frac{2}{3}\Delta_\pi - \frac{2}{9}\Delta_\eta + \frac{32}{9}\Delta_K) \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + (\frac{4}{3}\Delta_\pi - \frac{4}{9}\Delta_\eta - \frac{8}{9}\Delta_K) \begin{pmatrix} 1 \\ -\frac{1}{2} \\ -1 \\ 0 \end{pmatrix}$$

(6.3)

The first term represents again an SU_3 singlet term with octet-singlet degenerate coupling, the second term is an octet type breaking of the octet couplings. This is a rather surprising result because we have not specified any SU_3 structure of the input symmetry breaking. If we had a reason to postulate self-consistency between the input and output symmetry breaking we would be led to a pure octet type solution. This case is indeed given for the mass renormalization represented in Fig. 3 whose structure can be analyzed using the same techniques as for the inelastic shadow with a result of exactly the same form as (6.3). Here it is of course not obvious that the ladder approximation represents a dominant contribution. Another interesting feature of our result (6.3) is the lack of any breaking of the singlet octet degeneracy of the Pomeron couplings. This result being hardly accessible to an experimental test we leave it to the reader to consider it as a prediction or as a drawback of the model.

We conclude from our investigation that the structure of the inelastic shadow in the nonet model of Paton and Chan bears several features one would expect to find in a realistic model. The selection of an octet type symmetry breaking found for the modified model seems to us of particular interest.

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APPENDIX I

DERIVATION OF THE SUMMATION RESULTS IN THE BROKEN SYMMETRY CASE

The derivation of the sums $S^{(\mu)}$ defined in (4.4) follows closely the way outlined in Section 4 above Eq. (4.5). The equations corresponding to (4.5) now read ^{*})

$$\begin{aligned}
 S_e^{(1)}(\xi) &= \frac{1}{2} \sum_L \binom{e-n-L}{L} \binom{n+L-1}{n} \left(2 - \frac{4}{3} \Delta + \xi\right)^n \times \\
 &\quad \times \left(4 + \frac{8}{3} \Delta + \xi\right)^{e-n-2L} 8^L \left(1 - \frac{1}{2} \Delta\right)^{2L} \\
 S_e^{(2)}(\xi) &= \sum_L \binom{e-n-L}{L} \binom{n+L-1}{n} \left(4 + \frac{8}{3} \Delta + \xi\right)^n \times \\
 &\quad \times \left(2 - \frac{4}{3} \Delta + \xi\right)^{e-n-2L-1} 8^L \left(1 - \frac{1}{2} \Delta\right)^{2L} \\
 S_e^{(3)}(\xi) &= 2 \sum_L \binom{e-n-L-1}{L} \binom{n+L}{n} \left(2 - \frac{4}{3} \Delta + \xi\right)^n \times \\
 &\quad \times \left(4 + \frac{8}{3} \Delta + \xi\right)^{e-n-2L-1} 8^L \left(1 - \frac{1}{2} \Delta\right)^{2L+1} \\
 &= S_e^{(4)}(\xi)
 \end{aligned}$$

(4.5')

where we have introduced the variable $\xi = \frac{2}{3} \Delta_X$.

It is easily shown that

$$\frac{d}{d\xi} S_e^{(\mu)}(\xi) = e S_{e-1}^{(\mu)}(\xi)$$

(I.1)

^{*}) We specialize here to an octet type symmetry breaking (see Table III).

This allows the following representation of the $S_\ell^{(\mu)}(\xi)$ as a function of ξ :

$$\begin{aligned} S_e^{(\mu)}(\xi) &= S_e^{(\mu)}(\xi_0) + \sum_k \frac{1}{k!} \left(\frac{d}{d\xi} \right)^k S_e^{(\mu)}(\xi_0) (\xi - \xi_0)^k \\ &= S_e^{(\mu)}(\xi_0) + \sum_k S_{e-k}(\xi_0) \binom{\ell}{k} (\xi - \xi_0)^k \end{aligned} \quad (I.2)$$

We choose now ξ_0 in such a way that the expression in the bracket raised to the power M in (4.5') vanishes so that only the $M=0$ part contributes. Restricting ourselves to $S^{(1)}$ in the following we have

$$S_e^{(1)}(-2 + \frac{4}{3}\Delta) = \sum_L \binom{\ell-L}{L} (2+4\Delta)^{\ell-2L} 8^L (1-\frac{1}{2}\Delta)^{2L} \quad (I.3)$$

which satisfies a recursion relation of the form

$$S_{e+1} = S_e + q S_{e-1} ; \quad q = (2+4\Delta)^{-2} 8(1-\frac{1}{2}\Delta)^2 \quad (I.4)$$

For $q=1$ this is a sequence of Fibonacci ⁴⁾ and can be represented in the form

$$S_e = S_0 \frac{\left(\frac{1+\sqrt{5}}{2}\right)^{e+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{e+1}}{\sqrt{5}}$$

Trying a similar ansatz for $q \neq 1$ we obtain

$$S_e = (2+4\Delta)^e \frac{\left(\frac{1+\sqrt{1+4q}}{2}\right)^{e+1} - \left(\frac{1-\sqrt{1+4q}}{2}\right)^{e+1}}{\sqrt{1+4q}}$$

Inserting this result into (I.2) we obtain two binomial expressions for the power series in $(\xi - \xi_0)$ and are led to the final result

$$S_e^{(1)} = \frac{F_+(F_+ + 2 - \frac{4}{3}\Delta + \frac{2}{3}\Delta x)^\ell - F_-(F_- + 2 - \frac{4}{3}\Delta + \frac{2}{3}\Delta x)^\ell}{2(F_+ - F_-)}$$

$$S_e^{(2)} = \frac{F_+ (F_- + 2 - \frac{4}{3}\Delta + \frac{2}{3}\Delta x)^e - F_- (F_+ + 2 - \frac{4}{3}\Delta + \frac{2}{3}\Delta x)^e}{F_+ - F_-}$$

$$S_e^{(3)} = 2(1 - \frac{1}{2}\Delta) \frac{(F_+ + 2 - \frac{4}{3}\Delta + \frac{2}{3}\Delta x)^e - (F_- + 2 - \frac{4}{3}\Delta + \frac{2}{3}\Delta x)^e}{F_+ - F_-}$$

where $F_{\pm} = 1 + 2\Delta \pm \sqrt{9 - 4\Delta + 6\Delta^2}$; $F_+ \approx 4$; $F_- \approx -2$ for small Δ . The brackets containing F_+ replace the value 6 for the symmetric case, the brackets containing F_- are of the order of the symmetry breaking and give the δ_{l0} terms in the symmetric case. Again we have not cared about the special situations occurring in the scattering of neutrals. The formulae for the contributions to $M_i M_j$ scattering are again obtained from Table III apart from the last mentioned modifications which are seen to contribute terms of the type

$$\left(-\frac{4}{3}\Delta + \frac{2}{3}\Delta x\right)^e \quad \text{and} \quad \left(\frac{2}{3}\Delta + \frac{2}{3}\Delta x\right)^e$$

which can be neglected for $l \neq 0$ for small symmetry breaking.

APPENDIX II

THE "TWISTING" CORRECTIONS $T_{\ell\ell'}^{M_1 M_2}$

We give, here, without derivation the results for the "twisting" corrections

$$T_{\ell\ell'}^{M_1 M_2} = \sum_{\lambda, \lambda'} \left(\tau \prod_{k=1}^1 N_{k'}^{\lambda'} M_2 \prod_{k=2}^1 N_k^{\lambda} M_1 \right) \left(\tau \left(M_2 \prod_{k=1}^{\ell} N_k^{\lambda} M_2 \prod_{k'=1}^{\ell'} N_{k'}^{\lambda'} \right) \right) \quad (2.6)$$

for different pairs of external particles (M_1, M_2) .

$$\begin{aligned} T^{K^+ K^+} &= T^{K^- K^-} = T^{K^0 K^0} = T^{\bar{K}^0 \bar{K}^0} = T^{\pi^+ \pi^+} = 2^{e+e'} (1+(-1)^e) (1-(-1)^{e'}) \\ T^{\pi K} &= T^{\pi \bar{K}} = T^{K^+ K^0} = T^{K^- K^0} = T^{K^+ \bar{K}^0} = T^{K^- \bar{K}^0} = 0 \\ T^{\pi^+ \pi^-} &= T^{K^+ K^-} = T^{K^0 \bar{K}^0} = (1+(-1)^e) (1+(-1)^{e'}) 2^{e+e'} \\ T^{\pi^+ \pi^0} &= 3 T^{\pi \gamma} = -\frac{3}{2} T^{K \gamma} = -\frac{3}{2} T^{\bar{K} \gamma} = \frac{3}{2} T^{K X} = \frac{3}{2} T^{\pi X} = \\ &= (1-(-1)^{e+e'}) 2^{e+e'} \\ T^{\gamma \gamma} &= -3 T^{\gamma X} = \frac{3}{2} T^{X X} = (1+(-1)^{e'+e}) 2^{e+e'} \end{aligned}$$

In the broken symmetry case $1 \rightarrow (1 + \epsilon)^{\ell}$ and $(-1)^{\ell} \rightarrow (-1 + \epsilon')^{\ell}$ in the brackets multiplying $2^{\ell+\ell'}$ where ϵ and ϵ' are linear expressions in the symmetry breaking Δ 's (different for different processes).

The ratio between the contributions of the $T_{\ell\ell'}^{M_i M_j}$ and the $S_{\ell\ell'}^{M_i M_j}$ depends on the intermediate particle multiplicity as $(\frac{1}{3})^{\ell+\ell'}$. If the multiplicity increases as $\ln s$ and we identify the $S_{\ell\ell'}^{M_i M_j}$ contribution with the Pomanchuk term, then the $T_{\ell\ell'}^{M_i M_j}$ part has an energy dependence as $s^{0.4}$.

$M_1 \backslash M_2$	η	π^+	π^-	π^0	K^+	K^0	$\bar{K}^0$	K^-
η	$\frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 4 & 4 & 4 \end{pmatrix}$	$\frac{1}{\sqrt{3}} \pi^+$	$\frac{1}{\sqrt{3}} \pi^-$	$\frac{1}{\sqrt{3}} \pi^0$	$-\frac{2}{\sqrt{3}} K^+$	$-\frac{2}{\sqrt{3}} K^0$	$\frac{1}{\sqrt{3}} \bar{K}^0$	$\frac{1}{\sqrt{3}} K^-$
π^+	$\frac{1}{\sqrt{3}} \pi^+$	0	$2 \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$	π^+	0	0	0	$\sqrt{2} \bar{K}_0$
π^-	$\frac{1}{\sqrt{3}} \pi^-$	$2 \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$	0	$-\pi^-$	0	0	$\sqrt{2} K^-$	0
π^0	$\frac{1}{\sqrt{3}} \pi^0$	$-\pi^+$	π^-	$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$	0	0	$-\bar{K}_0$	K^-
K^+	$\frac{1}{\sqrt{3}} K^+$	0	$\sqrt{2} K_0$	K^+	0	0	0	$2 \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$
K^0	$\frac{1}{\sqrt{3}} K^0$	$\sqrt{2} K^+$	0	$-K_0$	0	0	$2 \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$	0
$\bar{K}_0$	$-\frac{2}{\sqrt{3}} \bar{K}_0$	0	0	0	$\sqrt{2} \pi^+$	$2 \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$	0	0
K^-	$-\frac{2}{\sqrt{3}} K^-$	0	0	0	$2 \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$	$\sqrt{2} \pi^-$	0	0

Table I : Multiplication table for $M_1 \cdot M_2$, normalization: $\text{tr } M_i M_j = 2 \delta_{ij}$

M_1	$C_{\ell}^{(m)}$	M_2	$C_{\ell'}^{(m')}$
π	$C^{(1)}$	π	$C^{(1)}$
π	$C^{(3)}$	K	$C^{(1)}$
π	$C^{(1)}$	$\bar{K}$	$C^{(4)}$
K	$C^{(3)}$	K	$C^{(3)}$
K	$C^{(1)}$	$\bar{K}$	$C^{(2)}$
$\bar{K}$	$C^{(4)}$	$\bar{K}$	$C^{(4)}$
π	$C^{(1)}$	η, X	$C^{(4)}$
π	$C^{(3)}$	η, X	$C^{(4)}$

M_1	C	M_2	C
K	$C^{(1)}$	η, X	$C^{(3)}$
K	$C^{(3)}$	η, X	$C^{(2)}$
$\bar{K}$	$C^{(2)}$	η, X	$C^{(4)}$
$\bar{K}$	$C^{(4)}$	η, X	$C^{(1)}$
η, X	$C^{(1)}$	η, X	$C^{(1)}$
η, X	$C^{(2)}$	η, X	$C^{(2)}$
η, X	$C^{(3)}$	η, X	$C^{(4)}$
η, X	$C^{(4)}$	η, X	$C^{(3)}$

Table II : Intermediate particle configurations for different pairs M_1, M_2 of external particles

Each configuration contributes a term $F(M_1)F(C_1^{(m)})F(M_2)F(C_2^{(m)})$ in Eq. (4.3)

M				internal		external:	rev.prod.
						F(M)	
00000	K	----		2	$(1+\Delta_K)$	2	$2 (1+\Delta_K)$
-----	$\bar{K}$	0000		2	$(1+\Delta_K)$	2	$2 (1+\Delta_K)$
-----	$\pi^\pm$	----		2	$(1+\Delta_K)$	2	$2 (1+\Delta_\pi)$
-----	π^0	----		$\frac{1}{3}$	$(1+\Delta_\pi)$	2	$(-1)(1+\Delta_\pi)$
-----	η	----		$\frac{1}{3}$	$(1+\Delta_\eta)$	$\frac{1}{3}$	$\frac{1}{3} (1+\Delta_\eta)$
00000	η	00000		$\frac{4}{3}$	$(1+\Delta_\eta)$	$\frac{4}{3}$	$(-\frac{2}{3})(1+\Delta_\eta)$
-----	X	-----		$\frac{2}{3}$	$(1+\Delta_X)$	$\frac{4}{3}$	$\frac{2}{3} (1+\Delta_X)$
00000	X	00000		$\frac{2}{3}$	$(1+\Delta_X)$	$\frac{2}{3}$	$\frac{2}{3} (1+\Delta_X)$

Table III : Factors contributed by individual mesons to the squares of traces and products of traces with their reversed trace.

$\Delta_\pi = -\Delta_\eta = -2\Delta_K = \Delta$ for octet type symmetry breaking.

R E F E R E N C E S

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FIG. 1

FIG. 2

FIG. 3