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THE $K\ell_3$ PUZZLE AND CURRENT ALGEBRA

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A B S T R A C T

Sum rules for the on-shell scalar form factor $[f_0(q^2)]$ in $K\ell_3$ decay show that the experimentally observed negative slope of $f_0(q^2)$ may be explained if chiral $SU(2)$ symmetry is nearly exact. Our analysis further suggests that $f_0(q^2)$ should turn over and increase outside the physical region.

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Different methods of measuring $\xi(0) = f_-(0)/f_+(0)$, the ratio of the form factors in Kl_3 decay

$$\begin{aligned} \langle \pi^0(p) | V_\mu^K | K^+(k) \rangle &= \langle \pi^0(p) | V_\mu^4 + i V_\mu^5 | K^+(k) \rangle \\ &= \frac{1}{\sqrt{2}} \left[(k+p)_\mu f_+(q^2) + (k-p)_\mu f_-(q^2) \right] \quad (1) \end{aligned}$$

at $q^2 \equiv (k-p)^2 = 0$, have in the past yielded conflicting results ¹⁾. From the recent X_2 experiments ²⁾, however, where $\xi(0)$ has been measured by all the three methods (branching ratio, polarization and spectra), a consistent picture has emerged and a value $\xi(0) \simeq -1$ has been obtained. A large negative value of $\xi(0)$ implies that the slope λ_0 of the scalar form factor $f_0(q^2)$ defined by

$$f_0(q^2) = f_+(q^2) + q^2 f_-(q^2)/(m_K^2 - m_\pi^2) \quad (2)$$

is negative. This is evident from the relation

$$\lambda_0 = \lambda_+ + \xi(0)/(m_K^2 - m_\pi^2) \quad (3)$$

obtained from Eq. (2) with λ_0, λ_+ given by

$$f_\varphi = f_+(0) \left[1 + \lambda_\varphi q^2/m_\pi^2 + \dots \right] \quad (4)$$

Indeed an over-all fit to the X_2 data ²⁾ with linear $f_0(q^2)$ and $f_+(q^2)$ yields

$$\lambda_0 = -0.030 \pm 0.012 \quad (5)$$

A negative slope of $f_0(q^2)$ in the singularity-free region $[q^2 < (m_K + m_\pi)^2]$ is obviously incompatible with the pole-dominant model. But the puzzle, at least from the point of view of current algebra, is that a negative slope appears to be in conflict also with the Callan-Treiman ³⁾ relation ^{*})

$$f_+(m_K^2, 0) + f_-(m_K^2, 0) = F_K/F_\pi \quad (6)$$

Indeed, assuming in the spirit of PCAC

$$f_+(m_K^2, m_\pi^2) + f_-(m_K^2, m_\pi^2) \approx f_+(m_K^2, 0) + f_-(m_K^2, 0) \quad (7)$$

one obtains from Eqs. (2) and (6)

$$f_0(m_K^2) \approx F_K/F_\pi = (1.23 \pm 0.02) f_+(0) \quad (8)$$

which shows that λ_0 is positive if $f_0(q^2)$ is approximately linear all the way up to the Callan-Treiman point $q^2 = m_K^2$. It is also clear that the constraint imposed by Eq. (8) would lead to positive λ_0 and hence a small $|\xi(0)|$ even if one assumes a once subtracted form $[M > (m_K + m_\pi)]$

$$f_0(q^2) = f_+(0) \left[1 + \lambda_0 \frac{q^2}{m_\pi^2} \left(1 - \frac{q^2}{M^2} \right)^{-1} \right] \quad (9)$$

^{*}) The second argument in $f_\pm(m_K^2, 0)$ refers to the invariant mass p^2 of the off-shell pion. In our notation $\langle 0 | A_\mu^3 | \pi^0(p) \rangle = i p_\mu F_\pi / \sqrt{2}$, with similar definition for F_K .

essentially because such a form allows only a monotonic behaviour for $f_0(q^2)$ in the region $q^2 < (m_K + m_\pi)^2$. This has led to the suggestion ^{4),5)} that the Callan-Treiman relation cannot be extrapolated smoothly onto the mass shell. It has also been argued ^{4),5)} that a negative slope of $f_0(q^2)$ may be reconciled with current algebra results if $SU(3)$ rather than $SU(2) \times SU(2)$ is a very good symmetry of the hadronic Hamiltonian.

It should, however, be observed that the apparent contradiction with the Callan-Treiman relation disappears if $f_0(q^2)$ has a dip near $q^2 = (m_K - m_\pi)^2$ and then rises. In this note we show that such a q^2 dependence of the on-shell scalar form factor admits of a natural explanation if chiral $SU(2)$ is a very good symmetry of the Hamiltonian, i.e., in the notation of Gell-Mann et al. ⁶⁾ (GMOR) $\sqrt{2} + c \approx 0$. If, instead, $SU(3)$ is a much better symmetry, as suggested in Refs. 4) and 5), our analysis shows that $f_0(q^2)$ should sharply increase and pass through a peak near the physical region in contradiction with experimental evidences.

A convenient framework for relating current algebra constraints to the q^2 dependence of on-shell $f_0(q^2)$ is provided by dispersion theoretic sum rules in the co-linear configuration ^{7),8)} $p = zk$. Following Ademollo et al. ⁸⁾ we define

$$W(z) \equiv \int d^4x e^{ip \cdot x} \theta(x_0) \langle 0 | [\partial^\nu A_\nu^3(x), \partial^\mu V_\mu^K(0)] | K^+(k) \rangle \quad (10)$$

in the rest frame of the K meson, i.e.,

$$p = zk = (zm_K, 0, 0, 0) . \quad (11)$$

We may conclude à la Bjorken ⁹⁾ that $W(z)$ vanishes asymptotically (as $z \rightarrow \infty$) provided only the commutator appearing in Eq. (10) exists at equal time ($x_0 = 0$). From the ETCR's

$$\delta(x_0) [A_0^3(x), V_\mu^K(0)] = \frac{1}{2} \delta(x) A_\mu^K \quad (12)$$

$$\delta(x_0) [\partial^\nu A_\nu^3(x), V_0^K(0)] = \frac{1}{2} \frac{\sqrt{2}+c}{\sqrt{2}-\frac{c}{2}} \delta(x) \partial^\nu A_\nu^K \quad (13)$$

one now obtains ⁸⁾ the sum rules ^{*)}

$$W(0) = \frac{1}{\pi} \int \frac{\text{Im} W(z)}{z} dz = -\frac{1}{2} m_K^2 F_K \left(1 - \frac{\sqrt{2}+c}{\sqrt{2}-\frac{c}{2}}\right) \quad (14)$$

$$W(1) = \frac{1}{\pi} \int \frac{\text{Im} W(z)}{z-1} dz = \frac{1}{2} m_K^2 F_K \frac{\sqrt{2}+c}{\sqrt{2}-\frac{c}{2}} \quad (15)$$

Notice that the presence of Schwinger terms in the ETCR's (12) and (13) does not affect the sum rules. The parameter c is defined by the piece of the Hamiltonian

$$H' = u_0 + e u_8 \quad (16)$$

*) The authors of Ref. 8) considered only the particular combination $W(1) - W(0) = \frac{1}{2} m_K^2 F_K$ of the sum rules. Moreover, their assumption of pole-dominance for $f_0(q^2)$ automatically excluded the possibility of obtaining a negative slope.

which breaks chiral $SU(3)$ symmetry and is assumed ⁶⁾ to transform according to the $(3, \bar{3}) + (\bar{3}, 3)$ representation. In the limit of chiral $SU(2)$ invariance $\sqrt{2} + c = 0$, whereas $SU(3)$ invariance implies $c = 0$.

One pion intermediate states contribute to the sum rules (14) and (15) at two different values of z viz., $z = \pm m_\pi/m_K$, corresponding to connected and semi-disconnected diagrams. Thus the sum rules provide us with relations for the on-shell form factor $f_0(q^2)$ at $q^2 = (m_K \pm m_\pi)^2$. Only 0^- and 0^+ intermediate states with the quantum numbers of pion and kaon respectively contribute to our sum rules. Their contributions may be parametrized in terms of an effective X meson which contributes at $z = (m_K \mp M)/m_K$, where M is the X mass. Thus we obtain after some simplifications

$$\begin{aligned} \left(1 + \frac{m_\pi}{m_K}\right) f_0((m_K - m_\pi)^2) &= \frac{F_K}{F_\pi} + \frac{F_K}{F_\pi} \frac{m_K}{m_\pi} \frac{\sqrt{2} + c}{\sqrt{2} - \frac{c}{2}} \\ &+ \frac{F_X}{\sqrt{2}} \frac{m_\pi}{m_K} \left[\frac{g_X((M - m_K)^2)}{(M - m_K)(M - m_K + m_\pi)} \right. \\ &\quad \left. + \frac{g_X((M + m_K)^2)}{(M + m_K)(M + m_K - m_\pi)} \right] \end{aligned} \quad (17)$$

$$\begin{aligned} \left(1 - \frac{m_\pi}{m_K}\right) f_0((m_K + m_\pi)^2) &= \frac{F_K}{F_\pi} - \frac{F_K}{F_\pi} \frac{m_K}{m_\pi} \frac{\sqrt{2} + c}{\sqrt{2} - \frac{c}{2}} + \\ &+ \frac{F_X}{\sqrt{2}} \frac{m_\pi}{m_K} \left[\frac{g_X((M - m_K)^2)}{(M - m_K)(M - m_K - m_\pi)} + \frac{g_X((M + m_K)^2)}{(M + m_K)(M + m_K + m_\pi)} \right] \end{aligned} \quad (18)$$

These are our basic equations. Here F_K and g_K are defined by

$$\langle 0 | V_\mu^K | K^+(q) \rangle = -F_K q_\mu \quad (19)$$

$$\langle K^+(q) | \partial^\nu A_\nu^3 | K^+(q) \rangle = \frac{F_\pi m_\pi^2}{\sqrt{2}} \cdot \frac{g_K (q-k)^2}{(q-k)^2 - m_\pi^2} \quad (20)$$

There are considerable uncertainties regarding the K parameters. As regards F_K theoretical estimates ¹⁰⁾ are in the range $0.4 \leq F_K/F_\pi \leq 0.6$. Assuming $F_K = 0.5 F_\pi$ we present in the Table our estimates ^{*)} for the K contribution in Eq. (17) based on the parameters (M, Γ_K) suggested by the analysis of Lovelace ¹¹⁾ and of Trippe et al. ¹²⁾. The corresponding contribution in Eq. (18) is of opposite sign and larger in magnitude.

If $SU(3)$ symmetry is nearly exact $c \approx 0$. In this case the c dependent term in Eq. (17) is positive and larger than our estimate for K contribution by an order of magnitude. Therefore, in this scheme ^{4),5)} the on-shell scalar form factor would increase sharply in the physical region in contradiction with the experimental evidences. One also concludes from Eq. (18) that $f_0(q^2)$ is negative at $q^2 = (m_K + m_\pi)^2$ and, therefore, passes through a peak near the physical region.

^{*)} In making these estimates we have neglected, in the spirit of PCAC, the variation of $g_K(p^2)$. As an independent check, we have also calculated from our estimates of the K contribution the correction term in the particular sum rule considered by Ademollo et al. ⁸⁾ who have obtained the same corrections in terms of the πK scattering length. The agreement of the final results in the two approaches is quite satisfactory.

A negative slope of $f_0(q^2)$ would be rather easy to realize if $\sqrt{2} + c \approx 0$, i.e., if chiral $SU(2)$ symmetry is nearly exact. In this case the χ contribution may cancel the c dependent term or together they may even yield a negative contribution in Eq. (17). It then follows from Eq. (18) that $f_0(q^2)$ should increase outside the physical region ¹³⁾. To be specific, let us assume that χ contribution cancels the c dependent term in Eq. (17). According to our estimate in the Table, this would be achieved if $c \approx -1.35$, which agrees, within errors, with the determination $c \approx -1.25$ by GMOR ⁶⁾.

A quadratic fit

$$f_0(q^2) = f_+(0) \left[1 + \lambda_0 (q^2/m_K^2) + \alpha (q^2/m_K^2)^2 \right] \quad (21)$$

to Eqs. (17) and (18) with our assumed cancellation and with the parameters given in Ref. 11) yields

$$\lambda_0 = -0.028 \quad , \quad \alpha = 0.0033 \quad (22)$$

Since α is small the agreement of this determination of λ_0 with the experimental result (5) obtained from linear parametrization of $f_0(q^2)$ is quite encouraging. In the figure we display the curves for $f_0(q^2)$ obtained from quadratic fits to our sum rules with $c = -1.28$ (curve I) and $c = -1.39$ (curve II) respectively. The χ parameters are the same as before ¹¹⁾. For the purpose of comparison we have also presented the envelope of fits to $f_0(q^2)$ obtained from all linear fits to $\xi(q^2)$ which are within one standard deviation of the best fit to the X_2 data for $\lambda_+ = 0.029$.

It is interesting to note that at the Callan-Treiman point $q^2 = m_K^2$ our results (22) yield $f_0(m_K^2) = 1.22 f_+(0)$. This restores our confidence in the smoothness assumption implied by PCAC.

Our sum rules (17) and (18) provide some insight into the results obtained from different models in the effective Lagrangian approach. For example, if the Lagrangian does not contain χ meson, as in the model of Lee ¹⁴⁾, $f_0(q^2)$ would, in general, be linear in q^2 . From our sum rules we obtain directly $\lambda_0 = m_\pi^2 \delta / (m_K^2 - m_\pi^2)$ where δ is defined by $F_K/F_\pi f_+(0) = 1 + \delta$. This is exactly the result of Lee ¹⁴⁾. As for c in this model we obtain from our sum rules $(\sqrt{2}+c)/(\sqrt{2}-\frac{c}{2}) = (1-\delta) m_\pi^2 / (1+\delta) m_K^2$ which agrees with the determination by GMOR ⁶⁾. Introduction of a χ meson would not help in realizing a negative λ_0 unless one allows for q^2 dependence at the $\chi\pi K$ vertex. This is essentially the reason why Chang and Leung ¹⁰⁾ obtain a small value for $|\xi(0)|$.

To conclude, our analysis shows that a negative slope of $f_0(q^2)$ admits of a natural explanation within the scheme proposed by GMOR ⁶⁾ for breaking chiral $SU(3)$ symmetry. A crucial test of current algebra, however, consists in determining whether $f_0(q^2)$ shows the proposed turn-over behaviour. More precise knowledge of the experimental situation regarding this aspect is, therefore, desirable.

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Source	χ Mass (MeV)	χ Width (MeV)	χ Contribution	c
Lovelace ¹¹⁾	900	214	- 0.15	-1.34
Trippe et al. ¹²⁾	1 100	~ 400	- 0.11	-1.36

Numerical estimates of the χ contribution in Eq. (17) with $F_{\chi} \approx 0.5 F_{\pi}$ and of the symmetry breaking parameter c (deduced from the assumed cancellation).

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FIGURE CAPTION

q^2 dependence of $f_0(q^2)$ obtained from quadratic fits to sum rules (17) and (18) with $c = -1.28$ (curve I), and $c = -1.39$ (curve II). The α parameters are taken from Ref. 11). The broken curves represent the envelope of fits to $f_0(q^2)$ obtained from all linear fits to $\xi(q^2)$ which are within one standard deviation of the best fit to the X_2 data ²⁾ for $\lambda_+ = 0.029$.