CM-P00058228

Ref.TH.1111-CERN

Archives

A CROSSING SYMMETRIC DESCRIPTION OF DATA WITH
THE GENERALIZED VENEZIANO MODEL

Chan Hong-Mo, R.O. Raitio ^{*)},
G.H. Thomas ^{**)} and N.A. Tornqvist ^{*)}

CERN - Geneva

A B S T R A C T

The generalized Veneziano model is applied to study the reactions :

- (A) $K^+ p \rightarrow K^0 \pi^+ p$
- (B) $K^- p \rightarrow \bar{K}^0 \pi^- p$
- (C) $\pi^- p \rightarrow K^0 K^- p$

which are related by crossing. It is found that with the same formula and essentially only one free parameter, one is able to give a good global description of all three reactions at energies ranging from 2.5 to 13 GeV. The possibility of extending the study to further reactions is also briefly considered.

^{*)} On leave from the Institute of Nuclear Physics, University of Helsinki, Helsinki, Finland.

^{**)} Postdoctoral NATO Fellow.

1. INTRODUCTION

The discovery of the generalized Veneziano model¹⁾, and the apparent success of its application to the reaction $K^- p \rightarrow \Lambda \pi^+ \pi^-$ ²⁾ have opened up exciting new possibilities in the field of multiparticle phenomenology. One may well hope to construct on these premises a quite reliable model for three-body reactions in general. If successful, the resultant model would give, in terms of very few parameters, a unified description not only of mechanisms previously regarded as separate, such as "resonance production", "double-peripherality", and "background", but even different reactions which are related by crossing.

A particularly good testing ground for such a model is the class of reactions represented by the graph of Fig. 1, where all the lines have been taken as ingoing. Because of the quantum numbers, many of the graphs obtained by permuting the external lines give rise to exotic channels. This means that of the usual 12 terms in the five-point Veneziano amplitude, such exotic terms can, to a first approximation, be neglected. Moreover, since all the external lines are distinct, there are a large number of different reactions related by crossing symmetry. These afford ample tests for the assumed crossing invariance of the Veneziano amplitude.

Figure 1 represents altogether 14 charge configurations. Using charge conjugation and charge symmetry, one can relate some of these, leaving five distinct groups:

- (I) $\pi^0 K^+ \bar{K}^- p \bar{p}$
- (II) $\pi^- K^+ \bar{K}^0 p \bar{p}$
- (III) $\pi^- K^+ \bar{K}^- p \bar{n}$
- (IV) $\pi^0 K^0 \bar{K}^- p \bar{n}$
- (V) $\pi^0 K^0 \bar{K}^0 p \bar{p}$

Among these, groups I, II, and III are most amenable to experimental investigations, each consisting of several reactions for which some data are available. We list these below:

$$(I) \quad K^+ p \rightarrow K^+ \pi^0 p \quad (1)$$

$$K^- p \rightarrow K^- \pi^0 p \quad (2)$$

$$\bar{p} p \rightarrow K^+ K^- \pi^0 \quad (3)$$

$$(II) \quad K^+ p \rightarrow K^0 \pi^+ p \quad (4)$$

$$K^- p \rightarrow \bar{K}^0 \pi^- p \quad (5)$$

$$\pi^+ p \rightarrow K^+ \bar{K}^0 p \quad (6)$$

$$\pi^- n \rightarrow K^0 \bar{K}^- n \quad (6')$$

$$\pi^- p \rightarrow K^- K^0 p \quad (7)$$

$$\pi^+ n \rightarrow \bar{K}^0 K^+ n \quad (7')$$

$$K^+ n \rightarrow K^0 \pi^+ n \quad (8)$$

$$K^- n \rightarrow \bar{K}^0 \pi^- n \quad (9)$$

$$\bar{p} p \rightarrow K^- K^0 \pi^+ \quad (10)$$

$$(III) \quad K^+ p \rightarrow K^+ \pi^+ n \quad (11)$$

$$K^- p \rightarrow K^- \pi^+ n \quad (12)$$

$$\pi^+ n \rightarrow K^+ K^- p \quad (13)$$

$$\pi^- p \rightarrow K^0 \bar{K}^0 n \quad (13')$$

$$\pi^- p \rightarrow K^+ K^- n \quad (14)$$

$$\pi^+ n \rightarrow K^0 \bar{K}^0 p \quad (14')$$

$$K^+ n \rightarrow K^+ \pi^- p \quad (15)$$

$$K^- n \rightarrow K^- \pi^- p \quad (16)$$

$$\bar{p} n \rightarrow K^+ K^- \pi^- \quad (17)$$

Some qualitative features of these reactions are known already from previous phenomenological analyses, especially of K^* production in the $K^\pm p$ initiated reactions. For charged K^* production in reactions (1), (2), (4), and (5), it was found that the cross-section decreases rapidly with increasing incoming energy up to the highest available energy³⁾. This indicates that the contribution of the Pomeron, which may in principle

be coupled to the $\bar{p}p$ system, is somehow suppressed. In theoretical language, this may be understood by regarding the Pomeron as a "diffractive effect". The incoming K meson has unnatural parity, while the outgoing $K\pi$ system must necessarily have natural parity. In current models of diffractive dissociation⁴⁾, such a change of parity is not allowed. This apparent absence of the Pomeron coupling to the $\bar{p}p$ system is a crucial fact for our considerations, since the Pomeron has no natural place in a dual model.

However, the effect just mentioned does not eliminate the Pomeron problem in group I and III reactions, since there the $K\bar{K}$ system may also be coupled to the Pomeron. We have little empirical evidence either for or against this coupling. Also, present theoretical considerations are not very helpful for resolving this question. We shall therefore leave the question open for the moment, pending further investigation.

Again from K^* production, it is known empirically from the decay correlations that charged K^* production in reactions (1), (2), (4), and (5) is dominated by " ω exchange", while neutral K^* production in reaction (12) is dominated by " π exchange"⁵⁾. These empirical statements, accepted for the moment at their face value, suggest that one needs:

- i) a vector exchange model only for group II reactions;
- ii) a vector + a Pomeron exchange model for group I reactions;
- iii) a pion + a Pomeron exchange model for group III reactions.

Since the Pomeron is patently non-dual⁶⁾ and the dual nature of the pion is at least unclear, we have chosen the group II reactions as our first objective which is to be discussed in this paper. We have in mind, of course, also the fact that the vector-exchange model has already proved quite successful in Ref. 2. The assumption of pure vector exchange in group II reactions is, of course, only an approximation. Indeed, one can estimate from the cross-sections of group III reactions the amount of π exchange in group II using isospin symmetry. This yields a "correction" of approximately 20 per cent in the vector-exchange model for group II at ~ 4.5 GeV⁷⁾. The need for such a correction will be apparent when later on we compare our calculations with experimental data. However, we shall postpone our consideration of these effects until after they have been investigated in group III where they are known to dominate. In this paper, pion-exchange effects are neglected.

As we shall see, with this simple model one is already able to give a good qualitative description of the existing data. We have chosen for special study the reactions (4), (5), and (7) for which good data are available. The only parameter entirely at our disposal is the constant normalization of the amplitude which we fixed empirically. From this, we are then able to predict the following for all three reactions and for p_{lab} ranging from 2.5 to 13 GeV/c:

- i) the absolute cross-sections;
- ii) single particle distributions of the three final particles;
- iii) effective mass distributions;
- iv) the amount of resonance productions and their dependence on energy and momentum transfer;
- v) decay correlations of the main resonances.

Before we go on to describe our model in detail, we wish to remind our readers of two basic difficulties of the Veneziano model in general:

- i) the absence of a reliable method for dealing with fermions;
- ii) the unwanted degeneracies such as that between π and η enforced by isospin symmetry and the absence of exotic states⁸⁾.

We made no attempt to overcome these difficulties in this paper. Our fermions are treated as bosons, while the over-stringent conditions imposed by isospin symmetry are simply ignored.

2. DETAILS OF THE MODEL

First, we list again for easy reference the three reactions under consideration in this paper:

- (A) $K^+ p \rightarrow K^0 \pi^+ p$
- (B) $K^- p \rightarrow \bar{K}^0 \pi^- p$
- (C) $\pi^- p \rightarrow K^0 \bar{K}^- p$.

These reactions are supposed to be described by the same Veneziano amplitude which is given as a linear combination of 12 terms corresponding to the 12 non-equivalent orderings of the external lines. It can readily be checked that among these, only four are free from exotic channels, corresponding to the orderings in the four diagrams of Fig. 2.

One limitation of the Veneziano model in its present form is that one is allowed in practice to take account of only one trajectory in each channel. We have therefore inserted in each case what we believe to be the dominant trajectory, arriving at the result shown in Fig. 2. For the channels in which we have inserted the K^* , A_2 and Δ trajectories, the quantum numbers are such as to leave no reasonable alternative. For the other cases, our choices are arrived at through the following considerations:

i) The ω in the $\bar{p}p$ channel is dictated by the empirical evidence in K^* production in reactions (A) and (B), as discussed in Section 1.

ii) The Y_1^{*+} in the $\bar{K}^0 p$ channel we took to be the $Y_1^*(1385)$, since according to Schmid⁹⁾ this is the only Y_1^* strongly coupled to the $\bar{K}N$ system.

iii) The Y^{*0} in the $K^+ \bar{p}$ channel can in principle be either the Λ or the $Y_1^*(1385)$ according to Schmid⁹⁾. However, empirically, in reaction (C) where this Y^{*0} can be produced, there is strong evidence for the production of Λ -resonances but no indications for the recurrences of $Y_1^*(1385)$. We have therefore inserted here the Λ trajectory.

iv) As far as the quantum numbers are concerned, there are many choices for the N^{*0} trajectory in the $\pi^- p$ channel. In the reaction (B) where this N^{*0} can be produced, the present experimental data are inconclusive for the production of any resonance. Thus in the absence of any clear preference, we have inserted the N_α trajectory in the belief that this is going to dominate in the exchange channels, as is the case in πN backward elastic scattering¹⁰⁾.

One notices that in all the graphs of Fig. 2, the trajectory in every channel starts with a p-wave resonance. The situation is thus similar to that of Ref. 2, and suggests that we write a similar form for each of the four graphs in Fig. 2; namely:

$$(P) = \epsilon_{\mu\nu\rho\sigma} p_\pi^\mu p_{\bar{K}}^\nu p_K^\rho p_{\bar{p}}^\sigma \times \\ \times B_5(1 - \alpha_{K^*}, 1 - \alpha_A, \frac{1}{2} - \alpha_\Lambda, 1 - \alpha_\omega, \frac{1}{2} - \alpha_N) \quad (18)$$

$$(Q) = \epsilon_{\mu\nu\rho\sigma} p_{\pi}^{\mu} p_{\bar{K}}^{\nu} p_K^{\rho} p_{\bar{p}}^{\sigma} \times \\ \times B_5(1 - \alpha_{K^*}, \frac{1}{2} - \alpha_{\Lambda}, 1 - \alpha_{\omega}, \frac{3}{2} - \alpha_{Y^*}, 1 - \alpha_{K^*}) \quad (19)$$

$$(R) = \epsilon_{\mu\nu\rho\sigma} p_{\pi}^{\mu} p_{\bar{K}}^{\nu} p_K^{\rho} p_{\bar{p}}^{\sigma} \times \\ \times B_5(1 - \alpha_{K^*}, 1 - \alpha_{\Lambda}, \frac{3}{2} - \alpha_{Y^*}, 1 - \alpha_{\omega}, \frac{3}{2} - \alpha_{\Delta}) \quad (20)$$

$$(S) = \epsilon_{\mu\nu\rho\sigma} p_{\pi}^{\mu} p_{\bar{K}}^{\nu} p_K^{\rho} p_{\bar{p}}^{\sigma} \times \\ \times B_5(\frac{3}{2} - \alpha_{\Delta}, \frac{1}{2} - \alpha_{\Lambda}, 1 - \alpha_{\Lambda}, \frac{3}{2} - \alpha_{Y^*}, \frac{1}{2} - \alpha_N) . \quad (21)$$

The formulas (18) to (21) will give poles at the correct resonance masses and also the correct "orbital angular momenta" of the resonances. However, since we have not taken into account kinematic factors appropriate to the fermions, our formulas will give the wrong asymptotic behaviour for baryon Regge exchanges. In order to remedy this, we again follow Ref. 2 and simply shift those arguments of the B_5 function corresponding to exchanged baryons by one-half. This, of course, is meant only as a makeshift method for obtaining the correct asymptotic behaviour. Since the baryon trajectories have, in general, quite low intercepts and give only small contributions when Reggeized, it is probable that our approximation will not be too drastic. The only likely exception is the Δ trajectory, which has an intercept almost as high as the K^* . Indeed, there are several problems connected with the Δ in our calculation, which we shall have to discuss later in some detail.

Strictly speaking, the Veneziano model is supposed to be valid only for real linear trajectories all with the same slopes. For phenomenological applications, however, this stringent condition has to be slightly relaxed in order to comply with reality. Empirical knowledge of a trajectory function $\alpha(x)$ for $x < 0$ comes from high-energy scattering data in reactions where α is exchanged. These are in general insufficient to determine α accurately. For this region, then, we may keep both the assumptions of real

linearity and universal slope. For $x > 0$, however, $\alpha(x)$ gives rise to resonances for which ample data exist. As we shall see -- for most of the cases under consideration -- present knowledge of the resonance parameters as compiled in the Rosenfeld tables is sufficient for determining α quite well in this region.

The parameters of the trajectories actually used in our calculation are summarized in the table. The slope of the real part of α above threshold and the intercept are determined from the Chew-Frautschi plot of the known resonances using the masses given in the Rosenfeld tables¹¹⁾, where we have assumed the usual exchange degeneracies of the ω - A_2 , the Λ , and the Y_1^* trajectories. In all cases, the determination is quite unambiguous. When exchanged, all trajectories are given the same slope of 0.9 GeV^{-2} .

The imaginary part of α above threshold is obtained from the total widths of known resonances given in the Rosenfeld tables, using the formula:

$$\text{Im } \alpha = \alpha' M_{\text{res}} \Gamma_{\text{res}} . \quad (22)$$

For the baryon resonances, usually several recurrences are known. Those relevant to our calculation are shown in Fig. 3. One notes that in all cases the imaginary parts of α are reasonably well fitted by straight lines with a common slope of $\sim 0.14 \text{ GeV}^{-2}$. Again, the only outstanding exception is the $\Delta(1236)$ resonance which, according to phase-shift analysis¹²⁾ has a width of 120 MeV. Our interpolation of $\text{Im } \alpha$, however, yields by Eq. (22) a width of only 70 MeV. Nonetheless, it turns out that in our calculation, our formula yields about the right width for the observed Δ in reaction (A). Whereas if we had taken the phase shift value in Eq. (22), one would obtain too wide a Δ compared with the data. Indeed, a direct Breit-Wigner fit to the data in this reaction yields a Δ width of only 82 MeV¹³⁾. This may be due to the usual ambiguity in width determinations near thresholds¹⁴⁾, to which our formula (22) is subject.

For the meson trajectories, few recurrences are known. Fortunately, we need the imaginary parts of only two meson trajectories in our calculation, namely the K^* and the A_2 . For the K^* trajectory, we fit a straight line through the $K^*(890)$, the $K^{**}(1400)$, and the $K\pi$ threshold. For the A_2 ,

Trajectory parameters below and above the threshold x_{th}

$\alpha(x) = \theta(x_{th}-x) [\alpha_0 + \delta + \alpha'_-x] + \theta(x-x_{th}) [(\alpha_0 + \alpha'_+x) + i a(x-x_0)]$							
Trajectory	α_0	δ	α'_+	α'_-	a	x_0	
ω -A ₂	0.48	0	0.9	0.9	0.07	0.18	
K*	0.35	0	0.82	0.9	0.07	0.29	
N	-0.37	$-\frac{1}{2}$	1	0.9	0.14	1.0	
Δ	0.13	$\frac{1}{2}$	0.9	0.9	0.14	1.0	
Λ	-0.68	$-\frac{1}{2}$	0.95	0.9	0.14	2.1	
Y ₁ *	-0.22	$\frac{1}{2}$	0.9	0.9	0.14	1.57	

we draw a straight line through the resonance and the 3π threshold. The width of A_2 was taken to be 90 MeV, as given in the Rosenfeld table, which is much wider than the 50 MeV for the A_2 observed in reaction (C)¹⁵⁾. However, we made no attempt to correct for this, since the situation for the A_2 in general is far from clear.

The parameters of our trajectories now being fixed as in the table, each graph in Fig. 2 is then completely specified in the physical regions of all three reactions (A), (B), and (C). However, we have yet to determine with what relative weights and signs to combine them in order to form our full amplitude T . These remaining parameters are fixed through the following considerations. Since the Δ resonance is strongly produced in reaction (A), either the R graph or the S graph or both must be present. These graphs differ, however, only by the interchange of the π^- and $\bar{p}$ lines; their relative weights and signs thus control the signature of the Δ trajectory. Now, as far as we know, there is no experimental evidence anywhere for the existence of a recurrence of the Δ with the opposite signature (e.g. a Δ recurrence with $J^P = 5/2^-$). This suggests then that we combine R and S with equal weights with a relative sign such as to cancel all wrong-signature poles. For (R) and (S) as defined in formulas (20) and (21), we have thus:

$$(R) : (S) = 1 : 1 . \quad (23)$$

Next, since the charged K^* is produced in both reactions (A) and (B), either P or Q or both must be present in T . One notes, however, that Q differs from P only by an interchange of the $\pi^- \bar{K}^0$ lines and from R only by an interchange of the $\pi^- K^+$ lines. Thus, if one assumes the exchange degeneracy in residue of both the K^* trajectories, in keeping with evidence from other sources, Q must be omitted from the full amplitude. One remarks also that the graph Q is forbidden by the duality graphic rules of Harari and Rosner¹⁶⁾. There is yet another ratio to determine, namely that between (P) and (S). Now, these two graphs differ only by the interchange of the π^- and p lines. If we wish the internal N_α trajectory to have definite signature (absence of N_α^- recurrence with $J^P = 3/2^-$), we would require again:

$$(P) : (S) = 1 : 1 . \quad (24)$$

This choice is further supported by the consideration of charge conjugation and the isospin relations between groups I and II. The counterparts of the graphs (P) and (R) in group I are charge conjugates of each other and must by C-invariance have equal weights. If we accept that in both I and II, the ω exchange ($I = 0$) dominates in the $\bar{p}p$ channel, then isospin implies the same ratio also between (P) and (R) for group II. By formula (21), this then leads to formula (22).

The arguments presented above for fixing the ratios between the various graphs are admittedly weak, since one knows already that strict applications of exchange degeneracy and even isospin leads sometimes to inconsistencies elsewhere within the Veneziano model. None the less, such arguments do restrict the ratios within rather narrow limits. Indeed, attempts have been made to optimize these ratios by varying them to fit the actual data under consideration. The result is not very different from that given above, although the accuracy of this last statement is limited by the crudeness of our Monte Carlo techniques. One notes also that similar arguments have been applied in Ref. 2, obtaining quite satisfactory results¹⁷⁾. One striking difference, however, is that here the selected graphs are those satisfying the Harari-Rosner rule, whereas in Refs. 2 and 17, one has exactly the reverse.

If these assumptions are accepted, one has then for the full amplitude:

$$T = \beta[(P) + (R) + (S)] , \quad (25)$$

with (P), (R), (S) fully specified by formulas (18), (20), and (21), and the table. The only remaining free parameter is the constant β with which we hope to describe all the three reactions (A), (B), and (C) for all incoming laboratory energies greater than, say, 2 GeV.

3. COMPARISON WITH EXPERIMENTAL DATA

All the calculations reported below were performed with the Monte Carlo technique. For $p_{lab} < 5$ GeV/c, generally the CERN library program FOWL¹⁸⁾ was used. For $p_{lab} > 5$ GeV/c, in order to improve the efficiency we used the Helsinki version¹⁹⁾ which generates events according to a specified distribution favouring low-momentum transfers. Sufficient statistics were accumulated to give reasonably smooth distributions. The curves shown in the figures have been smoothed by hand from the computer output. We wish to emphasize that our aim is only a global description of the data. At the present stage of the model, a detailed fit seems to us unjustified.

3.1 Cross-sections

Figure 4 shows the total cross-sections of reactions (A), (B), and (C) as functions of the incoming laboratory momenta. The theoretical curves have been calculated by normalizing β in Eq. (25) to the 5 GeV/c point for reaction (A). One obtains reasonable agreement both in the energy dependence and in the absolute value of the cross-sections. We note in particular the following points:

i) In reactions (A) and (B) the agreement is generally better at higher energies. Below 3 GeV/c, the calculated curves tend to fall below the data by as much as a factor of 2. This may be an indication of the pion exchange contribution which we have neglected, and which is expected to be more important at lower energies.

ii) In reaction (C) also, the energy dependence is roughly correct, as far as one can judge from the poorer data. One notes that the curve here is quite a bit different from those of (A) and (B), which can qualitatively be understood from the following kinematical considerations. Firstly, there is the threshold effect due to the higher mass of the K mesons produced. Secondly, again due to the mass difference between π and K, the momentum transfer $t_{\pi K} < t_{\min} < 0$, where $|t_{\min}|$ decreases with the incoming energy. The strong dependence of our amplitude on t then helps to explain the different energy behaviour³⁰⁾.

iii) The predicted absolute value of the cross-section for reaction (C) is off by about a factor of 2. In our opinion, even this crude agreement is significant considering the large difference ($\sim$ factor 20) between this reaction and the reaction (A) on which our curve is normalized. The smallness of the cross-section for (C) relative to (A), especially at high energy, cannot be understood only by the kinematic arguments given in (ii). It has to depend on such things as coupling strengths of resonances and residues of Regge poles.

(iv) The prediction of the cross-sections of both (B) and (C) from (A) we regard as a significant test of crossing symmetry for our amplitude. For example, in reaction (A), the charged K^* trajectory gives rise to resonances, while in (C) it is an exchanged Regge pole. To predict (C) from (A) would thus need an extrapolation of the residue function of the charged K^* from $t > 0$ to $t < 0$. This seems to have been achieved by the

amplitude T to within a factor of 2 in cross-sections in both (B) and (C), which is much better than expected. A similar phenomenological test of crossing in two-body collisions can be applied at the moment only to baryon trajectories, and the result is not very satisfactory. For example, an attempt at extrapolating the residue of the Δ trajectory from the negative t region in backward elastic $\pi^- p$ scattering to the Δ mass, leads to a discrepancy by a factor of 2000³¹⁾! Even the much simpler line reversal symmetry between reactions such as KN charge exchange at high energy has been tested only to within a factor of 2³²⁾.

3.2 Single particle distributions

These do not afford very sensitive tests of the present model. Indeed, even the much cruder double-Regge models already give, in general, a reasonable description of single particle distributions³³⁾. Some improvement is achieved in our calculation because resonance production is included, but it is perhaps not too significant. We therefore quote only some examples in Figs. 5 and 6. One notes in particular the interesting discrepancy in the t distribution near $t = 0$. Since our model has pure vector exchange, the predicted distribution must vanish in the forward direction. There the experimental distributions also show a dip but are certainly non-vanishing. This discrepancy we ascribe again to the neglected effects of π exchange which is expected to be dominated at $t \approx 0$.

3.3 Effective mass distributions

One great advantage of the Veneziano model over the older Regge models is that it is supposed to describe automatically also the resonance effects. These show up most prominently in the effective mass distributions which the older versions fail completely to describe. We have therefore presented our results on these in full detail, in Figs. 7 to 9. One notes that the general features of the experimental effective mass plots are well reproduced by our calculation. We have also looked at the Dalitz plots predicted by the model, finding reasonable agreement with the data. They have, however, been omitted in this paper because of the difficulty in presentation.

The following points in the effective mass distributions are worth noting:

i) The relative amounts of $K^*(890)$ and $K^{**}(1400)$ produced in the reactions (A) and (B) are sensitive in the model to the ratio between the (P) and (Q) graphs. The assumption of exchange degeneracy in residue for the K^* trajectory, namely keeping either (P) or (Q) only in T, gives about the right relative amounts of the two K^* resonances.

ii) The relative strength of $K^*(890)$ or $K^{**}(1400)$ to Δ production in reaction (A) is sensitive to the ratio between the (P) and the (R) or (S) graphs. With the choice of parameters as listed in the table, assumption (24) gives about the correct relative strength between the K^* 's and the Δ .

iii) No Δ recurrence at around 1.7 GeV is at all visible in the πp effective mass plot of reaction (A). This requires, at least approximately, the ansatz (23).

iv) It has been known experimentally for some time that there is a constructive interference in the regions where the K^* bands and the Δ band overlap on the dalitz plot of reaction (A). This is correctly reproduced by our calculation. Had we taken (Q) instead of (P) or insisted on (24) with an opposite sign, we would instead obtain destructive interference for one or both of the K^* 's with the Δ .

v) The apparently spurious hump at ~ 1.3 GeV in the πp effective mass plot for reaction (B) is due to the daughter of the cancelled wrong-signature first recurrence of the nucleon. Its prominence in our calculation is connected with the high intercept of the exchanged Δ -trajectory, as we shall discuss in Section 4.

vi) The A_2 produced in reaction (C) has too wide a width and too low a mass in our calculation, compared with the data. This is because we have taken the Rosenfeld values of $M = 1293$ MeV and $\Gamma = 90$ MeV as compared with the values $M = 1315$ MeV and $\Gamma = 50$ MeV obtained experimentally for this reaction. In view of the present confusion about the A_2 resonance³⁴⁾, we have not attempted to correct for this. Any change of the A_2 parameters above threshold, however, will affect only this reaction.

vii) As explained in Section 2, our choice of parameters in the table yields via (22) a width of only 70 MeV for the Δ resonance, as compared with the 120 MeV given by phase-shift analysis. Had we taken the latter

value, we would obtain too wide a Δ compared with the data. One example is quoted in Fig. 7 at 5.0 GeV/c. Width determination seems to be a recurrent problem, and we can offer no satisfactory explanation. We note, however, that any change of the Δ trajectory parameters above threshold will affect only the reaction (A).

3.4 Resonance production

The concept of a resonance as a quasi-particle is strictly valid only in the zero width approximation. In its application to data analysis, therefore, one is invariably faced with such problems as "background subtractions" and "kinematic reflections" of other resonances which may not even be well-defined. In the present model, however, one claims to have constructed the amplitude for the actual three-body final state. The concept of quasi-particles then becomes merely a matter of convenience and no longer a necessity. Thus the ambiguities mentioned previously are entirely avoided.

In all the discussions which follow, we define a resonance, in both the experimental data and the theoretical calculations, as comprising all those events within a band on the effective mass plot of width 2Γ , centred at the resonance mass. This is different from what is usually done in phenomenological analyses, which normally involve fitting with various amounts of phase space and Breit-Wigner shapes, and depend on the quasi-particle concept. Some distributions presented in this paper may therefore look different from those published in earlier literature.

In Figs. 10 and 11, the amounts of production for the dominant resonances $K^*(890)$, $K^{**}(1400)$, and $\Delta(1236)$ in reactions (A) and (B) are given as functions of the incoming momentum and the momentum transfer. General agreement with the data is obtained, apart from some discrepancies at $p_{lab} \lesssim 3$ GeV/c and at momentum transfers near $t = 0$. Both these discrepancies we ascribe to the neglected π -exchange effects, as discussed in Sections 3.1 and 3.2.

3.5 Resonance decays

Some examples of resonance decay distributions obtained from our calculation are compared with the data in Figs. 12 to 15.

In our model decay distributions for the leading resonance on each trajectory depend essentially only on the kinematic factor $\varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$ in Eqs. (18) to (21) and thus merely check our basic assumption of dominant vector exchange. As seen in Fig. 12, the agreement with data is quite reasonable apart from expected discrepancies due to the neglect of π exchange.

The decay of higher resonances, such as $K^{**}(1400)$ and A_2 , however, depends on more details of the model and deserves somewhat more attention. In Fig. 13 are shown the decay distributions of the $K^{**}(1400)$ produced in reaction (A). One notes some variation in the experimental distributions with the incoming momentum. Although the agreement of our calculation with the data is only tenuous, the trend of the energy dependence is reproduced.

Whereas the $K^{**}(1400)$ in reaction (A) tends to decay with the K meson going forwards in the Jackson frame with respect to the incoming K, the same resonance in reaction (B) has just the opposite asymmetry, as can be seen in Figs. 13 and 14. This difference is reproduced in our calculation and has a simple qualitative explanation. The Jackson angle θ varies from 0 to π as one goes along the resonance band from one edge of the Dalitz plot to the other. For the $K^{**}(1400)$ in reaction (A), we have a strong enhancement of one edge due to the Δ , corresponding to $\theta = 0$. For the K^{**} in reaction (B), however, one has instead an enhancement due to the Y^* at the opposite edge corresponding to $\theta = \pi$.

Finally, for the decay of the A_2 in reaction (C), there is an apparent anomaly for which we have no explanation. At 4.0 GeV/c the experimental distributions are similar to the $K^{**}(1400)$ discussed above and are well reproduced by our calculation. At 3.1 GeV/c, however, although the calculated distributions still look the same, they fail completely to agree with experiment. Further data are needed in order to clarify the situation.

4. CONCLUDING REMARKS

We note first of all the following points:

i) As already observed previously, experimental evidence in π^-p elastic scattering seems to indicate an "anomalous" suppression of Δ exchange. One suggestion is that the Δ residue has an extra "dynamical" zero at the wrong-signature sense point with $\alpha_\Delta = \frac{1}{2}$, which could then explain this anomaly³⁵⁾. We tend to believe that Δ suppression operates also in our reactions, although its effects are masked by the much stronger meson exchanges and are therefore made less obvious than in elastic scattering. In order to investigate such effects in the present case, we performed the following calculation. Assuming that the Δ trajectory that is dominantly produced in reaction (A) becomes suppressed when exchanged, we repeated the calculation for (B) and (C) with the Δ intercept lowered by $\frac{1}{2}$. Then the resulting cross-sections for (B) and (C) calculated with the same normalization are lowered by about 10 per cent in each case, while the distributions remain qualitatively similar. The only significant change is in the π^-p effective mass distributions of reaction (B), where it is found that the spurious peak at ~ 1.3 GeV disappears, as shown by the dotted curves in Fig. 8.

ii) For the remaining reactions listed in Section 1 for group II, some data exist, though insufficient at present to justify a detailed investigation. A batch of data for reaction (6) is available at 8 GeV/c³⁶⁾ with 47 events, which gives the cross-section and the $K\bar{K}$ effective mass distribution. Using exactly the same parameters as in Section 2, we obtained a rough agreement with the scanty data, as shown in Fig. 16. The cross-section comes out to be 59 μb , a factor of 2 larger than the 26 μb obtained experimentally, which is similar to the situation found in reaction (C).

iii) The weakest point in our calculation is clearly the haphazard manner in which we have treated the baryon trajectories. It is therefore to be expected that we would have more trouble if we try to cross baryon lines. We had an indication of this when we tried to calculate the cross-section for reaction (10). With the same parameters as in Section 2, we obtained a cross-section that is an order of magnitude too large, e.g. 160 μb at 3.7 GeV/c³⁷⁾ instead of the 10 μb reported experimentally. Unfortunately, existing data do not allow one to investigate this point any further.

On the whole we believe that the general agreement -- despite some discrepancies -- of our calculation with experiment for the cases so far studied is significant, considering the little freedom we had. We are therefore encouraged to attempt to extend the study to the other groups of reactions mentioned in the introduction, which involve the Pomeron and the pion-exchange.

Acknowledgements

We are grateful to the Brussels-CERN Collaboration, who generously allowed us free access to their data summary tape for the K^+p reaction at 3.5, 5.0, and 8.2 GeV/c. This permits us to check our calculation against experiment in some details which are otherwise not available in the published data. We wish to thank also Dr. J. Hopkinson for his efficient program for the B_5 -function, which we have employed throughout our work. In addition, we have benefitted a great deal at various stages from discussions with the following colleagues: Drs. A. Bia~~l~~as, Y. Goldschmidt-Clermont, J.E. Paton and H. Satz.

Two of us (G.H.T. and N.A.T.) are grateful to Professors J. Prentki and W. Thirring for their hospitality in the Theoretical Studies Division of CERN, and a third (R.O.R.) to Professors. R. Armenteros and Ch. Peyrou in the Track Chambers Division.

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Figure captions

- Fig. 1 : The general class of reactions discussed in this paper, where all lines are taken as incoming.
- Fig. 2 : The four graphs that are free from exotic channels for the reactions of group II. The label between each pair of adjacent lines represents the trajectory used in this paper. As in Fig. 1, all lines are incoming.
- Fig. 3 : The imaginary part of $\alpha(S)$ versus S (in GeV^2) for the baryon trajectories used in this paper. The points are taken from the Particle Data Group (Ref. 11). Error flags with horizontal bars are used when the Particle Data Group table quotes an error. Otherwise, error flags without horizontal bars are used to represent the highest and lowest experimental value quoted in the Particle Data Group table. For easy reference, the mass and width used to calculate each point is also presented. The solid curve gives the imaginary part of α used in this paper, where the slope has been taken to be 0.14 GeV^{-2} .
- Fig. 4 : The total cross-sections in mb for reactions (A), (B), and (C) as functions of the incoming laboratory momentum. The solid curves are the predictions of amplitude (25) normalized at 5 GeV/c in reaction (A). The dotted curve is the prediction of (25) for reaction (C) scaled by a factor of $\frac{1}{2}$. The cross-sections for reaction (A) are given in Refs. 20-23, for (B) in Refs. 7 and 24-28, and for (C) in Refs. 15 and 29. For greater readability, some error bars for reaction (C) have been omitted. Those error bars presented are representative examples.
- Fig. 5 : The momentum transfer distributions in percentages for reaction (A) at 3.5 GeV/c (Ref. 21), 5 GeV/c and 8.25 GeV/c (Ref. 22). The solid curves are the predictions of amplitude (25).

Fig. 6 : Reaction (A) at 3.5 GeV/c (Ref. 21), 5.0 GeV/c and 8.25 GeV/c (Ref. 22). Percentage distributions in the c.m. longitudinal momenta p_L and in the cosine of the c.m. angle, $\cos \theta^*(\pi^+)$ between the incident K^+ and the final π^+ . The solid curves are the predictions of amplitude (25).

Fig. 7 : The effective mass distributions in percentages for reaction (A) at 3.5 GeV/c (Ref. 21), 5 GeV/c and 8.25 GeV/c (Ref. 22), and 12.7 GeV/c (Ref. 23). The solid curves are the predictions of amplitude (25). At 3.5 GeV/c, the lower curve in the $M(K^0\pi^+)$ distribution for the events in the Δ^{++} band $[1.12 \leq M(p\pi^+) \leq 1.36]$. It is seen that both K^* 's interfere constructively with the Δ^{++} . At 5 GeV/c in the $M(p\pi^+)$ distribution, the dotted curve shows the effect of putting $\text{Im } \alpha_\Delta = 0.36$ ($s - 1.0$) which gives a 120 MeV Δ width. Such a width requires an amplitude $0.6P + R + S$ and widths for higher Δ recurrences which are much too large.

Fig. 8 : The effective mass distributions in percentages for reaction (B) at 2.6-2.7 GeV/c (Ref. 25), 3.5 GeV/c (Ref. 26), 6.0 GeV/c (Ref. 27), and 10 GeV/c (Ref. 28). The solid curves are the predictions of amplitude (25). In the πp distributions, the dotted curves show the effect of lowering the exchanged Δ trajectory by $\frac{1}{2}$ in amplitude (25).

Fig. 9 : The effective mass distributions in percentages for reaction (C) at 3.1 GeV/c and 4.0 GeV/c (Ref. 15). The solid curves are the predictions of amplitude (25). The experimental histograms contain events covering a range of 0.4 GeV about the central values quoted. This is simulated by a Gaussian spread in energy in the theoretical calculations.

Fig. 10 : The fraction (in per cent) of resonances produced versus p_{lab} for reactions (A) and (B). The stars are the predictions of amplitude (25). Here no background subtractions of any kind are made, the resonance being defined as lying in the following bands (in units of GeV):

$$0.840 \leq M(K^{*\pm}) \leq 0.940;$$

$$1.33 \leq M(K^{**\pm}) \leq 1.51;$$

$$1.12 \leq M(\Delta^{++}) \leq 1.36.$$

Fig. 11 : The momentum transfer distributions in percentages of the resonances in reaction (A) at 5 GeV/c (Ref. 22) and in reaction (B) at 6 GeV/c (Ref. 27). The solid curves are the predictions of amplitude (25). No background subtractions are made, resonances being defined in the bands given in the caption to Fig. 10.

Fig. 12 : The decay distributions in percentages of the p-wave resonances for reaction (A) at 5 GeV/c (Ref. 22) and for reaction (B) at 6 GeV/c (Ref. 27). The solid curves are the predictions of amplitude (25). The Jackson angle θ and Treiman-Yang angle ϕ are the polar coordinates of the final $K(p)$ in the $K\pi$ (πp) rest frame where the z-axis is the incident $K(p)$ direction and the y-axis is the direction of the normal to the production plane of the resonance.

Fig. 13 : The decay distributions in percentages of the K^{**} resonance in reaction (A) at 3.5 GeV/c (Ref. 21), 5 GeV/c and 8.25 GeV/c (Ref. 22). The solid curves are the predictions of amplitude (25). The angles being defined as in Fig. 12.

Fig. 14 : The decay distributions in percentages of the K^{**} resonance in reaction (B) at 6 GeV/c (Ref. 27). The solid curves are the predictions of amplitude (25). The angles are defined as in Fig. 12.

Fig. 15 : The decay distributions in percentages of the A_2 resonance in reaction (C) at 3.1 GeV/c and 4.0 GeV/c (Ref. 15). The solid curves are the predictions of the amplitude (25). The A_2 is defined for the experimental points as lying in the band $1.267 \leq M(K\bar{K}) \leq 1.367$; for the predicted curve in the band $1.25 \leq M(K\bar{K}) \leq 1.34$. The Jackson angle θ and Treiman-Yang angle ϕ are the polar coordinates of the final K^0 in the $K\bar{K}^0$ rest frame where the z-axis is the incident π^- direction and the y-axis is the direction of the normal to the production plane of the A_2 resonance.

Fig. 16 : The $K^+\bar{K}^0$ effective mass distribution for the reaction $\pi^+p \rightarrow K^+\bar{K}^0p$, normalized to 47 events. The solid curve is the prediction of amplitude (25).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

$$K^- p \rightarrow \bar{K}^0 \pi^- p$$

Fig. 8

$$\pi^- p \rightarrow K^0 K^- p$$

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

$K^- p \rightarrow \bar{K}^0 \pi^- p$

Fig. 14

$\pi^- p \rightarrow K^0 K^- p$

Fig. 15

Fig. 16