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AMBIGUITIES IN REGGE ANALYSES AND COMPARISON
WITH PHASE SHIFTS

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ABSTRACT

We discuss a simple rotation in the flip non-flip plane, which is undetermined in conventional Regge fits (i.e., those without FESR), and which thus affects the conclusions about mechanism choosing at $\alpha = 0$. Our t independent effective pole analysis of $K^\pm p$ elastic scattering naturally continues down to $p_{\text{Lab}} = 1.45 \text{ GeV}/c$, where a rotation invariant comparison with the $K^+ p$ phase shifts is made.

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Regge pole phenomenology provides a framework for obtaining the amplitudes of high energy processes. However in conventional analyses [i.e., those without FESR ^{*)}], there are several features which should be discussed in detail :

- i) the uniqueness of the fits;
- ii) the zeros of the amplitudes and more specifically the mechanisms at $\alpha = 0$, etc.. The knowledge of these zeros is crucial for duality comparisons;
- iii) the predictive power of such fits.

Most fits have been done in a t -dependent manner which necessitated a priori assumptions about zeros and mechanisms for each pole residue. We want to determine such zeros in an unbiased way and therefore we perform our analysis at each fixed t value independently. We consider meson-baryon scattering ($0^{-\frac{1}{2}+}-0^{-\frac{1}{2}+}$). Within a model employing a fixed number of Regge poles, one can discuss the uniqueness of the fits. We shall show that :

- a) there is a simple rotation operation in the spin flip - spin non-flip plane which leaves unchanged the cross-section and polarization at each t separately, while intermixing the Regge residue functions. This operation affects all conventional Regge analyses (i.e., those without FESR). However, if one works modulo the rotation, one can investigate simply any remaining ambiguities in the analysis;
- b) the intermixing of Regge residues can modify the zeros in residue functions and change the interpretation of mechanisms;
- c) the effective Regge pole model has predictive power coming from the connection between the phase of the amplitude and its energy dependence. However the Regge predictions must be tested in a manner invariant under the rotation.

^{*)} We perform a Regge analysis without FESR for the following reasons :

- a) possibility to confront high energy results with FESR;
- b) for our application to $K^{\pm}p$ scattering, though they were useful when no polarization data existed ¹⁾, the FESR have poorly known contributions from the Born poles, unphysical region and $K^{\mp}p$ phase shift region.

When $d\sigma/dt$ and P data are available, one has only two real numbers to determine the four components of the two complex amplitudes A' and B . An analysis employing several Regge poles gives one further constraint since the phase is obtained from the energy dependence ^{*}). To remove the above rotation degeneracy, one must either measure R or A parameters or compare to amplitudes obtained through phase shift analysis.

We define amplitudes a and b ²⁾ such that

$$d\sigma/dt = C (|a|^2 + |b|^2)$$

$$P d\sigma/dt = -2C \operatorname{Im} a b^*$$

a is the conventional A' amplitude. For elastic scattering $C = (1-t/4M^2)/(16\pi p_L^2)$, where p_L is the meson lab. momentum, M the target mass. With q the centre-of-mass momentum, we have

$$b = \frac{\sqrt{-t} p_L B (1 + t/4q^2)^{1/2}}{2M (1 - t/4M^2)}$$

We now consider the (a, b) plane (and not the complex plane)

$$\vec{Re} = (\operatorname{Re} a, \operatorname{Re} b)$$

$$\vec{Im} = (\operatorname{Im} a, \operatorname{Im} b)$$

In vector notation the measured quantities are given by

$$d\sigma/dt = C (\vec{Re} \cdot \vec{Re} + \vec{Im} \cdot \vec{Im})$$

$$P d\sigma/dt = 2C \vec{Re} \times \vec{Im}$$

$d\sigma/dt$ is a scalar product, $P d\sigma/dt$ a vector product and both are invariant under a rotation in the spin flip - non-flip plane about the normal.

^{*}) When there is only one Regge pole (as in πN charge exchange), one may determine from only $d\sigma/dt$ data two extra quantities, the phases of A' and B . These phases are equal.

Reggeology gives restrictions on the phase (angle in the complex plane) from energy dependence, but this Regge phase is invariant under rotations in the (a,b) plane. Since for each Regge pole R_j we have

$$\overrightarrow{Re}_j = \frac{1 - \cos \pi \alpha_j}{\sin \pi \alpha_j} \overrightarrow{Im}_j$$

Then any sum of pole amplitudes may be rotated in the (a,b) plane without changing the complex phases. A Regge cut can in general be considered as a sum of effective poles and will also be invariant ^{*}).

The parameters A and R are related to $|a|^2 - |b|^2$ and $\text{Re}(ab^*)$ which are clearly not invariant under such rotations and will allow, within a Regge pole model, a complete determination of the amplitudes. However, they have only been measured in πN scattering ³⁾. Also near $t=0$ one may use limited t continuity arguments as in πN charge exchange ⁴⁾ to limit the rotation angle since the forward cross-section dip implies substantial spin flip.

Particular mechanisms such as "nonsense" and "Chew" for + signature trajectories at $\alpha = 0$ are not invariant concepts and can be rotated into one another. Therefore they cannot be determined in an unbiased way in a traditional Regge fit. Dynamical zeros may also be moved in a similar way.

In order to discuss any remaining ambiguities in a Regge fit, we must first arbitrarily fix the over-all orientation in the (a-b) plane. We choose this over-all orientation such that $b(P)=0$ at all t , which is not a physical constraint on the Pomeron, but is merely a convention; equally well we could have chosen $b=0$ for any other of the poles in the model. In the past, it was known that b_P was poorly determined ⁵⁾; we emphasize that our result has nothing to do with the Pomeron per se, but rotates all poles in the problem equally.

^{*}) However, a specific model, such as the absorption prescription for the cut discontinuity, is not invariant under these rotations.

As an illustration we shall now discuss the Regge analysis of $K^{\pm}p$ elastic scattering. We use $d\sigma/dt$ and P data ^{6),7)} between $P_L = 1.45$ and $15 \text{ GeV}/c$. The $d\sigma/dt$ data were smoothed in t and normalized at $t=0$ to the forward amplitude obtained from σ_{TOT} ⁸⁾ and dispersion relation real parts ⁹⁾. All data are interpolated to obtain points at t values in steps of 0.1 from $t=0$ to $t=-1.0 \text{ (GeV)}^2$. We use a conventional effective pole model, where effective poles represent the combined effects of poles and cuts. This implies that we cannot use factorization.

Since there are no good data on K^+n elastic scattering (in particular no polarizations) nor on K^0 regeneration on protons, charge exchange will not give any constraints on the $K^{\pm}p$ elastic amplitudes which we set out to determine. Therefore we do not include charge exchange data, and we only determine the definite linear combination of t channel isospins, which corresponds to $K^{\pm}p$ elastic scattering, namely, $\rho + \omega \equiv V$ and $A_2 + f_0 \equiv T$. We assume that the coupling of ϕ and f' to NN is weak and can be neglected. We are left with three poles: P, T, V . For simplicity, we assume that the trajectories of V and T are degenerate $\alpha_V = \alpha_T = \alpha_M$ ($M = \text{meson}$), but the residues must not be taken degenerate since such strong exchange degeneracy would not represent the data quantitatively ⁷⁾. For numerical stability, we fix the two trajectory functions, α_P and α_M at various values. Here, we shall show two cases; a non-zero P slope $[S]: \alpha_P = 1 + 0.3t, \alpha_M = 0.5 + 0.9t$; and a flat P $[F]: \alpha_P = 1, \alpha_M = 0.5 + t$. In a subsequent publication we shall discuss the evidence that the data prefer the former choice. As described above, we work at each t separately in order to remove prejudices about zeros of residues; we concentrate on $t \neq 0$. In order to fix the over-all orientation in the (a, b) plane we chose the convention $B_P \equiv 0$ at all t ; thus we are left with five unknown residues at each t value: $\gamma_P, \gamma_T, \gamma_V, \beta_T, \beta_V$ (where $\text{Im } A' = \gamma \nu^\alpha$, $\text{Im } B = \beta \nu^{\alpha-1}$ and $\nu = \omega_L + t/4M$).

For $0 \leq -t \leq 0.5$ and each α choice, we obtained fits at each t value which led to a unique solution in which each residue function was remarkably continuous in t . Using first a lowest momentum of $2.75 \text{ GeV}/c$ for the fits, we extended this to $1.45 \text{ GeV}/c$ and found substantially the same solution. Consequently we retained $1.45 \text{ GeV}/c$ as a lowest momentum and were

encouraged by the reasonable fits, particularly for K^+p , at this momentum. We analyzed the fitting procedure in order to understand the uniqueness; we found that γ_P was determined by the higher energy data in $\sigma(K^-)+\sigma(K^+)$, and then γ_V and β_T were determined in a unique way from the data on $\sigma(K^-)-\sigma(K^+)$ and $P\sigma(K^-)+P\sigma(K^+)$, respectively. γ_T and β_V are then identified from the lower energy data on $\sigma(K^-)+\sigma(K^+)$ and from $P\sigma(K^-)-P\sigma(K^+)$. γ_T and β_V are in general less well determined, but out to $|t|=0.5$ one obtains a unique solution. For $|t| > 0.5$, the ambiguities (in particular a four-fold ambiguity affecting β_V and γ_T) will be discussed in another publication.

We summarize our results for $-t \leq 0.5$ (with $\beta_P \equiv 0$ of course); γ_V has the usual cross over zero at $t \approx -0.15$, β_V has a zero at -0.45 ; both γ_T and β_T have ghost killing zeros at $\alpha_M = 0$; β_V and β_T are almost equal in agreement with exchange degeneracy.

We pass now to the comparison of our Regge predictions for the phase and spin dependence of the K^+p amplitudes at 1.45 GeV/c with those from phase shift analysis ¹⁰⁾. We show in Fig. 1 a typical comparison for the amplitudes in the (a,b) plane. Since we may rotate the Regge amplitudes about the normal to the plane, we must compare invariant quantities. Possible candidates are $|\vec{Re}|$, $|\vec{Im}|$ and $\Theta(\text{Re}, \text{Im})$. However, it is more convenient to compare first the two fitted quantities $d\sigma/dt \sim |\vec{Re}|^2 + |\vec{Im}|^2$ and $Pd\sigma/dt \sim |\vec{Re}| \cdot |\vec{Im}| \cdot \sin\Theta(\text{Re}, \text{Im})$. The real test is then the comparison of a third invariant which can be $\Theta(\text{Re}, \text{Im})$ or else $\phi = |\vec{Re}|/|\vec{Im}|$, the latter being the tangent of the "average Regge phase". Such a third invariant is the prediction of Regge in this context. In Fig. 2 these four quantities (of which only three are independent) are compared for both our sets of Regge trajectories S, F and for both phase shift solutions I, II ¹⁰⁾. The polarization data clearly show the need of higher partial waves in the phase shift analysis to give a sharper rise at small t . In the Θ plot, Fig. 2d, the Regge prediction is seen to be rather independent of the choice of trajectories, and the Regge prediction shows a preference for the phase shift solution II, which is the solution most in accord with exchange degeneracy expectations ⁷⁾. In Fig 2c, ϕ which is related to the phase and thus to the α values, shows a larger difference between the predictions from our two choices. These predictions are in qualitative agreement with the phase shift amplitudes except

at $t \approx 0$. Here, our δ values are obtained from fits to the energy dependence of σ_{TOT} for $K^{\pm}p$ and, at $t=0$, lie 0.1 higher than Carter's value ⁹⁾. The phase shift estimates ¹⁰⁾ of δ at $t=0$ do not vary smoothly with energy, and may change appreciably in more refined analyses.

Having seen that the invariant comparison to phase shift amplitudes is fairly good, one may now fix the orientation of the Regge solutions at each t value separately from the phase shifts. If solution II is essentially correct, the Regge solution should be rotated clockwise (see Fig.1) and the angle corresponds to $\beta_P/\delta_P \sim -0.5$. However, solution I would be compatible with β_P/δ_P as large as +2.0. This same angle intermixes the β and δ residues for both V and T poles as well.

The methods of amplitude extraction employed in phase shift analysis and in Regge analysis are at fixed s and t , respectively, and are quite independent of each other. It is most encouraging that when one is able to confront the two methods, they agree qualitatively. Moreover, the Regge approach even allows us to discriminate among phase shift solutions giving equally good fits to the data. Provided relevant quantities are asked of it, the predictive power of Regge pole analysis is evident.

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FIGURE CAPTIONS

Figure 1 : Amplitudes for K^+p elastic scattering at 1.45 GeV/c and $t = -0.4 \text{ GeV}^2$, in the spin flip - non-flip plane from the Regge pole model S described in the text, and from the phase shift solutions of Ref. 10). The Regge amplitudes $\vec{R}_e$ and $\vec{R}_m$ may be rotated simultaneously as described in the text; $\theta(\vec{R}_e, \vec{R}_m)$ is the angle between these two vectors.

Figure 2 : Comparison for K^+p elastic scattering at 1.45 GeV/c against t . Solid lines are phase shift solutions I and II of Ref. 10). The squares are our fixed t Regge predictions which are connected to guide the eye by a dashed line for trajectories S ($\alpha_P = 1 + 0.3t$, $\alpha_M = 0.5 + 0.9t$) and a dot-dash line for trajectories F ($\alpha_P = 1$, $\alpha_M = 0.5 + 1.0t$).

- a) Differential cross-section, data from Bettini et al. ⁶⁾.
- b) Polarization, data ⁶⁾ from Anderson et al. (black circles) and Asbury et al. (open circles).
- c) The ratio ϕ of lengths of the vectors $\vec{R}_e$ and $\vec{R}_m$.
- d) The angle θ between the vectors $\vec{R}_e$ and $\vec{R}_m$, see Fig. 1.

FIG.1

FIG.2