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UNITARITY BOUNDS TO T ODD CORRELATIONS
IN NEUTRINO REACTIONS

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A B S T R A C T

Rigorous bounds, to order $\alpha \simeq 1/137$, are derived in elastic neutrino reactions for the outgoing lepton or hadron transverse polarization induced by the final state interactions.

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Elastic neutrino-nucleon scattering, with subsequent analysis of the transverse polarization of the outgoing lepton or hadron has been suggested by various authors as a possible test of time-reversal invariance ¹⁾⁻⁵⁾. The test, however, must fail at a certain level because of the final state interactions. It is therefore of interest to have a reliable knowledge of this effect, so that any outstanding discrepancy could be attributed to intrinsic violation of time-reversal invariance. In this note, it is shown that to order $\alpha \simeq 1/137$ it is possible to find a useful bound to the transverse polarization of the outgoing lepton or hadron in elastic neutrino reactions. The bound depends only on observables which, to a large extent, have already been measured.

We are interested in reactions of the type

$$\nu n \rightarrow \mu^- p \quad (1)$$

and

$$\bar{\nu} p \rightarrow \mu^+ n \quad (2)$$

At a given c.m. squared energy s and squared momentum transfer t , the transverse polarization $\xi(s, t)$ of one of the outgoing particles in these reactions is defined as

$$\xi(s, t) = \frac{\frac{d\sigma^\uparrow}{dt} - \frac{d\sigma^\downarrow}{dt}}{\frac{d\sigma^\uparrow}{dt} + \frac{d\sigma^\downarrow}{dt}} \quad (3)$$

where $\frac{d\sigma^{\uparrow(\downarrow)}}{dt}$ denotes the differential cross-section for the production of the particle polarized up(down) along the normal to the scattering plane : $\vec{n} = \vec{k} \times \vec{p}_f / |\vec{k} \times \vec{p}_f|$, where $\vec{k}$ is the incident neutrino momentum and $\vec{p}_f$ the momentum of the final particle we are considering. The corresponding average transverse polarization $\bar{\xi}(s)$ is defined as

$$\bar{\xi}(s) = \frac{1}{\sigma(s)} \int dt \frac{d\sigma}{dt} \xi(s, t) \quad (4)$$

Measurements of the transverse polarization of the proton in reaction (1) and of the neutron and muon in reaction (2) will become statistically significant in the next generation of neutrino experiments ⁶⁾. In this note we are concerned with the contribution to these polarizations from the interference of the lowest order weak amplitude with its electromagnetic corrections. In fact, since the effect one is looking at is a T-odd correlation, it is sufficient to know the imaginary part of this interference. Unitarity allows to express the latter, to second order in the electric charge, in terms of on-shell amplitudes as illustrated in Fig. 1. We have explicitly evaluated the contribution from the elastic intermediate state (Γ = nucleon, in Fig. 1) and bounded the contribution from all the other possible intermediate states.

The contribution to $\bar{\xi}(s)$ from the elastic intermediate state is shown in Fig. 2a, for the proton in reaction (1) and for the neutron and muon in reaction (2). In the calculations, we have taken the electromagnetic form factors given by the usual dipole fit ^{*)}, and the weak vector and weak magnetic form factors as predicted by CVC. For the axial-vector form factor we have taken the expression

$$F_A = 1.2 \frac{1}{\left(1 - \frac{t}{0.76 \text{ GeV}^2}\right)^2} \quad (5)$$

^{*)} See, e.g., Rutherglen ⁷⁾.

consistent with the CERN elastic neutrino scattering data ⁸⁾. Terms involving the induced pseudoscalar form factor in the transverse polarization of the proton or the neutron are of order m_μ^2/s and hence have been neglected. This is no longer true in the muon case ; however, the whole effect is then proportional to $m_\mu/\sqrt{s}$ and practically negligible ^{*)}.

The exact calculation of the contribution to $\xi(s,t)$ from the inelastic intermediate states requires a knowledge of magnitudes and phases of electromagnetic and weak hadronic vertices which is clearly not available. However, using the Schwartz inequality, it is possible to bound $|\xi(s,t)|$ in terms of electromagnetic and weak inelastic cross-sections. We obtain ^{**))}

$$|\xi(s,t)| \leq \int_{(M+m_\pi)^2}^s dw^2 \int_{\text{Min } t_2}^0 dt_2 \int_0^{2\pi} \frac{d\varphi}{2\pi} \left(\frac{\frac{d\sigma(\nu N)}{dw^2 dt_1} \frac{d\sigma(\mu N')}{dw^2 dt_2}}{\pi \frac{d\sigma(\nu N \rightarrow \mu N')}{dt}} \right)^{1/2} \quad (6)$$

^{*)} The Dirac algebra in these calculations was performed with the aid of the computer program Schoonship developed by M. Veltman ⁹⁾.

^{**))} It is also possible to give better bounds of $|\xi(s,t)|$ in terms of the electromagnetic and weak structure functions of the nucleon ; however, they are less useful in the sense of requiring experimental knowledge that is not yet available. Details of the derivation of these bounds for weak as well as electromagnetic reactions will be published elsewhere.

where t_1 and t_2 are the squared momentum transfers in the weak inelastic $\nu N \rightarrow \mu \Gamma$ and electromagnetic inelastic $\mu N' \rightarrow \mu \Gamma$ reactions ; and φ is such that with

$$t = - \frac{(s-M^2)^2}{2s} (1 - \cos \theta) \quad \text{and} \quad t_{1,2} = - \frac{(s-M^2)(s-W^2)}{2s} (1 - \cos \theta_{1,2})$$

$\cos \theta$, $\cos \theta_1$ and $\cos \theta_2$ satisfy the triangular relation

$$\cos \theta_1 = \cos \theta \cos \theta_2 + \sin \theta \sin \theta_2 \cos \varphi$$

Notice that the bound is not attainable in the forward and backward directions where the transverse polarization must vanish ^{*}). Equation (6) applies separately to reactions (1) and (2), but in each reaction the bound is the same for the outgoing lepton or hadron. In order to make numerical estimates, we have made the following parametrizations.

- (i) - The weak elastic cross-sections are expressed in terms of the form factors mentioned above. This is consistent with present experimental data ⁸⁾.
- (ii) - For the inelastic cross-sections in the case of reaction (1) :
 - the muon-proton cross-section, assuming muon-electron universality, is taken from the SLAC data ¹⁰⁾ on inelastic electron-proton scattering ^{**)} ;

^{*}) The transverse polarization of the outgoing lepton or hadron is proportional to $m/\sqrt{s} \sin \theta$, where m is the mass of the particle whose polarization we consider.

^{**)} At large missing mass and momentum transfer, an extrapolation of the published 6° and 10° data is needed. This has been done assuming that the off-shell photoabsorption cross-section maintains its qualitative features, i.e., constancy as a function of W for fixed q^2 , q^{-8} decrease in the resonance region (fixed $W < 2$ GeV) and q^{-2} behaviour in the deep inelastic region (fixed $W > 2$ GeV).

- the neutrino-neutron cross-section is not directly obtainable from the CERN data, which is given for an approximately even admixture of protons and neutrons ¹¹⁾. However, since only an upper bound is needed, we have taken $\sigma(\bar{\nu}n \rightarrow \mu \bar{\nu}) < \sigma(\bar{\nu}p \rightarrow \mu \bar{\nu}) + \sigma(\bar{\nu}n \rightarrow \mu \bar{\nu}) \simeq 2\sigma(\text{Experimental})$

(iii) - For the inelastic cross-sections in the case of reaction (2) :

- we make the plausible assumption that $\sigma(\mu n \rightarrow \mu \bar{\nu}) \lesssim \sigma(\mu p \rightarrow \mu \bar{\nu})$. This will soon be not necessary when data on electron-neutron scattering are published ;
- the experimental data on antineutrino inelastic cross-sections are still very poor ; to the approximation where the parity mixing structure function of the nucleon vanishes, consistent with the CERN data ¹¹⁾, and neglecting strangeness changing processes, we can write $\sigma(\bar{\nu}p \rightarrow \mu \bar{\nu}) < \sigma(\bar{\nu}p \rightarrow \mu \bar{\nu}) + \sigma(\bar{\nu}n \rightarrow \mu \bar{\nu}) \simeq \sigma(\bar{\nu}p \rightarrow \mu \bar{\nu}) + \sigma(\bar{\nu}n \rightarrow \mu \bar{\nu}) \simeq 2\sigma(\text{Experimental})$

Notice that the numerical estimate in reaction (1) does not affect the rigour of the bound. On the contrary, reaction (2) is directly observable in hydrogen bubble chambers while data on reaction (1) have to be extracted from experiments on deuterium ^{*}). The numerical results for the bounds on $\bar{\xi}(s)$ obtained with the above parametrizations are shown in Fig. 2b.

^{*}) In fact, the interaction of the final proton with the expectator neutron of the deuteron can also give rise to a transverse polarization of the proton. However, the nuclear effects can be minimized by an appropriate momentum transfer cut-off, in a way which is unfortunately model dependent.

To compare our results with possible effects due to an intrinsic violation of T invariance, we have drawn in Fig. 2c the predictions of the Cabibbo theory ³⁾. In this theory, a transverse polarization is predicted due to the interference of first and second class hadronic currents. In the calculations ^{*}), we have taken an induced tensor form factor equal in magnitude to the weak magnetism but 90° out of phase with respect to the axial-vector, and neglected the induced scalar form factor as suggested by CVC. The induced pseudoscalar form factor F_{ps} , gives negligible contributions in the case of the proton and neutron. However, for the μ^+ in reaction (2), the transverse polarization is very sensitive to F_{ps} since it is proportional to the divergence of the axial-vector current ; i.e., to

$$F_A(t) + \frac{t}{4M^2} F_{ps}(t)$$

The μ^+ results in Fig. 2c correspond to the choice predicted by PDDAC ; but other choices of form factors ⁴⁾ can give much larger effects.

It can be seen from Fig. 2 that in the proton and neutron cases, the possible transverse polarization due to violation of T invariance can be much larger than the bounds on final state interaction effects ^{**)}. These bounds, at the relevant energies, are smaller than

^{*}) Calculations of this type have also been done by Adler ²⁾, where general formulae can be found ; by Berman and Veltman ⁴⁾ for the μ^- in reaction (1) ; and by Fujii and Yamaguchi ⁵⁾, for the proton in reaction (1).

^{**)} The reaction $\bar{\nu}_p \rightarrow \mu^+ \Lambda$ with subsequent analysis of the Λ transverse polarization is also of experimental interest as a test of time-reversal invariance. This is due to the fact that Λ decay is a good analyzer of its polarization. Unfortunately, it is not possible to give a bound to the final state interaction effect comparable in rigour to the one obtained for nucleons.

the expected accuracy in the next generation of neutrino experiments. In conclusion, measurements of the transverse polarization of the outgoing hadron in elastic neutrino reactions can be useful as a rigorous test of time-reversal invariance.

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FIGURE CAPTIONS

Figure 1 Unitarity diagram for $\text{Im } A(\nu N \rightarrow \mu N')$ to second order in the electromagnetic interactions.

Figure 2 Average transverse polarizations in % as a function of neutrino laboratory energy for the proton in reaction (1) and for the neutron and muon in reaction (2) :

- (a) is the contribution from the elastic intermediate state ;
- (b) is an upper bound to the contribution from the inelastic intermediate states (note the different energy scale) ;
- (c) is the prediction of the Cabibbo theory.

FIG1

FIG. 2