MUONIC ATOMS AND NUCLEAR STRUCTURE

Roland Engfer

Institut für Technische Kernphysik der T.H., Darmstadt, Germany, and
CERN, Geneva, Switzerland.

Geneva - September 1969

(Invited paper to be presented at the
International Conference on High-Energy Physics and Nuclear Structure
Columbia University, New York, 8-12 September, 1969.)

MUONIC ATOMS AND NUCLEAR STRUCTURE

Roland Engfer

Institut für Technische Kernphysik der T.H., Darmstadt,
Germany, and CERN, Geneva, Switzerland.

1. INTRODUCTION

A classical problem in nuclear structure is the question of the size of nuclei. During the last 15 years muons in muonic atoms have been used as probes to study the electromagnetic size of nuclei. The following properties have been determined:

- i) $\rho(r)$ -- the size and shape of the charge distribution -- from K-, L- and M-X-rays yielding $\langle r^2 \rangle$ for light and medium heavy nuclei and two parameters for heavy nuclei;
- ii) Q_0 -- the intrinsic quadrupole moment of deformed nuclei -- from the quadrupole hyperfine structure of K- and L-X-rays;
- iii) $\mu(r)$ -- the distribution of the magnetic dipole moment of spherical nuclei -- from a broadening of $2p_{1/2}-1s_{1/2}$ transitions and from the hyperfine structure of nuclear γ -rays in muonic atoms;
- iv) $\delta\rho(r)$ -- differences of charge distributions of nuclear ground and excited states (isomer shift) -- from energy shifts of nuclear γ -rays in muonic atoms.

Besides these nuclear parameters quantum electrodynamics (vacuum polarization) was tested by accurate energy measurements of muonic X-rays. It is not possible in the time allotted to go into all details of measurements and analysis yielding this information. Therefore, I have selected several recent papers with a rather subjective viewpoint to explain experimental techniques and results

from muonic atom studies. I should recommend a recent review on "Muonic Atoms" by Devons and Duerdoth as a very instructive reference book and summary [1].

2. RECENT EXPERIMENTAL IMPROVEMENTS

The principal experimental set-up, which was used in the first "classical" experiment by Fitch and Rainwater [2], has not changed very much. A plastic scintillator telescope system is used for the detection of the stopped muon; the NaI(Tl) crystal is replaced by high resolving Ge(Li)-detectors. To achieve an accurate calibration, γ -lines from radioactive sources are measured simultaneously with μ -X lines and stored in different parts of a multichannel analyser. Two different systems are used for this purpose:

- i) A coincidence of β -rays or a second γ -ray characterizes a calibration event, which needs a fairly high source intensity (e.g. Ref. [3]);
- ii) All events including background are accepted but vetoed by an incoming muon; this enables one to use weak radioactive sources but requires a low background. In addition, these events are gated with the time structure of the muon intensity providing an identical load for stored muonic and calibration events (e.g. Refs. [4-6]).

These calibration techniques have obtained an accuracy of ~ 30 eV neglecting uncertainties of the calibration energies.

A NaI(Tl) counter system surrounding the Ge-detector was used effectively in coincidence as a pair spectrometer or in anticoincidence to reduce low-energy Compton background (Fig. 1) [6].

A serious problem in detecting μ -X-rays or nuclear γ -rays in the 100-200 keV region are X-rays from μ -carbon and μ -oxygen emitted from the scintillators and surrounding materials. To reduce this background a NaI crystal can be installed in coincidence with the Ge-detector accepting only γ events of more than ~ 80 keV. Since a number of high-energy X-rays are emitted in heavy μ -atoms the coincidence efficiency is rather high ($\sim 50\%$). On the other hand the disturbing μ -K series of C and O are not in coincidence with high-energy (< 80 keV) X-rays; thus they are reduced by a factor of 120-140. An example of such a measurement is given in Fig. 2 [7]; a fluorine target is attached to the Ta-target to measure the reduction factor.

With the use of on-line computers the energy dependence of the Ge-detector timing was corrected on-line [6]. Spectra taken in

Fig. 1 Prompt μ spectrum in ^{206}Pb [6]. The X spectrum contains all prompt events, the X $\cdot$ A spectrum the prompt events in anti-coincidence with the NaI annulus.

different delay time intervals between μ -stop and γ -detection were used to measure isotope effects in the capture rates of muons in ^{151}Eu and ^{153}Eu [8]. Typical time resolutions are 80 nsec at 100 keV up to 10 nsec at 6 MeV for a 17 ccm coaxial detector [6] and 10 nsec at 100 keV up to 3 nsec above 300 keV for a 3.5 ccm planar detector with leading edge timing.

3. CORRECTIONS TO THE BINDING ENERGIES OF MUONIC ATOMS

Radiative correction, nuclear polarization and electron screening influence the binding energy of a muon in muonic atoms. For all three effects experimental evidence has been found or valuable tests were performed recently. Talks on the vacuum polarization will be given by Anderson and Picasso. Experimental evidence of nuclear polarization [6,23] will be discussed in Section 5.1. In the following, recent results on the electron screening [10] are given in some more detail.

Fig. 2 Prompt μ spectrum in ^{181}Ta [7]. μ -X-rays and nuclear γ -rays are identified. Upper spectrum: Ta and fluorine target without and lower spectrum with NaI coincidence. The 2p-1s μ -X-line of ^{16}O (133.6 keV) coincides with a weak magnetic hf-component of a nuclear γ -transition in μ - ^{181}Ta . In the lower spectrum the μ -0 line does not appear, but the hf-component was not observed ($< 1\%$ of the main hf-component) beyond the statistical errors.

Up to now electron screening of muonic levels has been neglected in the interpretation of muonic X-ray data. This is justified for the lowest muonic states since the electronic charge gives rise only to a constant potential in the region of the lowest muonic orbits. The remaining effect of the non-constant potential term was calculated to be 4.6 eV for the 1s state and 190 eV for the 5g state in Bi [11].

Recently transitions from muonic states up to $n = 16$ have been measured [10]. The energies of some transitions are shifted by more than 1 keV compared to the unscreened values (Table 1). With the assumption of a Thomas-Fermi model [12] for the electron potential a good agreement with the experimental data was found (Table 1). Small systematic deviations mainly for the 6-5, 9-6 and 10-6 transitions are not significant compared to the experimental errors; they could indicate an incomplete electron shell during the cascade of the muon caused by preceding Auger transitions (Section 4). But higher experimental accuracy is needed to support this effect. An accurate knowledge of the energies is necessary to identify nuclear γ -rays in the low-energy region (50-200 keV) of muonic spectra (see Section 7).

4. INTENSITIES OF MUONIC X-RAYS

Data on intensities of μ -X-rays up to levels with $n = 16$ [6,10,13-15] were measured. It is not clear whether chemical shifts of the K-series intensities [13,16] are due to different initial distributions or incomplete electron shells after the μ -capture and the first Auger transitions. For low and medium Z elements it was found that a modifying factor $\exp(a \cdot Z)$ of the

Table 1

Energies of μ -X-rays in natural osmium [10]. E_{exp} : measured energies; up to 10 fine structure components are taken into account, the energy of the strongest component is given. E_{th} : calculated energies including vacuum polarization [Eq. (2.2) in Ref. 11] and finite size. $\Delta E = E_{\text{exp}} - E_{\text{th}}$. ΔW : calculated electron screening with Thomas-Fermi model [12].

Transition n-n' Δn	E_{exp} [keV]	E_{th} [keV]	ΔE [keV]	ΔW [keV]
9-8 1	53.48 $\pm$ 0.11	53.49	-	-
7-6 1	120.28 $\pm$ 0.11	120.410	-0.13 $\pm$ 0.11	-0.22
6-5 1	200.11 $\pm$ 0.11	200.186	-0.08 $\pm$ 0.11	-0.19
9-7 2	131.02 $\pm$ 0.11	131.477	-0.46 $\pm$ 0.11	-0.53
8-6 2	197.77 $\pm$ 0.13	198.222	-0.45 $\pm$ 0.13	-0.52
11-8 3	119.02 $\pm$ 0.15	119.889	-0.87 $\pm$ 0.15	-0.81
9-6 3	251.13 $\pm$ 0.11	251.595	-0.47 $\pm$ 0.11	-0.81
12-8 4	140.29 $\pm$ 0.12	141.360	-1.07 $\pm$ 0.12	-1.09
11-7 4	196.64 $\pm$ 0.16	197.765	-1.13 $\pm$ 0.16	-1.10
10-6 4	289.02 $\pm$ 0.15	289.788	-0.77 $\pm$ 0.15	-1.10
13-8 5	156.93 $\pm$ 0.15	158.070	-1.14 $\pm$ 0.15	-1.37
12-7 5	218.55 $\pm$ 0.13	219.567	-1.02 $\pm$ 0.13	-1.38

initial statistical distribution can be compensated by incomplete Auger transitions (see Section 3) [17]. The absolute intensities of higher transitions in heavy μ -atoms are in agreement with a statistical initial population at $n = 14$ [10,15]. A distribution over n seems to be necessary to improve the agreement mainly for transitions from levels with $n \geq 12$.

5. DETERMINATION OF NUCLEAR CHARGE RADII

5.1 Comparison of μ -Atom Data and Elastic Electron Scattering Results

For a large number of elements, data on nuclear charge radii are available and they are collected in Ref. [18]. One interesting aspect of this information is the comparison with elastic electron scattering data. Deviations could indicate the influence of dynamical effects (nuclear polarization, dispersion effects) or prove the validity of radiation corrections. As explained elsewhere [19,20] a model-independent comparison can be performed with electron scattering data measured at an "equivalent momentum transfer" $q^2 \approx 6Z/(R \cdot a_\mu) \approx 0.1 \text{ fm}^{-2}$ ($R^2 = \frac{5}{3} \langle r^2 \rangle$, $a_\mu = 256 \text{ fm}$) at which elastic electron scattering cross-section and muonic atom data show the same sensitivity to higher moments of the charge distribution. A comparison of such low momentum data to muonic atom results is given in Fig. 3 for nuclei with $A = 20$ to 30 [21]. An excellent agreement is observed for both methods.

Fig. 3 Comparison of nuclear charge radii obtained from (e,e) experiments at $\sim 60 \text{ MeV}$ [21] and μ -atoms.

Fig. 4 Comparison of Fermi charge distribution parameters of natural lead from (e,e) experiments at 40-60 MeV [22] and μ -atoms. μA : [72], μAB : [63], $\mu A 69$: [6], BO : [73].

Results of higher Z are sensitive for two parameters of the charge distribution. This is shown in Fig. 4 for natural Pb [22]. The recent result on μ -atoms of Anderson et al. [6,23] shows an interesting aspect in this comparison. In the evaluation of the c, t parameters the $1s^{1/2}$ state was omitted, but the $2s^{1/2}$ state was taken into account. The calculated energy of the $1s^{1/2}$ level was found to be too high by 6.8 ± 2.3 keV which the authors interpret as being due to nuclear polarization. If the $1s$ state is fitted too, only c and t but not $\langle r^2 \rangle$ is changed. The better agreement of the new value with the low momentum transfer data of (e,e) scattering support but do not prove the polarization effect. It is unexplained whether the discrepancy of the high-energy (e,e) scattering data [73] is due to the model dependence or dispersion effects.

Fig. 5 2p-1s transition in muonic Ag [24]. Natural element with 51.35% ^{107}Ag and 48.65% ^{109}Ag . The double escape peaks are shown but their full energies, E , are indicated.

Fig. 6 2p-1s transition in muonic Tl [7]. Natural element with 29.5% ^{203}Tl and 70.5% ^{205}Tl .

5.2 Muonic Isotope Shift

With separated isotopes many data on the isotope shift are measured with μ -atoms (c.f. [18]). Two recent spectra of natural elements demonstrate the improvement of the resolution of Ge-detectors. Figure 5 shows the 2p-1s transition in natural Ag [24]. The shift of the isotopes $^{107}, ^{109}\text{Ag}$ clearly is resolved and yields $\delta\langle r^2 \rangle^{1/2} = (2.14 \pm 0.10) \times 10^{-2}$ fm. The intensity ratio of the fine structure components agree roughly with the expected value of $\sim 2 : 1$. The same is shown in Fig. 6 for the Thallium isotopes $^{203}, ^{205}\text{Tl}$ [7]. The isotope shift was analysed to $(9.2 \pm 0.3)\text{keV}$ for the $2p_{1/2}-1s_{1/2}$ transition and $(9.6 \pm 0.2)\text{keV}$ for the $2p_{3/2}-1s_{1/2}$ transition, and yields $\delta\langle r^2 \rangle^{1/2} = (1.06 \pm 0.02) \times 10^{-2}$ fm at $\Delta t = 0$. Here the fine structure intensity ratios are 1.6 ± 0.1 and 1.03 ± 0.07 in disagreement with the expected value 1.92. The anomalies are due to a strong excitation of nuclear levels (see Section 7.2).

6. HYPERFINE STRUCTURE OF MUONIC X-RAYS

6.1 Deformed Nuclei - Quadrupole hf-Splitting

Muonic X-ray spectra of deformed nuclei have been measured for a large number of nuclei ($Z = 62$ to 92) by several groups [25-33]. These spectra have been analysed in terms of a Fermi-type charge distribution modified by a deformation of the nucleus [34]. As shown by Wilets [35] and Jacobson [36] the strong quadrupole interaction of a deformed nucleus with the muon mixes nuclear and muonic states resulting in an excitation of rotational levels. Thus also in even-even nuclei strong quadrupole hyperfine

splittings have been observed. An analysis is complicated due to magnetic hf-splitting, isomer shift, nuclear polarization, and a quadrupole vacuum polarization term. Nevertheless, c, t parameters and the intrinsic quadrupole moment were determined with good reliability, but it should be noticed that the deduced quadrupole moment depends on the assumed rotation model. The experimental data seem to favour the "deformed model" rather than the "hard core model" [30].

Figure 7 shows an example for the $2p-1s$ transition in the muonic Eu isotopes [32,33]. These nuclei are at the beginning of the deformed region. ^{151}Eu has a small quadrupole moment and no rotational band. The dynamic excitation of the first excited state at 21.7 keV has to be included in the analysis to explain the $2p^{3/2}-1s^{1/2}$ structure whereas the broadening of the $2p^{1/2}-1s^{1/2}$ transition is explained by the magnetic hf-splitting (Section 6.2) ($\mu = 3.46 \text{ nm}$). ^{153}Eu has a large quadrupole moment Q_0 and because of this a complicated hf-structure. $Q_0(^{153}\text{Eu}) = (8.02 \pm 0.12)b$ was deduced from the data.

Fig. 7 The $2p-1s$ muonic transition in $^{151}, ^{153}\text{Eu}$ [32,33]. The solid curves represent the best fit to the data. The dotted line in (b) is the fit to ^{151}Eu without inclusion of excitation to the 21.7 keV state. The vertical lines give the positions and intensities of the hf-components.

6.2 Spherical Nuclei --Magnetic Hyperfine-Splitting

Magnetic hyperfine structures have been observed in the $2p_{1/2}$ - $1s_{1/2}$ transitions of muonic ^{141}Pr , ^{151}Eu and ^{209}Bi (see Table 2). Since the magnetic splitting is proportional to m^2 and the electric quadrupole splitting proportional to m^3 the magnetic splitting normally is covered by the quadrupole splitting and thus can be analysed separately only from the $2p_{1/2}$ - $1s_{1/2}$ transition of spherical nuclei. On the other hand the energy resolution of the Ge-detectors at the corresponding high energies is comparable with or less than the splitting, therefore, only a broadening of the line can be observed (see $K\alpha_2$ of ^{151}Eu in Fig. 7).

The displacement of the magnetic substates is given by

$$\Delta W_F = A_1(nlj)[F(F+1) - I(I+1) - j(j+1)]/2Ij .$$

I is the nuclear spin, j the angular momentum of the muonic state and F the total angular momentum of the sublevel. Experimental values of the hyperfine constant A_1 are collected in Table 2. These values are strongly reduced ($\sim 40\%$) compared to A_1^{point} (the value expected for a point magnetic moment) because of the spatial distribution of the nuclear magnetization density. This finite size effect was calculated by Bohr and Weisskopf [44] for electrons and was extended to muons by Le Bellac [45]. Results of calculations based on the shell model with configuration mixing as presented in Table 2 are in fair agreement with the experimental results. Since in odd nuclei the magnetic moment is mainly produced by the unpaired nucleon (g_l and g_s factors) the properties of this particle are tested.

Table 2

Magnetic hf-constants in μ -atoms. A_1 is given for the $1s_{1/2}$ state deduced from $2p_{1/2}$ - $1s_{1/2}$ transitions. For Tl the values were derived from hf-splittings of nuclear γ -rays.

Isotope	A_1^{exp} [keV]	Ref.	A_1^{theor} [keV]	Ref.	A_1^{point} [keV]
^{141}Pr	1.52 ± 0.06	37	1.47 ± 0.03 a)	37	2.41
^{151}Eu	0.80 ± 0.27	32,33	1.04 b)	32	2.04
^{203}Tl	0.66 ± 0.04	5,38	0.72 a)	39,40	1.16
^{205}Tl	0.57 ± 0.01	5,38	0.67 a)	39,40	1.16
^{209}Bi	2.5 ± 0.5	41	2.0 a)	41,42	3.1
	$2.1 \pm 0.5^{\text{c}}$	41			
	2.1 ± 0.2	43			

a) shell model with configuration mixing;

b) $g_R = Z/A$;

c) an isomer shift of 3 keV is considered.

7. STUDY OF NUCLEAR γ -RAYS IN MUONIC ATOMS

7.1 Muonic Isomer Shift in Deformed Nuclei

The nuclear excitation in muonic atoms (Section 6.1) leads to a system with the nucleus in an excited state and the muon in its $1s^{1/2}$ groundstate. The lifetime of the $1s^{1/2}$ muon (μ -capture) is $\sim 7 \times 10^{-8}$ sec for heavy nuclei, whereas the nuclear transitions are much faster, 10^{-9} - 10^{-11} sec; therefore, the nuclear γ -ray is emitted in the presence of the $1s^{1/2}$ muon. As explained by Devons [46] and Hüfner [47] the excitation energies are changed due to nuclear polarization, isomer shift (monopole interaction between muon and nucleus) and the magnetic hf-interaction. These small effects ≤ 1 keV influence the muonic X-rays too, but as a consequence of their high energies they are difficult to observe, whereas the low energy of nuclear γ -rays (~ 100 keV) renders the observation feasible.

For a large number of deformed nuclei, nuclear γ -rays in a μ -atom have been measured by the Columbia group and the Darmstadt group at CERN. The energy shifts ΔE_{exp} compared to the corresponding unshifted γ -line from a radioactive source are shown in Table 3. The isomer shift is given by

$$\Delta E_{\text{is}} = e \int |\psi_{1s^{1/2}}(r)|^2 \Delta V(r) d\tau ;$$

$\Delta V(r)$ is the difference in electrostatic potential of the excited and groundstate corresponding to $\Delta \rho(r)$, the difference in the charge distribution. Essentially this is the same shift as measured in Mössbauer experiments. To deduce an isomer shift from the measured energy shift the above mentioned effects have to be considered.

The nuclear polarization is in the order of several keV as estimated by several authors [52-59] and experimentally supported by Anderson and co-workers [23], but is expected to be an order of magnitude less for a nuclear transition. For a rotational band it can be shown to be zero but it may contribute in cases of a "non-ideal" rotational band. Since no accurate calculations or experimental evidence exist at the moment this effect is neglected.

The magnetic hf-interaction splits the excited and ground state (for $I \geq 1/2$) into a doublet (see Fig. 8). This splitting is strongly reduced by the Bohr-Weisskopf effect [44] (c.f. Table 2). Only in $^{203,205}\text{Tl}$ the splitting was resolved [5,38]. For the deformed nuclei the magnetic distribution was assumed to be proportional to the charge distribution, which is not confirmed experimentally. The excitation mechanism does not populate the hf-levels statistically [49,60]. In addition a fast M1-interdoublet

Table 3

Measured energy shifts ΔE_{exp} and isomer shifts ΔE_{is} of nuclear γ -rays in muonic atoms

Isotope	Transition E[keV] I-I'	ΔE_{exp} [eV]	ΔE_{is} [eV]	$\left(\frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle}\right) \times 10^4$ $\Delta t=0$	Ref.
^{150}Nd	130.2 2^+-0^+	+ 570 $\pm$ 120	+ 840 $\pm$ 120	5.8 $\pm$ 0.8	48
^{152}Sm	121.8 2^+-0^+	+ 560 $\pm$ 60	+ 920 $\pm$ 70	5.9 $\pm$ 0.4	48
		+ 500 $\pm$ 40	+ 770	4.8	49
^{154}Gd	123.1 2^+-0^+	+ 670 $\pm$ 150	+ 980 $\pm$ 150	5.9 $\pm$ 0.8	48
^{156}Gd	88.9 2^+-0^+	- 512 $\pm$ 200			a)
^{158}Gd	79.5 2^+-0^+	- 499 $\pm$ 150			a)
^{160}Gd	75.3 2^+-0^+	- 431 $\pm$ 150			a)
^{166}Er	80.6 2^+-0^+	- 350 $\pm$ 150	- 30 $\pm$ 150	-0.16 $\pm$ 0.8	48
^{182}W	100.1 2^+-0^+	- 320 $\pm$ 100	- 30 $\pm$ 100	-0.13 $\pm$ 0.5	48
		- 290 $\pm$ 90	- 60	-0.2	49
^{184}W	111.2 2^+-0^+	- 340 $\pm$ 100	- 25 $\pm$ 100	-0.11 $\pm$ 0.5	48
		- 350 $\pm$ 50	- 100	-0.4	49
^{186}W	122.6 2^+-0^+	- 350 $\pm$ 100	- 10 $\pm$ 100	-0.04 $\pm$ 0.5	48
		- 400 $\pm$ 40	- 150	-0.6	49
^{188}Os	155.0 2^+-0^+	- 400 $\pm$ 40	- 190	-0.8	49
^{190}Os	186.7 2^+-0^+	- 470 $\pm$ 40	- 360	-1.4	49
^{192}Os	205.8 2^+-0^+	- 610 $\pm$ 50	- 514	-2.0	49
^{194}Pt	328.5 2^+-0^+	+ 262 $\pm$ 45	+ 321	+1.3	50
^{194}Pt	293.6 2^+-2^+	+ 3 $\pm$ 50	- 144	-0.6	50
^{196}Pt	355.7 2^+-0^+	+ 390 $\pm$ 280			50
^{169}Tm	118.2 $5/2-1/2$	- 410 $\pm$ 65	- 457	-2.0	50
^{169}Tm	109.8 $5/2-3/2$	+ 57 $\pm$ 40	+ 68	+0.3	50
^{169}Tm	130.5 $7/2-3/2$	- 800 $\pm$ 700	- 480	-2.1	50
^{181}Ta	136.2 $5/2-7/2$	+ 134 $\pm$ 60	+ 110	+0.5	4
^{185}Re	125.3 $7/2-5/2$	+ 175 $\pm$ 35			50
^{187}Re	134.3 $7/2-5/2$	+ 174 $\pm$ 35			50
^{191}Ir	129.4 $5/2-3/2$	- 108 $\pm$ 45	+ 103	+0.4	50
^{193}Ir	139.3 $5/2-3/2$	- 266 $\pm$ 45	- 121	-0.5	50
^{203}Tl	279.1 $3/2-1/2$	-	+ 260 $\pm$ 180	+0.9	5,50
^{205}Tl	203.8 $3/2-1/2$	-	- 330 $\pm$ 150	-1.1	5,50
^{209}Bi	2741.4 $15/2-9/2^-$	+5600 $\pm$ 600	+6200		51
^{209}Bi	1132.5 $15/2-13/2$	+2700 $\pm$ 800	+3000		51
^{209}Bi	1608.9 $13/2-9/2^-$	+1900 $\pm$ 700	+2200		51
^{209}Bi	2563.9 $9/2^+-9/2^-$	+6600 $\pm$ 1100	+6600		51

a) preliminary result of the Darmstadt group at CERN.

Fig. 8 Level scheme of ^{205}Tl without (left side) and with (right side) a muon in its $1s_{1/2}$ state [5]. The magnetic hf-splitting is calculated from shell model with configuration mixing [40].

transition which is highly converted ($\alpha \sim 10^4$ - 10^5); depopulates the higher component (Fig. 8) in competition with the nuclear transition. This M1-transition first was observed in muonic capture rates by Winston and Telegdi [74,75].

The corrected values of the isomer shift ΔE_{is} are collected in Table 3. In this correction the initial population, the lifetime and the multipole mixing of the nuclear transition, the M1-interdoublet transition and the magnetic hf-splitting are taken into account. Because of uncertainties in the M1-interdoublet lifetime and the hf-splitting the derived isomer shifts may be incorrect besides the statistical errors up to 100 eV for the even nuclei and up to 150 eV for the odd nuclei.

The muonic isomer shift can be compared to Mössbauer data. But whilst a Mössbauer experiment measures a shift proportional to $\Delta\langle r^2 \rangle$, the muonic isomer shift depends strongly on the shape of $\Delta\rho(r)$. Figure 9 shows an interpretation of ^{152}Sm and ^{182}W results in terms of changes in $\Delta\langle r^2 \rangle$ and Δt assuming a two parameter Fermi charge distribution [49]. Obviously the sign of $\Delta\langle r^2 \rangle$ does not necessarily determine the sign of ΔE_{is} . However, if $\Delta\langle r^2 \rangle$ is known from a Mössbauer experiment [76,77], Δt can be derived from the muonic isomer shift. Corresponding to Fig. 9 the following $\Delta t/t$ values can be deduced: -0.49% (^{152}Sm), $+0.23\%$ (^{186}W), $+0.05\%$ (^{191}Ir), $+0.1\%$ (^{193}Ir) and $+1.6\%$ ($3/2^+$ level in ^{169}Tm). Such an interpretation gives a valuable information on the properties of nuclear states, but at the moment it is very questionable because of uncertainties of the Mössbauer data and the neglected muonic nuclear polarization. Nevertheless the large negative isomer shifts in W, Os, Tm and ^{205}Tl are confirmed. For $^{203}, ^{205}\text{Tl}$ a possible explanation of the

Fig. 9 Analysis of the ^{152}Sm and ^{186}W results in terms of the nuclear charge parameters $\Delta\langle r^2 \rangle$ and Δt [49]. Hatched areas include experimental errors and the limit of complete de-excitation of the upper hf-level. Horizontal lines are Mössbauer data [76,77].

different sign is given in [5]. For deformed nuclei no theoretical interpretation is known. Even the self-consistent cranking model [78] which contains a negative contribution to the isomer shift gives results which are too large and exclusively positive.

7.2 Spherical Nuclei

The nuclear excitation in μ -atoms is not restricted to deformed nuclei. An appreciable mixing of nuclear and muonic states can produce a nuclear excitation if the following condition is fulfilled [61]:

$$|(E_A - E_B)/M| \lesssim 1.$$

E_A and E_B are muonic states coupled to the nuclear ground state and excited state, respectively. $M = \langle \psi_A | H_1 | \psi_B \rangle$ is the matrix element due to the perturbation H_1 . In addition total angular momentum and parity of the states A and B have to be equal. The following resonances have been predicted or observed: E0 in Kr, Zn [62], E1 in Pb isotopes [6] and Bi [9], E2 and M1 in Tl [5,38,61] and I [63], E3 in Bi [51,64]. As an example the Bi and Tl spectra are shown in Figs. 10 and 11. In ^{209}Bi four nuclear γ -rays have been found [51]; their intensities are consistent with the anomalous intensity ratio of the muonic 3d-2p fine structure components. Isomer shifts of these high energetic levels

Fig. 10 $3d-2p$ transition in $\mu\text{-}^{209}\text{Bi}$ [10]. D.E.: double escape peak; γ : nuclear γ -rays; del.: delayed γ -rays after μ -capture.

Fig. 11 Nuclear γ -rays in $\mu\text{-Tl}$ [5]. μ -X transitions are labelled with $n-n'$.

(Table 3) of several keV have been determined, which can be explained by the deformation of the $3^{-} {}^{208}\text{Pb}$ core of the $15/2^{+}$ and $9/2^{+}$ septuplet states in ${}^{209}\text{Bi}$ [51].

In ${}^{203,205}\text{Tl}$ (Fig. 11) clearly resolved hf-components have been observed [5,38] which are due only to the hf-splitting of the ground state (c.f. Fig. 8). From this splitting the most accurate hf-constant was deduced (Table 2). A shell-model calculation with configuration mixing [39,40] is in a fair agreement with the experimental results. Although the magnetic moments of ${}^{207,203}\text{Tl}$ are nearly equal, an isotope effect is observed in the hf-constants which can be explained by a different nuclear radii of both isotopes [39]. However, the observed isomer shifts (Table 3) and isotope shift (Fig. 6) cannot be explained simultaneously by the same shell model parameters.

REFERENCES

1. S. Devons and I. Duerdoth, Adv. in Nucl. Phys. 2, 295 (1969).
2. V.L. Fitch and J. Rainwater, Phys. Rev. 92, 789 (1953).
3. S. Bernow et al., Phys. Rev. Letters 18, 787 (1967).
4. H. Backe et al., Hyperfine Structure and Nuclear Radiations, North-Holland, Amsterdam (1968), p.65.
5. R. Engfer, Habilitationsschrift, T.H. Darmstadt (1969).
6. H.L. Anderson, C.K. Hargrove, E.P. Hincks, J.D. McAndrew, R.J. McKee, R.D. Barton and D. Kessler, to be published.
7. H. Backe et al., unpublished results of the Darmstadt group.
8. H. Backe, R. Engfer, U. Jahnke, E. Kankleit, K.H. Lindenberger, C. Petitjean, W.U. Schröder, H.K. Walter and K. Wien, Contributed paper to this conference.
9. C.K. Hargrove et al., Phys. Letters 23, 215 (1969).
10. H. Backe, Thesis, Inst. f. Techn. Kernphysik, T.H. Darmstadt (1969).
11. R.C. Barrett et al., Phys. Rev. 166, 1589 (1968).
12. P. Gombas, Hdb. der Physik XXXVI, p. 109, Springer Verlag Berlin-Göttingen-Heidelberg (1956).
13. D. Kessler et al., Phys. Rev. Letters 18, 1179 (1967).
14. H. Koch et al., Phys. Letters 28B, 280 (1968).
15. H. Backe, R. Engfer, E. Kankleit, W.V. Schröder, H.K. Walter and K. Wien, contributed paper to this conference.
16. L. Tauscher, Phys. Letters 27A, 581 (1968).
17. A. Suzuki, Phys. Rev. Letters 19, 1005 (1967).
G. Backenstoss, private communication.
18. L.R.B. Elton, Landolt-Börnstein tables, Springer-Verlag, Berlin (1967), New Series, group I, vol. 2.
19. R. Engfer, Proc. of Int. School of Physics "Enrico Fermi", Varenna (June 1966), p.64.
20. R. Engfer et al., Proc. of the Int. Conf. on Electromagnetic Sizes of Nuclei, Ottawa (1967), p.184.

21. H. Theissen, H. Fink and H.A. Bentz, Z. Physik, to be published.
22. G.J.C. van Niftrik, Nucl.Phys., to be published.
23. H.L. Anderson et al., Phys.Rev.Letters 22, 221 (1969).
24. R.A. Carrigan Jr. et al., Phys.Letters 27B, 622 (1968).
25. R.D. Ehrlich et al., Phys.Rev.Letters 13, 550 (1964).
26. G. Backenstoss et al., Nucl.Phys. 62, 449 (1965).
27. H.L. Acker et al., Nucl.Phys. 62, 477 (1965).
28. S. Raboy et al., Nucl.Phys. 73, 353 (1965).
29. R.E. Coté et al., Phys.Letters 19, 18 (1965).
30. S.A. de Wit et al., Nucl.Phys. 87, 657 (1967).
31. D. Hitlin, Proc. of the International Conference on Electromagnetic Sizes of Nuclei, Ottawa (1967), p.254.
32. M.N. Suzuki, Thesis, Carnegie-Mellon University, (Jan. 1968).
33. R.A. Carrigan et al., Phys.Rev.Letters 20, 874 (1968).
34. H.L. Acker, Nucl.Phys. 87, 153 (1966).
35. L. Wilets, Kgl. Dan. Vid. Selsk. Mat.-Fys. Medd. 29, No.3 (1954).
36. B.A. Jacobsohn, Phys.Rev. 96, 1637 (1957).
37. J.A. Johnson et al., Phys.Letters 29B, 420 (1969).
38. R. Baader et al., Phys.Letters 27B, 428 (1968).
39. F. Scheck, contributed paper to this conference.
40. R. Engfer and F. Scheck, Z. Phys. 216, 274 (1968).
41. R.J. Powers, Phys.Rev. 169, 1 (1968).
42. J. Johnson and R.A. Sorensen, Phys.Letters 26B, 700 (1968).
43. R.A. Carrigan et al., Bull.Am.Phys.Soc. 13, 65 (1968).
44. A. Bohr and V.F. Weisskopf, Phys.Rev. 77, 94 (1950).
45. M. le Bellac, Nucl.Phys. 40, 645 (1963).
46. S. Devons, Proc. of the Rutherford Jubilee Int. Conf., Manchester 1961, Academic Press N.Y., p.624.
47. J. Hüfner, Nucl.Phys. 60, 427 (1964).
48. S. Bernow et al., Phys.Rev.Letters 21, 457 (1968).
49. R. Baader et al., Phys.Letters 27B, 425 (1968).
50. H. Backe, R. Engfer, E. Kankeleit, W.U. Schröder, H.K. Walter and K. Wien, contributed paper to this conference.
51. H. Backe, R. Engfer, E. Kankeleit, W.U. Schröder, H.K. Walter and K. Wien, contributed paper to this conference.
52. L.N. Cooper and E.M. Henley, Phys.Rev. 92, 801 (1953).
53. E. Nuding, Z. Naturforschung 12a, 187 (1957).
54. W. Greiner, Z. Physik 164, 374 (1961).
55. W. Greiner and H. Marschall, Z. Physik 165, 171 (1961).
56. F. Scheck, Z. Physik 172, 239 (1963).
57. R.K. Cole Jr., Phys.Letters 25B, 178 (1967); Phys.Rev. 177, 164 (1968).
58. M.Y. Chen, Thesis, Princeton Univ. (1968); Phys.Rev. to be pub.
59. J.M. McKinley, Phys.Rev. to be published.
60. A. Gal et al., Phys.Rev.Letters 21, 453 (1968).
61. J. Hüfner, Z. Physik 190, 81 (1966).
62. E.M. Henley and L. Wilets, Phys.Rev.Letters 20, 1389 (1968).
63. H.L. Acker et al., Nucl.Phys. 87, 1 (1966).

64. J. Hüfner, Phys.Letters 25B, 189 (1967).
65. C.S. Johnson et al., Phys.Rev. 125, 2102 (1962).
66. W. Frati and J. Rainwater, Phys.Rev. 128, 2360 (1962).
67. H.L. Anderson and C.S. Johnson, Phys.Rev. 130, 2468 (1963).
68. D. Quitmann et al., Nucl.Phys. 51, 609 (1964).
69. J.A. Bjorkland et al., Nucl.Phys. 69, 161 (1965).
70. T.T. Bardin et al., Progress Report, Columbia University (1966), unpublished.
71. R. Engfer, Z. Physik 192, 29 (1966).
72. H.L. Anderson, Proc. of Int. Conf. on Electromagnetic Sizes of Nuclei, Ottawa (1967), p.53.
73. J.B. Bellicard and K.J. van Oostrum, Phys.Rev.Letters 19, 242 (1967).
74. R. Winston and V.L. Telegdi, Phys.Rev.Letters 7, 104 (1961).
75. R. Winston, Phys.Rev. 129, 2766 (1963).
76. W. Henning et al., Verhandlungen der Deutschen Physikalischen Gesellschaft 4, 324 (1968).
77. P. Kienle et al., Hyperfine Structure and Nuclear Radiations, North-Holland, Amsterdam (1968), p.971.
78. E.R. Marshalek, Phys.Rev.Letters 20, 217 (1968).