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K^- NUCLEAR INTERACTIONS AND YIELDS OF KAONIC X-RAYS

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A B S T R A C T

We re-examine the K^- nuclear interaction in kaonic atoms in the light of recent presumed evidence for an anomalous neutron distribution in the nuclear surface. A K^- nuclear potential, proportional to the KN scattering lengths and nuclear density, reproduces observed X-ray yields within experimental uncertainty. With a nuclear matter distribution proportional to the known proton distribution, no additional freedom is needed at present. The variation of the capture width with radius and surface thickness parameters are discussed in detail. Estimates for energy shifts in the kaonic levels are given. Dynamical hyperfine and Stark mixing of kaonic orbits are discussed. Distortion effects of the K^- wave function by the strong interactions are important and may well dominate all other effects. The renormalization of the KN scattering length by multiple scattering in nuclear matter is estimated. Approximations and limitations of the model are discussed.

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1. INTRODUCTION

In a recent pioneering experiment, Wiegand has given a first survey of intensities of kaonic X-rays through the periodic system¹⁾. From the relative intensity of the last X-ray transition observed in a given element, he derived the probability for K^- capture in its upper orbit. From these data and the observation of the charge number Z for which a particular transition in the kaonic cascade becomes experimentally unobservable, it was concluded that "nuclear matter (probably neutrons) extends above conventional nuclear surfaces". It was argued, in particular, that the parameters of the neutron density should be drastically modified as compared to those of the proton distribution. In view of the importance of this conclusion, we have re-examined the whole question of the K^- nuclear strong interaction at threshold, and its dependence on nuclear density distributions and on the strong interaction parameters.

Our conclusion is that at present there is no evidence from kaonic X-rays for an anomalous matter distribution in nuclei. We show below that conventional nuclear matter distributions (as known from electron scattering and muonic atoms) and a simple and natural approximation to the K^- nuclear (absorptive) interaction describe the data adequately, without free parameters.

Furthermore, it is widely believed that owing to the strong absorption of kaons in nuclear matter, kaonic X-rays depend very strongly on nuclear size, with a critical charge number Z_c for which there is a sharp cut-off in the yield of a transition. We show that such a behaviour is not expected theoretically nor is it established by the available data. On the contrary, as long as the real part of the kaon nuclear strong interaction is zero or repulsive [as expected from low-energy kaon-nucleon scattering²⁾], the yield of a particular X-ray transition decreases smoothly as a function of Z , and should be observable with attenuated but still measurable intensity for a wide range of nuclei^{*)}. This has the important

*) This picture would change only in the case of an attractive real part of the equivalent potential. The absorption widths grow rapidly with increasing attraction by the real potential; beyond a critical charge number, the X-ray lines then become quickly too broad to be experimentally observable. Although this possibility cannot be ruled out on the basis of the existing data, we argue that it is rather unlikely that the KN interaction is converted from repulsion into attraction in the nuclear surface by binding and multiple scattering corrections.

consequence that it should be possible to measure widths and possibly also shifts of K-mesic levels due to strong interaction, very much as in pionic atoms^{3,4)}.

The absorption of K^- from bound states in kaonic atoms has been studied previously by Jones⁵⁾, Wilkinson⁵⁾, Eisenberg and Kessler⁶⁾, Rook⁷⁾ and Burhop⁸⁾. Its characteristic physical features have been discussed in detail by Wilkinson⁵⁾. These authors assume the absorption to occur predominantly on single nucleons with average $\bar{K}N$ cross-sections characteristic of nuclear Fermi momenta, and they calculate the capture rates of relevant levels in first-order perturbation theory. As repeatedly emphasized by Wilkinson, a characteristic feature of K^- absorption from high orbits is that the main interaction occurs several fermis outside the central nuclear density, i.e. in a region of tenuous nuclear matter.

In this article we investigate further some of the problems of K^- absorption in nuclei. Guided by the previous studies, we make use throughout of the low-density approximation for the K^- nuclear interaction, and relate the equivalent potential to the nuclear density and $\bar{K}N$ scattering lengths. We then proceed in several ways. (a) Fixing the density parameters from the known proton distribution we calculate widths and yields perturbatively, in the general spirit of previous studies (Section 2). The available experimental data¹⁾ are well described by this approach (Section 3). (b) We explore, within the low-density approximation, the sensitivity of the capture width to nuclear parameters and to the distortion of the kaonic wave function by the real part of the strong interaction potential (Sections 4.1 and 4.2). (c) Quadrupole hyperfine and Stark mixing of kaonic orbits, which could strongly enhance the absorption, are discussed in Sections 4.3 and 4.4. (d) Finally, we discuss the effective field and binding corrections to the equivalent potential as well as corrections from p-wave $\bar{K}N$ scattering (Section 5). Section 6 contains some conclusions and remarks.

2. SEMI-PHENOMENOLOGICAL ABSORPTIVE POTENTIAL FOR KAONS IN NUCLEAR MATTER

The dominant absorption mode of K^- in nuclei is the absorption on a single nucleon, i.e. the process

$$K^- + N \rightarrow \pi + Y, \quad (1)$$

where Y stands for a Λ or Σ baryon. The absorption by two correlated nucleons $K^- + (N+N) \rightarrow Y + N$ is experimentally known to account for about 15% of the total absorption⁹⁾ and will be neglected throughout our discussion. This is in contrast to the case of pions, which for kinematical reasons are predominantly absorbed on two nucleons.

For reaction (1) it is usually assumed that the absorption is proportional to the local nuclear matter density $\rho_A(\vec{r})$. In the low-density limit, and near threshold, the $\bar{K}$ nuclear interaction is related to the s-wave part of the free $\bar{K}N$ scattering amplitude. As is well known, the $\bar{K}N$ amplitude can be parametrized at threshold by two complex scattering lengths A_0 and A_1 for the $I = 0$ and $I = 1$ isospin channels^{*)}. For the numerical studies we have chosen Kim's solution^{2) **)}:

$$\begin{aligned} A_0 &= (-1.674 + i 0.722) \text{ fm} \\ A_1 &= (-0.003 + i 0.688) \text{ fm} . \end{aligned} \quad (2)$$

Within these approximations the local part of the K^- nuclear equivalent potential is analogous to the π nuclear potential of pionic atoms^{3,4)}. It is given by

$$2\bar{m}_K V = -4\pi \left(1 + \frac{m_K}{m_N} \right) \left[A_1 \rho_n(\vec{r}) + \frac{1}{2}(A_0 + A_1) \rho_p(\vec{r}) \right], \quad (3)$$

where $\bar{m}_K$ is the reduced mass of the kaon and the nucleus; $\rho_n(\vec{r})$ and $\rho_p(\vec{r})$ are the neutron and proton densities, normalized to the number of neutrons N and protons Z, respectively.

*) An alternative treatment could make use of the K-matrix parametrization of low-energy $\bar{K}N$ scattering¹⁰⁾. Since the kaonic binding energies are quite close to the physical scattering region, where the two methods give numerically undistinguishable answers, we do not expect drastic changes when using the K-matrix method.

**) The following other numerically very similar values have been found:

$$\begin{aligned} A_0 &= (-1.65 \pm 0.04) + i(0.73 \pm 0.02) \text{ fm}; & A_1 &= (-0.13 \pm 0.02) + i(0.51 \pm 0.03) \text{ fm}^{11a)} \\ A_0 &= (-1.57 \pm 0.04) + i(0.54 \pm 0.04) \text{ fm}; & A_1 &= (-0.24 \pm 0.05) + i(0.43 \pm 0.05) \text{ fm}^{11b)} \\ A_0 &= (-1.66 \pm 0.02) + i(0.69 \pm 0.02) \text{ fm}; & A_1 &= (-0.09 \pm 0.03) + i(0.54 \pm 0.02) \text{ fm}^{11c)} \\ A_0 &= (-1.74 \pm 0.03) + i(0.70 \pm 0.02) \text{ fm}; & A_1 &= (-0.07 \pm 0.02) + i(0.62 \pm 0.03) \text{ fm}^{11d)} \end{aligned}$$

Our conclusions depend but little on the precise values for the scattering lengths.

Present experimental data on kaonic atoms are of a qualitative nature only. We can therefore use a rather crude theoretical model so that Eq. (3) should be considered only as a guide for the K^- nuclear elastic interaction. In particular, since there are no cancellations in the s-wave scattering lengths (unlike the case of pionic atoms), we expect the isospin and gradient terms in the equivalent potential to be of minor importance. For the moment, we also neglect renormalization of the scattering lengths by effective field corrections. We come back to these questions in Section 5, where approximations and limitations of the model are discussed in more detail.

It is instructive to re-express the scattering length (2) in energies according to Eq. (3). For a uniform distribution of an equal number of neutrons and protons, for instance, and for a nuclear radius $R_0 = 1.1 \times A^{1/3}$ fm, one finds from Eq. (3):

$$V \approx (-57 + i 96) \text{ MeV} .$$

The potential is hence repulsive with a predominant imaginary part.

Using perturbation theory, the absorption width $\Gamma_{n\ell}^{\text{capt}}$ for a K-mesic level with radial quantum number n and orbital angular momentum ℓ is then given by

$$\Gamma_n^{\text{capt}} = \frac{4\pi}{\bar{m}_K} \left(1 + \frac{m_K}{m_N} \right) \int d\vec{r} |\phi_{n\ell}|^2 \left[\text{Im } A_1 \rho_n(\vec{r}) + \text{Im } \frac{A_0 + A_1}{2} \rho_p(\vec{r}) \right] . \quad (4)$$

In order to be specific and to eliminate as many arbitrary parameters as possible, we have assumed in the following analysis that the neutron density has the same shape as the proton density, about which information exists from electron scattering and muonic atoms¹²⁾. In terms of the nuclear matter density $\rho_A(\vec{r})$ we hence assume

$$\rho_n(\vec{r}) : \rho_p(\vec{r}) : \rho_A(\vec{r}) = N : Z : A .$$

We further make the conventional assumption of a Fermi distribution, so that

$$\rho_A(\vec{r}) = N_A \left\{ 1 + \exp \left(\frac{(r-c)}{t} 4 \ln 3 \right) \right\}^{-1} , \quad (5)$$

where c denotes the radius parameter, t the surface thickness, and N_A the normalization constant, such that $\int \rho_A(\vec{r}) d\vec{r} = A$. One may, of course, improve on the form of the matter distribution, for instance by using the densities obtained from realistic nuclear structure calculations (Hartree-Fock wave functions or the like). The widths are indeed sensitive to the shape and in particular to the tail of $\rho_A(\vec{r})$. We shall, however, show (Section 4) that the widths depend even more sensitively on the real part of the strong interaction potential V , i.e. on distortion effects in the kaon wave function $\phi_{n\ell}$, and on the renormalization of the effective $\bar{K}N$ -scattering length in nuclear matter. As long as these latter effects are not better known quantitatively, it seems premature to improve on the matter distribution only, without investigating the strong interaction potential.

Since according to Eq. (2) the imaginary parts of A_0 and A_1 are numerically very close, we have to a good approximation

$$\Gamma_{n\ell}^{\text{capt}} \approx \frac{4\pi}{\bar{m}_K} \left(1 + \frac{m_K}{m_N} \right) \text{Im } \bar{A} \int |\phi_{n\ell}|^2 \rho_A(\vec{r}) d\vec{r} . \quad (6)$$

with $\text{Im } \bar{A} \approx 0.7 \text{ fm}$. Here we have set $\hbar = c = 1$; hence $m_K^{-1} = 0.399 \text{ fm}$, Eq. (6) giving the width in fm^{-1} ($1 \text{ fm}^{-1} = 197.3 \text{ MeV}$).

3. COMPARISON WITH THE BERKELEY DATA^{*)}

As discussed by Wilkinson⁵⁾, the atomic X-ray cascade proceeds predominantly by circular orbits ($n, \ell = n - 1$). We shall make use of this below. The yields of the X-ray transitions are determined by the competition between the well-known radiative dipole transition probability and the probability for strong absorption and Auger transitions. The latter are negligible for the cases we consider.

In a first approach we calculate the absorption width $\Gamma_{n,n-1}^{\text{capt}}$ from Eq. (6) and the radiative width $\Gamma_{n,n-1}^{\text{rad}}$ using relativistic Coulomb wave functions for a point charge Ze .

*) We follow the standard practice of atomic spectroscopy in labelling the atomic states. This differs from Wiegand's notation¹⁾. Our notation ... k, ℓ, m, n, o , ... corresponds to his notation ... j, k, ℓ, m, n ,

We now define the critical orbit ($n_c, \ell_c = n_c - 1$), for a given element, as the highest state from which X-ray transitions are unobservable because of strong absorption. Thus, the transition ($n_c + 1, \ell = n_c$) $\rightarrow$ ($n_c, \ell = n_c - 1$) is the last X-ray transition of the cascade with, say, more than 5% yield.

We have calculated the width Γ_{nl}^{capt} as well as the yield of the last observable X-ray transition

$$Y_{n_c+1 \rightarrow n_c} \equiv \frac{\Gamma_{n_c+1, \ell=n_c}^{rad}}{\Gamma_{n_c+1, \ell=n_c}^{rad} + \Gamma_{n_c+1, \ell=n_c}^{capt}} \quad (7)$$

for a large number of nuclei, according to formula (6). The parameters c and t of the matter density are taken from electron scattering or muonic atoms whenever known¹²⁾; in other cases we have taken $c = 1.1 \times A^{1/3}$ fm, $t = 2.4$ fm for spherical nuclei, and $t = 2.3$ fm for deformed nuclei^{*)}.

The results are compared to experiments in Figs. 1 to 6. Experimental yields have been defined as the ratio of the last X-ray transition intensity to the intensities of (observed) transitions feeding the level ($n_c + 1$):

$$Y_{exp} = \frac{I_{n_c+1 \rightarrow n_c}}{\sum_{n_i > n_c+1} I_{n_i \rightarrow n_c+1}} \quad (8)$$

(In practice, for medium and heavy nuclei, $n_i = n_c + 2$ will dominate the sum.) The corresponding values are given in Table 1, which contains the experimental yields as well as our calculated values for yields, capture and radiation widths (columns 4 to 7).

The most remarkable feature illustrated by Fig. 1 is that a given transition should be observable with attenuated but still sizeable intensity for a wide range of nuclei. As an example, attenuation of the transition ($6h \rightarrow 5g$) can be studied from $_{33}\text{As}$ up to about $_{53}\text{I}$. There is no

*) The latter value is a rough average of results from muonic atoms with deformed nuclei, according to the recent analysis by Chen¹³⁾ where nuclear polarization is taken into account.

sudden disappearance of the X-ray transition at a critical value of the nuclear charge.

From Figs. 2 to 6 it can immediately be seen whether it is reasonable to expect the absorption width of the lower level to be observable for a certain yield. From Fig. 4, for example, one immediately sees that $\Gamma_{5g}^{\text{capt}}$ should be about 7 keV with a yield for the $6h \rightarrow 5g$ transition of about 15% for $Z = 53$.

Calculated yields are generally in agreement with Wiegand's data¹⁾. There is no indication of an anomalous matter distribution $\rho_A(r)$. This is the more remarkable, as we have used only the most crude approximation to the equivalent potential so far, and no free parameters. Conventional density parameters, as they are known from electron scattering and muonic atoms, together with the uncorrected $\bar{K}N$ scattering lengths, already give a satisfactory explanation of the data.

The only cases of possible discrepancies are the following:

- a) The apparent low yield of the $(7i \rightarrow 6h)$ transition in ${}_{66}\text{Dy}$ and ${}_{68}\text{Er}$ (Fig. 5). The predicted yields are 40-50%, but the transition is not reported in the Berkeley experiment. No upper limit is given.
- b) The anomalously low yield of the $(8k \rightarrow 7i)$ transition in lead (Fig. 6). Note, however, that the same transition is observed with comparable yield in ${}_{92}\text{U}$. The anomaly is therefore no simple matter of absorption strength and, if it is confirmed, would be difficult to understand theoretically.

Except for the $(3d \rightarrow 2p)$ and $(4f \rightarrow 3d)$ transitions, the experimental data are still compatible with a somewhat sharper drop-off of the theoretical yields. A careful measurement of yield and width in the tail of one of the yields curves would be very desirable.

We conclude this section with a few remarks on the energies of the kaonic X-ray transitions. In the last two columns of Table 1 we list the Klein-Gordon energies as well as the vacuum polarization correction for the transitions investigated in the Berkeley experiment. The vacuum polarization is calculated for the field of a point nucleus and with relativistic hydrogen-like wave functions. The results have, however, been checked against a more precise calculation and are accurate to within

a few per cent. For medium and heavy elements, this correction is seen to be quite important. We do not fully acknowledge Wiegand's statement that the measured energies agree with the Klein-Gordon values without further corrections within the quoted experimental error of ± 1 keV. This is because the vacuum polarization correction alone is in most cases well outside this error, and it has a definite sign^{*)}. From the published data it is hence impossible to determine whether the critical level in medium and heavy elements shows a strong energy shift or not. Conversely, such a shift can certainly not be excluded from the present data. Since the largest energy shifts are expected to occur in those elements which have largest widths, a careful determination of the energy of such a line is highly desirable. (The expected magnitude of such shifts will be discussed below, see also Table 2.)

4. SENSITIVITY OF CAPTURE WIDTHS TO PARAMETERS AND TO MIXING OF ORBITS

The theoretical interpretation of the experimental absorption data depends crucially on a proper understanding of the variation of capture widths with model parameters. Further, since the assumption of pure circular orbits is very important, the question of possible mixing of states of different angular momentum due to a non-central field must be investigated.

Wilkinson⁵⁾ has illustrated how the centrifugal barrier for the relevant orbits keeps the kaon out of the central density region of the nucleus, so that the probability of absorption [integrand of Eq. (6)] is sharply peaked in the nuclear surface a few fermis outside the region of nearly uniform nuclear density. Thus the capture probability depends sensitively on the size characterized by the radius parameter c and on the surface thickness t . On the other hand, distortion of the kaonic wave function by the strong interaction with the nucleus could also produce important changes in the widths. We will now discuss these effects one by one.

*) The systematic deviation between the listed values and the calculated ones presumably reflects a somewhat larger experimental error than the one quoted, rather than indicating strong interaction shifts.

4.1 Dependence on radius and surface thickness

We have explored the dependence of Γ^{capt} on c and t numerically for various nuclei within reasonable intervals around the experimental proton values c_0 and t_0 . Empirically we find to a good approximation

$$\Gamma^{\text{capt}}(c, t) \propto e^{-\alpha(c-c_0)-\beta(t-t_0)},$$

where α and β are positive constants which are numerically rather similar. These constants vary, of course, from nucleus to nucleus and with the capturing state. We have found this simple dependence for various nuclei and states. An illustrative example is shown in Fig. 7, which gives both the functional relation of c versus t for constant Γ^{capt} and the dependence of Γ^{capt} on c and t with the other variable fixed.

It is characteristic that

- i) for a given transition, c and t cannot be determined independently, since the functional dependence is about equally strong on both.
- ii) since α and β are characteristically of order 1 fm^{-1} , a change in c or t by 0.7 fm changes widths by about a factor of two. The sensitivity to the nuclear surface is therefore only moderate.

4.2 Distortion effects on the kaon wave function

So far we have replaced the true kaon wave function ϕ by relativistic Coulomb functions and have neglected the finite size correction to the Coulomb potential as well as the effect of the real part of the strong interaction potential (3). Whereas the former effect is entirely negligible, distortion of ϕ by the real part of V is not, and it must be taken into account. To investigate their joint effect, we integrate numerically the kaonic Klein-Gordon equation

$$\nabla^2 \phi + \left[(E - V_c)^2 - \bar{m}_K^2 \right] \phi = 2\bar{m}_K V \phi \quad (9)$$

corresponding to the multiple scattering potential (3) for bound states. It is well known from pionic atoms that the distortion effects are produced predominantly by the real part of V ⁴⁾. We therefore neglect $\text{Im } V$ in Eq. (9) and calculate the absorption width from Eq. (4) using the distorted (real) wave functions. The use of the real part of V in Eq. (9) is a major computational simplification with little loss of accuracy.

The numerical values for the free $\bar{K}N$ scattering lengths (2) correspond to a repulsive potential (3) with the strength

$$\text{Re } (\bar{A}) = \left\{ \frac{N}{A} \text{Re } (A_1) + \frac{Z}{A} \text{Re } \left(\frac{A_0 + A_1}{2} \right) \right\} \approx -0.84 \frac{Z}{A} \text{ fm} . \quad (10)$$

The effective value for the scattering lengths in nuclear matter may, however, be renormalized as discussed in the next section. Therefore, for the moment we will treat $\text{Re } \bar{A}$ as a free parameter and explore the sensitivity of yields and widths to it, both for attraction and repulsion.

As a typical example for these calculations we show in Fig. 8 the transition $6h \rightarrow 5g$ versus Z . The effect on the yield, which depends on the width of the upper state $6h$, is not very pronounced as compared to the large experimental errors (see Fig. 1), while the width of the $5g$ level depends strongly on the parameter $\text{Re } (\bar{A})$. In particular, for the case of attraction, the moderate value $\text{Re } (\bar{A}) = +0.2 \text{ fm}$ enhances the width as much as the repulsive value $-0.84 Z/A \text{ fm}$ decreases it, although the latter is twice as large in magnitude. This dependence is exhibited more clearly in Fig. 9 which shows the width $\Gamma_{n\ell}$ for $(n = 6, \ell = 5)$, $(n = 7, \ell = 6)$, and $(n = 8, \ell = 7)$ in tungsten, $Z = 74$, as a function of $\text{Re } \bar{A}$. Whereas for repulsive interaction the widths vary only moderately, they increase very steeply with increasing attraction.

This rapid increase of $\Gamma_{n\ell}$ with increasing attraction has a simple explanation. For a critical strength of $\text{Re } \bar{A}$, the short-range nuclear attraction becomes sufficient to bind a kaon of given ℓ in the nuclear field independently of the long-range Coulomb potential. The probability of the presence of the kaon in the nucleus then increases violently, and of course also the value of the overlap integral in Eq. (6). The critical attraction occurs already for quite moderate values of $\text{Re } \bar{A}$. This is explicitly illustrated for $\text{Re } \bar{A} = +0.4 \text{ fm}$ in Fig. 10 where we show the centrifugal, electromagnetic, and nuclear potentials for $\ell = 5$ in tungsten. In this specific case, $\ell = 5$ is already bound in the nucleus. The wave function has its maximum at $r \approx 5.8 \text{ fm}$ inside the nucleus. For the same $\text{Re } \bar{A}$, the higher states with $\ell > 5$ are not yet bound by the nuclear potential alone.

Although we find it unlikely that the nuclear $\bar{K}N$ interaction is so strongly renormalized in the nuclear periphery as to convert repulsion into attraction, it is clearly very important to establish experimentally that repulsion occurs. Even a slight attraction could drastically change the absorption pattern (in which case the perturbative approach to the width has to be improved). The distortion by the real potential, whether repulsive or attractive, has such a large influence that at present it causes larger uncertainties than those due to matter distribution.

This question of the real potential can in part be settled by a good experimental study of the width of the critical level in selected nuclei. A more direct way is clearly to determine at least the sign of energy shifts. Since the knowledge of the real interaction is vital to the question of distortions, we have tabulated strong interaction shifts as obtained from a solution of Eq. (9) with a purely real interaction in Table 2 as a guide to experimentalists for the magnitude expected. Note that for attraction the shifts tend to be small compared to the width of the same state, which might cause observational difficulties.

4.3 Mixing of kaonic orbits by dynamical quadrupole interaction

The capture width $\Gamma_{n\ell}$ varies very strongly with the orbital angular momentum ℓ , but is almost independent of the radial quantum number n . Typically, $\Gamma_{n\ell}$ increases by a factor 10^2 to 10^3 when going from ℓ to $\ell - 1$, for fixed n . As a consequence, already a slight mixing of different orbital states, with the same n , could drastically change the absorption. The two obvious mixing mechanisms are the dynamical quadrupole interaction which mixes ℓ and $\ell - 2$, and the Stark effect which mixes ℓ with $\ell - 1$. We show that both effects are small and can be neglected as compared to present accuracy in the analysis of kaonic atoms.

The dynamical quadrupole mixing is well known from muonic atoms where it is an important effect. We consider, for simplicity, an even-even strongly deformed nucleus. The quadrupole interaction of the kaon and the nucleus causes a mixing of the nuclear ground and first excited state, with spin-parity 2^+ , and of the kaonic orbits $(n, \ell = n - 1)$ and $(n, \ell = n - 3)$. Since the absorption increases by a factor of 10^4 to 10^5 when going from ℓ to $\ell - 2$, a few per cent admixture (in amplitude) could be enough to change the absorption by a factor of the order of ten.

The admixture coefficient a in the perturbed kaon wave function $\sqrt{1 - a^2} \phi_{n,\ell=n-1} + a \phi_{n,\ell=n-3}$ is given by

$$a = \frac{\langle 0^+; n, n-1 | H_Q | 2^+; n, n-3 \rangle}{E_{0^+} - E_{2^+} + E_{n, n-1} - E_{n, n-3}}, \quad (11)$$

where H_Q is the quadrupole interaction of the kaon and the nucleus. Expanding the matter density in the nuclear intrinsic system as usual in the form

$$\rho(\vec{r}) = \rho_0(r) + \rho_2(r) Y_{20}(\theta) \quad (12)$$

and with the rotator model for the deformed nucleus, the matrix element of H_Q is found to be

$$\begin{aligned} \langle 0^+; n, n-1 | H_Q | 2^+; n, n-3 \rangle &\approx \frac{e^2 Q_0}{2} \langle n, n-1 | \left| \frac{1}{r^3} \right| n, n-3 \rangle \times \\ &\times (-)^{I_f + \ell_f} \sqrt{2\ell_f + 1} \begin{pmatrix} I_f & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_f & \ell_i & 2 \\ 0 & 0 & 0 \end{pmatrix}, \end{aligned} \quad (13)$$

where Q_0 is the intrinsic quadrupole moment of the nucleus, related to ρ_2 by $2\sqrt{4\pi/5} \int \rho_2 r^4 dr = Q_0$; and where $I_f = 2$, $\ell_i = n - 1$, and $\ell_f = n - 3$. The effect of the finite size of the nucleus on the quadrupole interaction H_Q is strongly suppressed by the centrifugal barrier in all cases of practical interest. For non-relativistic hydrogen-like states of the kaon, the matrix element (13) is exactly zero. This is due to the "selection rule" for the radial matrix element

$$\int_0^\infty R_{n\ell} \frac{1}{r^3} R_{n(\ell-2)} r^2 dr = 0 \quad (14)$$

which holds for non-relativistic hydrogen wave functions. This is a special case of the following more general multipole selection rule for the non-relativistic Coulomb field: a (dynamical) multipole interaction of order λ , in the absence of finite size effects, has no first-order matrix elements between states (n, ℓ) and (n, ℓ') for a non-relativistic Coulomb interaction, if $|\ell - \ell'| = \lambda$, i.e. maximal. The proof is given in the Appendix.

For the case of relativistic Coulomb wave functions, this matrix element is not zero. Also, if the strong interaction potential has a real part, attractive or repulsive, the inner wave function $(n, \ell - 2)$, which penetrates deeper into the nucleus, is more distorted than the outer one (n, ℓ) ; this distortion further increases the matrix element. Both effects give $\langle 1/r^3 \rangle$ different from zero but still leave it numerically small. We exemplify this with the transition $7i \rightarrow 6h$ in ${}_{64}\text{Gd}$ which is a strongly deformed nucleus. The yield for this transition is determined by the capture width in the state $7i$. If we take the real interaction to be repulsive with its expected value $\text{Re } \bar{A} = -0.84 \text{ Z/A fm}$, the matrix element of H_Q is $7 \times 10^{-2} \text{ keV}$. The energy denominator in expression (11) contains a negative part $E_{0+} - E_{2+}$ which is 123.1 keV. The difference of the kaon energies $E_{7,6} - E_{7,4}$ is more difficult to estimate. For a point charge in the kaonic Klein-Gordon equation it is positive and equal to 2.4 keV. For the repulsive real part of V with $\text{Re } \bar{A} = -0.84 \text{ Z/A fm}$ it is -4.2 keV. The admixture coefficient a , Eq. (11), is then about $a \approx 5 \times 10^{-4}$, and the yield is changed by less than 10^{-4} . The width of the same transition is determined by Γ_{6h} . For this state the quadrupole matrix element is about 1 keV, giving $a \approx 8 \times 10^{-3}$. The width is increased by about 30%. Since absorption broadening of lines has so far not been experimentally observed, this is at present academic, but the effect should become observable in the future. (Note, however, that in the latter case the large width of the admixed state $6f$ makes the perturbative approach unreliable).

In the unlikely case of an attractive real part of the strong potential, the enhancement of $\langle 1/r^3 \rangle$ and the possibility of cancellations in the energy denominator would require a more careful estimate.

Thus, but for this latter case, the effects of quadrupole mixing are of minor importance as compared with present experimental precision^{*)}. The absorption width then depends only on the spherically symmetric part $\rho_0(r)$ of the matter distribution (12). It is this part we have inserted in the expression for the width with the parameters known from muonic atoms.

4.4 Stark effect

It is difficult to obtain the strong field necessary for an appreciable Stark mixing. As an example, a 3% admixture (in amplitude) of the

*) In muonic atoms the quadrupole interaction mixes muonic orbits with different total angular momenta j but with same n and ℓ . This is the reason why the effect is much larger there than in kaonic atoms.

($n = 7, \ell = 5$) state into the ($n = 7, \ell = 6$) state for ${}_{64}\text{Gd}$ requires an electric field of $3 \times 10^{13} \text{ V/cm} = 0.3 \text{ MV}/10^{-8} \text{ cm}$, where we have scaled it to atomic distances. A field of this magnitude can, for instance, be found in the unscreened Coulomb field $Z\alpha/r$ at a distance of $5 \times 10^{-10} \text{ cm}$ from the centre of a heavy nucleus. This corresponds to the $n = 3$ electron shell. Since atoms interpenetrate only rarely in such a way as to affect the deep shells, the Stark mixing is extremely unlikely in medium and heavy nuclei.

5. LIMITATIONS AND CORRECTIONS

In the previous investigation we were guided by low-density approximation for the equivalent $\bar{K}$ nuclear interaction [Eq. (3)]. Empirically the comparison with the experimental yields showed that the approximation for the absorptive interaction is satisfactory. Concerning the real part of V , unfortunately, the Berkeley data for kaonic atoms give no information so far on its magnitude or even on its sign. (The indirect evidence that bound $\bar{K}$ -nuclear states do not occur limits attraction to about $\text{Re } \bar{A} \lesssim +0.5 \text{ fm.}$) It is then natural to enquire how well this simple model is expected to work.

There are three types of corrections that can change the character of the low-density limit:

- i) The structure of the $\bar{K}N$ interaction itself and in particular its range. There are bound states in the $\bar{K}N$ interaction with a Y_0^* at only 27 MeV and a Y_1^* at 48 MeV below threshold. Therefore the basic $\bar{K}N$ interaction is attractive; even so, the low-density limit predicts a repulsive interaction near threshold. Important corrections may therefore occur.
- ii) The effective field corrections (i.e. the effects of nuclear polarization) which occur even in the limit when the $\bar{K}N$ scattering can be described by scattering lengths only¹⁴⁾.
- iii) The contributions of partial waves higher than s-waves, to the low-density limit. For π nuclear interactions, it is well known that p-waves in the πN system give very important contributions.

5.1 The $\bar{K}N$ interaction

Concerning the first effect, the detailed investigation of which would extend beyond the present study, we argue that it is not unreasonable to ignore the existence of the bound states below threshold and to treat the $\bar{K}N$ interaction in kaonic atoms in the constant scattering-length limit.

The crucial point in the argument is that the $\bar{K}$ nuclear interaction in the kaonic atom occurs in high angular momentum states. The centrifugal barrier therefore strongly suppresses contributions from the nuclear interior. The dominant interaction region for relevant kaonic levels lies in the nuclear outskirts where the density is typically only 10% to 20% of its value at the centre of the nucleus, as has been emphasized by Wilkinson. Our conclusions below are therefore invalid for the central region of the nucleus^{*)}.

The mass density in the nuclear surface is mainly due to the weakest bound nucleons. The "lab. kinetic energy" $T = [s - (m_K + m_N)^2]/2m_N$ between a kaon at rest and a bound nucleon of binding energy B_N is in the nucleon rest frame

$$T = -(m_K/m_N)B_N .$$

Since the binding of the last nucleon is typically $B_N \approx 8$ MeV, the equivalent kaon kinetic energy is

$$T \approx -4 \text{ MeV} .$$

This energy is reasonably small in magnitude compared to the resonant energy -27 MeV for Y_0^* , which is the nearest isobar, and it is unimportant for Y_1^* . It is thus not unreasonable to believe that the $\bar{K}N$ amplitudes can be considered to be constant in the periphery of heavy nuclei. Note, however, that the binding energies typical of the nuclear interior are much larger, so that the corresponding approximation is obviously incorrect there.

*) There is some weak evidence from K^- scattering in nuclear emulsions for an attractive K^- nuclear interaction at low energy¹⁵⁻¹⁷). This is, however, not in conflict with our repulsive potential (3), since the former refers to low partial waves, while the latter is characteristic of high partial waves.

Some independent information on this assumption is obtained from branching ratios in the absorption¹⁸⁾. In particular, the comparison of $E_2 = (\Sigma^- \pi^+ / \Sigma^+ \pi^-)$ for heavy nuclei and for the proton shows a much smaller modification of this ratio than for light elements⁸⁾, and it is not inconsistent with our approximation^{*)}.

5.2 Effective field (nuclear polarization)

By virtual excitation of the nuclear medium, the wave incident on an individual nucleon in the medium is renormalized to the effective local field value^{3,14)}. For a point interaction of the type $A_I \delta(\vec{r} - \vec{r}_i)$ as used in the derivation of Eq. (3), the scattering lengths A_I should be replaced by the effective scattering lengths A_I^{eff} for the two diagonal $\bar{K}N$ channels of isospin 0 and 1 properly averaged. Hence

$$\bar{A} = \frac{1}{A} [Z A_p + N A_n] \rightarrow \bar{A}^{\text{eff}} = \frac{1}{A} \left[Z \frac{A_0^{\text{eff}} + A_1^{\text{eff}}}{2} + N A_1^{\text{eff}} \right].$$

In the long wavelength limit and for negligible nuclear excitation energies, the effective field corrections for s-waves are known from pionic atoms, where one obtains¹⁴⁾

$$A_I^{\text{eff}} \approx \frac{A_I}{1 + \langle 1/r \rangle_{\text{corr}} A_I}. \quad (15)$$

Here $\langle 1/r \rangle_{\text{corr}}$ is the average value of $1/r$ over the pair correlation function in the nucleus. Since the kaon is heavier than the pion, and since in addition the average internucleon distance is larger at the surface than in the central region, the excitation energies of the intermediate states cannot be neglected. If we replace the excitation energies by an effective value $\bar{\epsilon} = \bar{\kappa}^2/2m_K$, then the outgoing spherical wave $1/r$ must be replaced by the damped wave $e^{-\bar{\kappa}r}/r$ in Eq. (15); we then have^{**)}

$$A_I^{\text{eff}} \approx \frac{A_I}{1 + \left\langle \frac{\exp(-\bar{\kappa}r)}{r} \right\rangle_{\text{corr}} A_I}. \quad (16)$$

*) Unfortunately, it is not known in which atomic state absorption occurs predominantly, since only about 30% of the kaons reach low levels¹⁾. Presumably the remainder are absorbed in states of high n .

**) Note that for complex amplitudes A_I the renormalized A_I^{eff} remains unitary.

The correlations are expected to be predominantly Pauli correlations with $\bar{\epsilon} \approx (10 \text{ to } 20) \text{ MeV}$. For central nuclear densities $\langle 1/r \rangle_{\text{corr}} \approx 0.7 \text{ fm}^{-1}$. A density of one-eighth of the central value reduces this by a factor of 2. The damped propagation gives a further reduction by a factor of about 2 to 3. An estimate for the quantity $X \equiv \langle e^{-\bar{k}r}/r \rangle_{\text{corr}}$ characteristic for kaon absorption is therefore

$$X \equiv \left\langle \frac{e^{-\bar{k}r}}{r} \right\rangle_{\text{corr}} \approx (0.1 \text{ to } 0.2) \text{ fm}^{-1}. \quad (17)$$

In view of the uncertainty of this estimate, it is instructive to explore the dependence of $\bar{A}^{\text{eff}}$ with X , considering X as a free parameter (see Fig. 11). For large values of X , the long-wavelength approximation breaks down and the conclusions become unreliable. It is, however, interesting to note that the imaginary part stays within reasonable limits while the real part even changes sign. In the region of the expected value of X [Eq. (17)] however, the renormalization effects are small. Therefore effective field corrections should be unimportant in the peripheral $\bar{K}$ nuclear interaction.

5.3 Contributions from p-wave $\bar{K}N$ scattering

In states of high angular momentum the π -nuclear interaction is dominated by p-wave πN scattering. For kaons the p-wave $\bar{K}N$ scattering is much weaker in relation to s-wave scattering, and it is not dominant. The basic reason for this is the large $\bar{K}N$ scattering lengths, which do not have the large cancellations typical of the π -nuclear system (soft pion theorems). This can be expressed more precisely in terms of a velocity-dependent $\bar{K}$ -nuclear potential analogous to the π -nuclear potential (effective field corrections will be neglected). We will neglect differences in neutron and proton densities as before, and assume the nucleus to have $N = Z$. Then

$$2\bar{m}_K V = -4\pi \left(1 + \frac{m_K}{m_N} \right) \left[\bar{A}^{\ell=0} \rho(\vec{r}) - 3 \left(1 + \frac{m_K}{m_N} \right)^{-2} (\vec{\nabla} \cdot \bar{A}^{\ell=1} \rho(\vec{r}) \vec{\nabla}) \right]. \quad (18)$$

Here $\bar{A}^{\ell=0}$ is the previous average of the s-wave scattering lengths, while $\bar{A}^{\ell=1}$ is the spin-isospin averaged p-wave scattering lengths:

$$\bar{A}^{\ell=1} = \frac{1}{12} [A_{11} + 3A_{31} + 2A_{13} + 6A_{33}]. \quad (19)$$

These scattering lengths are only partly known. A recent analysis of Martin and Ross¹⁹⁾ gives

$$\begin{aligned} A_{31} &= -(0.067 \pm 0.037) \text{ fm}^3 + i(0.0024 \pm 0.0010) \text{ fm}^3 \\ A_{33} &= (-0.060 \pm 0.037) \text{ fm}^3 + i(0.037 \pm 0.0012) \text{ fm}^3 . \end{aligned}$$

For the $I = 0$ amplitudes, which have lower statistical weight, no direct evidence exists, although there is no evidence of p-wave contributions below 260 MeV/c. This argues strongly for a small A_{13} , but does not rule out that A_{11} could be non-negligible. We will however neglect it in view of its small coefficient. Then

$$\bar{A}^{\ell=1} \approx -(0.047 \pm 0.028) \text{ fm}^3 + i(0.0025 \pm 0.0009) \text{ fm}^3 .$$

Note the nearly negligible value of the $\text{Im } \bar{A}^{\ell=1}$, which implies a nearly real gradient interaction. Further, the negative sign of $\text{Re } \bar{A}^{\ell=1}$ implies repulsion.

The relative importance of s- and p-wave real $\bar{K}N$ interaction in a state of angular momentum ℓ is roughly given by the ratio²⁰⁾

$$\left(\frac{P}{S} \right)_{\ell} \approx \frac{\ell(2\ell + 1)}{\left(1 + \frac{m_K}{m_N} \right)^2} \frac{3 \text{ Re } \bar{A}^{\ell=1}}{\text{Re } \bar{A}^{\ell=0}} \frac{\langle r^{2(\ell-1)} \rangle}{\langle r^{2\ell} \rangle} . \quad (20)$$

Assuming the interaction to be so peripheral that the density can be approximated by $\rho(r) \sim \exp \{-r/a\}$ with $a \equiv t\{4 \ln 3\}^{-1}$, this ratio can be analytically expressed as

$$\left(\frac{P}{S} \right)_{\ell} \approx \frac{\ell}{\ell + 1} \frac{1}{2 \left(1 + \frac{m_K}{m_N} \right)^2} \frac{3 \text{ Re } \bar{A}^{\ell=1}}{\text{Re } \bar{A}^{\ell=0}} a^{-2} . \quad (21)$$

For $a = 0.55 \text{ fm}$ and the values for $\text{Re } \bar{A}^{\ell=0}$ and $\text{Re } \bar{A}^{\ell=1}$ given above, this implies $R_{\ell=\infty} \approx 0.25$.

This correction will therefore be important in the description of energy shifts with 25% or better accuracy, but it is unimportant with present precision.

6. CONCLUSION

Our discussion of kaonic X-ray yields shows clearly that there is no need for any anomaly in the neutron or matter distribution in nuclei on the basis of existing data. There are not even any indications of such anomalies at present. On the other hand, our investigation does not preclude that future, more detailed, studies of yields, absorption widths, and energy shifts can be an interesting source of information on the nuclear surface. Since distortion effects have an important influence on the kaonic absorption (as we have shown) it is, however, first necessary to have a clear picture of the nature of the strong $\bar{K}$ nuclear interaction. This should, on the one hand, be obtained from experiments by studies of the energy shifts, for example. At present, it is not even established whether the potential is attractive or repulsive, so that even the sign of these shifts would be important information. On the theoretical side, the present investigation is based on the assumption that the presence of the Y_0^* resonance not far below threshold does not drastically modify the low-density approximation (3) to the potential, at least in the nuclear surface. This assumption is not a very strong one for the absorptive interaction, but it is considerably less certain for the real interaction which according to Eq. (2) is dominated by the rapidly varying amplitude A_0 .

Since our study applies only to peripheral interactions in nuclei, that is, in practice, to medium and heavy nuclei, there is no flagrant contradiction between the experimental branching ratio $E_2 \approx (\Sigma^- \pi^+) / (\Sigma^+ \pi^-)$ in heavy nuclei and for the proton. The dramatic modifications of this ratio occur in light elements with non-peripheral interactions. All our approximations are then invalid, so that utterly different results are possible. A simultaneous quantitative understanding of both kaonic atoms and the branching ratios is still lacking.

We have further shown that the effective field renormalization of the $\bar{K}N$ scattering lengths is small in the nuclear periphery. The velocity dependence of the $\bar{K}$ nuclear potential is shown to be much smaller than for the pionic case, but it will not be an entirely negligible effect in future precision studies.

Apart from the central problem of the nuclear potential and the distortion effects, we have also shown that functional dependence of the absorption widths on c and t are moderate with a remarkably simple pattern. Of other effects we have shown that Stark mixing in medium and heavy nuclei is negligibly down by magnitudes. Dynamical quadrupole mixing of kaonic states are unimportant with present experimental accuracy, but may become observable in future precision experiments.

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Table captions

Table 1 : Experimental¹⁾ and theoretical yields (cols. 4 and 5) for the transition $(n + 1, n) \rightarrow (n, n - 1)$, and capture and radiation widths for $(n, n - 1)$ (cols. 6 and 7) in keV. Column 8 shows the vacuum polarization correction to the transition energies, col. 9 gives the full energy difference $E_{n+1,n} - E_{n,n-1}$ in keV, for a point-like nucleus and including the vacuum polarization. Yields and widths are calculated perturbatively using $\text{Im } \bar{A} = 0.7$ fm and point Coulomb wave functions.

Table 2 : Sample values for yields, widths, and shifts for attraction $\text{Re } \bar{A} = +0.2$ fm, for $\text{Re } \bar{A} = 0$ and for repulsion $\text{Re } \bar{A} = -0.84 Z/A$ fm, expected the values of Eq. (2). The widths and shifts are in keV. Results obtained perturbatively using distorted wave functions and $\text{Im } \bar{A} = 0.7$ fm.

Table 1

Z (1)	Element (2)	Transition $n+1, \ell+1 \rightarrow n, \ell$ (3)	Y_{exp} (4)	Y_{th} (5)	$\Gamma_{n\ell}^{\text{capt}}$ (6)	$\Gamma_{n\ell}^{\text{rad}}$ (7)	Vac. pol. (8)	$E_{\text{K-G}} + \text{vac. pol.}$ (9)
16	S	5g-4f 4f-3d	- -	0.99 0.13	3.83×10^{-3} 2.87	5.67×10^{-4} 2.66×10^{-3}	0.33 0.99	74.88 162.15
17	Cl	5g-4f 4f-3d	0.083 ± 0.039	0.09	7.60×10^{-3} 5.09	7.23×10^{-4} 3.40×10^{-3}	0.39 1.16	84.68 183.37
19	K	6h-5g 5g-4f	1.0 ± 0.23	0.94	2.30×10^{-5} 2.36×10^{-2}	3.48×10^{-4} 1.13×10^{-3}	0.21 0.53	57.48 105.99
20	Ca	6h-5g 5g-4f	0.50 ± 0.18 0.57 ± 0.20	0.89	5.52×10^{-5} 4.43×10^{-2}	4.28×10^{-4} 1.39×10^{-3}	0.24 0.60	63.72 117.51
22	Ti	6h-5g 5g-4f	1.0 ± 0.35	0.79	1.63×10^{-4} 1.14×10^{-1}	6.28×10^{-4} 2.04×10^{-3}	0.32 0.77	77.30 142.56
28	Ni	7i-6h 6h-5g 5g-4f	0.67 ± 0.40 0.50 ± 0.35	0.99 0.37	4.08×10^{-6} 2.79×10^{-3} 1.11	6.38×10^{-4} 1.65×10^{-3} 5.37×10^{-3}	0.286 0.61 1.43	75.62 125.60 231.69
29	Cu	7i-6h 6h-5g	?	0.99	7.44×10^{-6} 4.87×10^{-3}	7.35×10^{-4} 1.90×10^{-3}	0.32 0.67	81.19 134.85
30	Zn	7i-6h 6h-5g	1.20 ± 0.25	0.99	1.26×10^{-5} 7.57×10^{-3}	8.42×10^{-4} 2.18×10^{-3}	0.35 0.73	86.94 144.40
32	Ge	7i-6h 6h-5g	1.0 ± 0.7	0.96	4.06×10^{-5} 1.98×10^{-2}	1.09×10^{-4} 2.83×10^{-3}	0.42 0.87	99.01 164.46
33	As	7i-6h 6h-5g	1.0 ± 0.7	0.96	5.40×10^{-5} 2.59×10^{-2}	1.23×10^{-3} 3.20×10^{-3}	0.45 0.94	105.34 174.98
34	Se	7i-6h 6h-5g	1.0 ± 0.7	0.94	8.80×10^{-5} 3.91×10^{-2}	1.39×10^{-3} 3.61×10^{-3}	0.49 1.02	111.88 185.85
42	Mo	9l-8k 8k-7i 7i-6h 6h-5g	0.50 ± 0.24	0.66	6.24×10^{-9} 3.90×10^{-6} 1.69×10^{-3} 4.54×10^{-1}	7.35×10^{-4} 1.46×10^{-3} 3.25×10^{-3} 8.42×10^{-3}	0.25 0.46 0.86 1.74	75.96 110.92 171.13 284.35
53	I	10m-9l 9l-8k 8k-7i 7i-6h	1.0 ± 0.28	0.95	1.04×10^{-9} 5.28×10^{-7} 1.96×10^{-4} 4.86×10^{-2}	1.02×10^{-3} 1.87×10^{-3} 3.72×10^{-3} 8.26×10^{-3}	0.29 0.49 0.86 1.58	86.66 121.27 177.11 273.34
56	Ba	9l-8k 8k-7i 7i-6h	1.0 ± 0.47	0.90	1.58×10^{-6} 5.16×10^{-4} 1.11×10^{-1}	2.33×10^{-3} 4.63×10^{-3} 1.03×10^{-2}	0.57 0.99 1.81	135.48 197.87 305.40

Table 1 (cont.)

Z (1)	Element (2)	Transition n+1, l+1 → n, l (3)	γ_{exp} (4)	γ_{th} (5)	$\Gamma_{\text{nl}}^{\text{capt}}$ (6)	$\Gamma_{\text{nl}}^{\text{rad}}$ (7)	Vac. pol. (8)	$E_{\text{K-G}} + \text{vac. pol.}$ (9)
62	Sm	10m-9l 9l-8k 8k-7i 7i-6h	0.60 ± 0.39	0.73	2.65×10^{-8} 9.69×10^{-6} 2.53×10^{-3} 4.27×10^{-1}	1.91×10^{-3} 3.50×10^{-3} 6.97×10^{-3} 1.55×10^{-2}	0.46 0.75 1.30 2.34	118.77 166.24 242.82 374.84
63	Eu	10m-9l 9l-8k 8k-7i 7i-6h	0.50 ± 0.16	0.73	2.85×10^{-8} 1.06×10^{-5} 2.79×10^{-3} 4.73×10^{-1}	2.04×10^{-3} 3.73×10^{-3} 7.43×10^{-3} 1.65×10^{-2}	0.48 0.79 1.35 2.44	122.65 171.67 250.75 387.11
64	Gd	10m-9l 9l-8k 8k-7i 7i-6h	1.00 ± 0.35 0.50 ± 0.16	0.70	3.80×10^{-8} 1.37×10^{-5} 3.48×10^{-3} 5.67×10^{-1}	2.17×10^{-3} 3.98×10^{-3} 7.92×10^{-3} 1.76×10^{-2}	0.50 0.82 1.41 2.54	126.59 177.18 258.82 399.57
66	Dy	10m-9l 9l-8k 8k-7i	1.0 ± 0.7	0.99	1.03×10^{-7} 3.21×10^{-5} 7.07×10^{-3}	2.46×10^{-3} 4.50×10^{-3} 8.96×10^{-3}	0.54 0.89 1.52	134.67 188.50 275.37
68	Er	10m-9l 9l-8k 8k-7i	0.5 ± 0.28	0.99	1.38×10^{-7} 4.32×10^{-5} 9.46×10^{-3}	2.77×10^{-3} 5.07×10^{-3} 1.01×10^{-2}	0.59 0.97 1.64	143.00 200.16 292.41
74	W	10m-9l 9l-8k 8k-7i	1.0 ± 0.9	0.97	7.77×10^{-7} 2.12×10^{-4} 3.89×10^{-2}	3.89×10^{-3} 7.12×10^{-3} 1.42×10^{-2}	0.74 1.21 2.04	169.50 237.28 346.68
77	Ir	10m-9l 9l-8k 8k-7i	0.50 ± 0.35	0.94	2.33×10^{-6} 5.13×10^{-4} 7.80×10^{-2}	4.56×10^{-3} 8.35×10^{-3} 1.66×10^{-2}	0.83 1.34 2.26	183.60 257.03 375.57
80	Hg	10m-9l 9l-8k 8k-7i	0.67 ± 0.20	0.90	5.03×10^{-6} 1.06×10^{-3} 1.51×10^{-1}	5.31×10^{-3} 9.73×10^{-3} 1.94×10^{-2}	0.92 1.48 2.49	198.27 277.58 405.63
81	Tl	11n-10m 10m-9l 9l-8k 8k-7i	0.80 ± 0.25	0.89	2.21×10^{-8} 6.24×10^{-6} 1.28×10^{-3} 1.77×10^{-1}	3.25×10^{-3} 5.58×10^{-3} 1.02×10^{-2} 2.04×10^{-2}	0.61 0.95 1.53 2.57	150.24 203.29 284.61 415.91
82	Pb	12o-11n 11n-10m 10m-9l 9l-8k 8k-7i	0.25 ± 0.14	0.89	7.13×10^{-11} 2.49×10^{-8} 6.78×10^{-6} 1.36×10^{-3} 1.85×10^{-1}	2.10×10^{-3} 3.42×10^{-3} 5.87×10^{-3} 1.07×10^{-2} 2.14×10^{-2}	0.41 0.63 0.98 1.58 2.65	117.01 153.99 208.37 291.72 426.32
92	U	12o-11n 11n-10m 10m-9l 9l-8k 8k-7i	0.40 ± 0.20	0.55	2.38×10^{-9} 5.66×10^{-7} 1.05×10^{-4} 1.43×10^{-2} 1.30	3.33×10^{-3} 5.42×10^{-3} 9.31×10^{-3} 1.71×10^{-2} 3.40×10^{-2}	0.57 0.86 1.33 2.12 3.53	147.45 194.07 262.64 367.78 537.62

Table 2

Z		Re $\bar{A} = +0.2$	Re $\bar{A} = 0$	Re $\bar{A} = -0.84$ Z/A
24	Y_{5g-4f}	0.67	0.70	0.75
	Γ_{4f}	0.389	0.238	0.142
	ϵ_{4f}	0.04	0	-0.05
28	Y_{5g-4f}	0.33	0.37	0.43
	Γ_{4f}	2.00	1.11	0.609
	ϵ_{4f}	0.20	0	-0.23
32	Y_{5g-4f}	0.10	0.13	0.17
	Γ_{4f}	13.14	5.38	2.50
	ϵ_{4f}	1.12	0	-0.96
42	Y_{6h-5g}	0.61	0.66	0.71
	Γ_{5g}	0.785	0.454	0.271
	ϵ_{5g}	0.08	0	-0.09
46	Y_{6h-5g}	0.37	0.43	0.50
	Γ_{5g}	2.63	1.32	0.723
	ϵ_{5g}	0.25	0	-0.25
50	Y_{6h-5g}	0.18	0.22	0.28
	Γ_{5g}	8.93	3.74	1.92
	ϵ_{5g}	0.76	0	-0.65
62	Y_{7i-6h}	0.67	0.73	0.78
	Γ_{6h}	0.765	0.427	0.261
	ϵ_{6h}	0.08	0	-0.08
64	Y_{7i-6h}	0.61	0.70	0.73
	Γ_{6h}	1.176	0.567	0.364
	ϵ_{6h}	0.12	0	-0.12
68	Y_{7i-6h}	0.44	0.52	0.59
	Γ_{6h}	2.80	1.31	0.723
	ϵ_{6h}	0.26	0	-0.23

APPENDIX: A DYNAMICAL MULTIPOLE SELECTION RULE

A (dynamical) multipole interaction of order λ , in the absence of finite size effects, has no first-order matrix elements between states (n, ℓ) and (n, ℓ') with $\lambda = |\ell - \ell'|$

$$\int_0^\infty e^{-\rho} R_{n\ell}(\rho) \rho^{-\lambda-1} R_{n(\ell-\lambda)}(\rho) \rho^2 d\rho = 0; \quad (\rho = \frac{2Z}{na_B} r) \quad (A.1)$$

for non-relativistic Coulomb functions.

Proof: The wave functions are related to Laguerre polynomials by

$$R_{n\ell} \sim \rho^\ell L_{n-\ell-1}^{2\ell+1}(\rho) e^{-\rho/2},$$

where the polynomial part is of order $n - 1$ and $\rho = (2Z/na_B)r$. The radial integral (A.1) is then

$$\int_0^\infty \rho^{\ell+\ell'-\lambda+1} L_{n-\ell-1}^{2\ell+1} L_{n-\ell'-1}^{2\ell'+1} e^{-\rho} d\rho.$$

For $\lambda = |\ell - \ell'|$ this integral vanishes by completeness and orthogonality for associated Laguerre polynomials²¹⁾:

$$\int_0^\infty \rho^\alpha L_k^\alpha L_{k'}^\alpha e^{-\rho} d\rho = 0 \quad \text{for } k \neq k'. \quad (A.2)$$

Any polynomial of degree k can be expanded in terms of these functions

$$[\text{polynomial of degree} = k] = \sum_{i=0}^k a_i^\alpha L_i^\alpha(\rho) \quad (A.3)$$

with certain constants a_i^α (completeness).

We can always choose $\ell > \ell'$ above. We can then take $\alpha = 2\ell' + 1$ in Eqs. (A.2) and (A.3): the power in ρ is $\ell + \ell' - \lambda + 1 \equiv 2\ell' + 1$. Since the Laguerre polynomial $L_{n-\ell-1}^{2\ell+1}$ is of degree $(n - \ell - 1) < (n - \ell' - 1)$, it has no component $L_{n-\ell'-1}^{2\ell'+1}$ when expanded according to Eq. (A.3). Hence

$$\int_0^\infty R_{n\ell} R_{n\ell'} \rho^{-\lambda+1} d\rho = 0 \quad \text{for } |\ell - \ell'| = \lambda. \quad \text{Q.E.D.}$$

REFERENCES

- 1) C.E. Wiegand, Phys. Rev. Letters 22, 1235 (1969). For the data on light nuclei, see C.E. Wiegand and D.A. Mack, Phys. Rev. Letters 18, 685 (1967).
- 2) J.K. Kim, Phys. Rev. Letters 14, 29 (1965); *ibid* 19, 1079 (1967); see also Ref. 11.
- 3) M. Ericson, C.R. Acad. Sci. Paris, t. 257, 3831 (1963); *ibid.* 258, 1471 (1964).
M. Ericson and T.E.O. Ericson, Ann. Phys. 36, 323 (1966).
- 4) T.E.O. Ericson and M. Krell, Nuclear Phys. B11, 521 (1969).
- 5) P.B. Jones, Phil. Mag. 3, 33 (1958).
D.H. Wilkinson, Phil. Mag. 4, 215 (1954); Proc. Int. Conf. on Nuclear Structure, Kingston (1960), p. 20; Proc. Rutherford Jubilee Int. Conf. (1961), p. 339; Proc. Int. Conf. on Nuclear Structure, Tokyo (1967), p. 469.
- 6) Y. Eisenberg and D. Kessler, Phys. Rev. 130, 2352 (1963).
- 7) J.R. Rook, Nuclear Phys. 39, 479 (1962); *ibid.* B9, 441 (1968).
- 8) E.H.S. Burhop, Nuclear Phys. B1, 438 (1967).
- 9) G.B. Chadwick, S.A. Durrani, P.B. Jones, J.W.G. Wignell and D.H. Wilkinson, Phil. Mag. 3, 1193 (1958).
Helium Bubble Chamber Collaboration, Proc. Rochester Conf. on High-Energy Physics (1960) (Univ. of Rochester), p. 426.
- 10) R.H. Dalitz and S.F. Tuan, Ann. Phys. 10, 307 (1960).
- 11) a) J.K. Kim and F. von Hippel, Phys. Rev. Letters 20, 1303 (1968).
b) W. Kittel, G. Otter and I. Wacek, Phys. Letters 21, 349 (1966).
c) B.R. Martin and M. Sakitt, Phys. Rev. 183, 1352 (1969).
d) A.D. Martin and G.G. Ross, Durham preprint. For compiled values see G. Ebel, H. Pilkuhn and F. Steiner, Nuclear Phys. B17, 1 (1970).
- 12) L.R.B. Elton, Nuclear charge distributions; in Landolt-Börnstein, Numerical Data and Functional Relationships in Science and Technology, New Series, ed. by K.H. Hellwege, Group I, Vol. 2 (Springer Verlag, Berlin-Heidelberg-New York, 1967), p. 1;
R. Hofstadter and H.R. Collard, Nuclear Radii Determined by Electron Scattering, *ibid.*, p. 21.
- 13) Min-Yi Chen, Nuclear Polarization in muonic atoms, Columbia Univ. Preprint 1969.
- 14) T. Ericson, Third Int. Conf. on High-Energy Physics and Nuclear Structure, Columbia (1969). CERN preprint TH 1083.
T. Ericson, School of Physics, Herceg Novi 1969 (Gordon and Breach, to be published). CERN preprint TH 1093.

- 15) P.B. Jones, Phys. Rev. Letters 4, 35 (1960).
- 16) M. Melkanoff, D.J. Prowse and D.H. Stork, Phys. Rev. Letters 4, 183 (1960).
- 17) R.D. Hill, J.H. Hetherington and D.G. Ravenhall, Phys. Rev. 122, 267 (1961).
- 18) D.H. Davis et al., Nuclear Phys. B1, 434 (1967).
S.D. Bloom, M.H. Johnson and E. Teller, Phys. Rev. Letters 23, 28 (1969).
- 19) A.D. Martin and G.G. Ross, Durham preprint. See also Ref. 11.
- 20) M. Ericson, T.E.O. Ericson and M. Krell; Phys. Rev. Letters 22, 1189 (1969).
- 21) I.S. Gradshteyn and I.M. Ryzhik, Table of Integrals, Series and Products (Academic Press, Inc., New York and London, 1965).

Figure captions

- Fig. 1 : Yields of kaonic X-rays versus Z . The solid curves are the theoretical values using the $\bar{K}N$ scattering lengths and a matter density proportional to the proton density obtained from Ref. 12. For simplicity they have been smoothed out in this figure. The data points are taken from Ref. 1, and are defined according to Eq. (8). Errors are added quadratically. Calculated values obtained perturbatively using point Coulomb wave functions and $\text{Im } \bar{A} = 0.7$ fm.
- Fig. 2 :: Yields of the transitions (3d - 2p) and (4f - 3d), as well as theoretical widths of the states 2p and 3d versus Z . The theoretical points are joined by straight lines. The left-hand scale refers to yields; the right-hand scale to widths. Calculated values obtained perturbatively using point Coulomb wave functions and $\text{Im } \bar{A} = 0.7$ fm.
- Fig. 3 : Yields of the transition (5g - 4f) and widths of 4f, versus Z . Otherwise as in Fig. 2.
- Fig. 4 : Yields of the transition (6h - 5g) and widths of 5g, versus Z . Otherwise as in Fig. 2.
- Fig. 5 : Yields of the transition (7i - 6h) and widths of 6h, versus Z . Otherwise as in Fig. 2.
- Fig. 6 : Yields of the transition (8k - 7i) and widths of 7i, versus Z . Otherwise as in Fig. 2.
- Fig. 7 : Sensitivity of the capture width Γ_{5g} , for $Z = 50$, to the parameters c and t :
- a) Γ_{5g} for varying t and fixed c ;
 - b) Γ_{5g} for fixed t and varying c ;
 - c) lines of constant width in a c - t diagram.

For simplicity $\text{Im } \bar{A}$ in Eq. (6), has been taken to be 1 fm. Other values of $\text{Im } \bar{A}$ are obtained by simple scaling. The values corresponding to $\text{Im } \bar{A} = 0.7$ fm [i.e. Eq. (2)] are obtained by multiplication by 0.7, for example. Calculated values obtained as in Fig. 1

- Fig. 8 : Yields and width of the transition (6h - 5g) for $\text{Re } A = +0.2$ fm, $\text{Re } \bar{A} = 0$, and $\text{Re } \bar{A} = -0.84 Z/A$ fm. (The yield Y_{6h-5g} depends on Γ_{6h} , whereas the width of the transition is dominated by Γ_{5g} .) Calculated values obtained perturbatively using distorted wave functions and $\text{Im } \bar{A} = 0.7$ fm.
- Fig. 9 : Variation of capture widths with the real part of the strong interaction potential, for $Z = 74$. The expected value of $\text{Re } \bar{A}$ lies around -0.34 fm.
- Fig. 10 : Kaonic potential for $\ell = 5$ in $Z = 74$, with and without attractive strong interaction potential ($\text{Re } \bar{A} = +0.4$ fm). Actual sign and strength of potential at present not experimentally established.
- Fig. 11 : Dependence of real and imaginary part of the effective scattering length $\bar{A}$ on the correlation parameter X defined in Eq. (17). The solid curves correspond to the full renormalization of A_0 and A_1 [cf. Eq. (16)]. The dashed lines are the linear approximations to these curves expected to be valid for small values of X only. Most likely value for X is $(0.1 - 0.2) \text{ fm}^{-1}$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

a

b

c

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11