CM-P00058203

Archives

Ref.TH.1085-CERN

ON THE BREMSSTRAHLUNG IN CRYSTALS

B. Ferretti *)

CERN - Geneva

and

G. Gamberini

Istituto Fisica
University di Bologna, Bologna

Ref.TH.1085-CERN

8 October 1969

In an old paper by one of us ¹⁾ about the bremsstrahlung in crystals, the Born approximation was used but it was also predicted that, for unitarity reason, at very high energy the Born approximation had to break down completely. To our knowledge this aspect of the question was not treated however in any of all the subsequent rather numerous theoretical and experimental papers on the subject ²⁾, with the exception of a short discussion in a stimulating paper by Schiff ³⁾.

We have decided therefore to clarify the point beginning with the most easy part of the problem, that is investigating the effect of a more correct treatment of the motion of the electron (of positron) in the crystal, on the probability of emission of radiation.

The interaction with the radiation field instead, even in the present work, has been treated in Born approximation. To get a sufficiently good wave function for the electron in the crystal, a certain version of the W.K.B. method has been developed, which appears to be increasingly suitable with increasing energy of the particle ⁴⁾. It may be shown then that for electrons, whose average momentum makes a very small angle θ with some of the principal crystal plane Π (and this is the interesting case), the wave function can be approximately derived by averaging the effects of the potential due to all the atoms belonging to the same crystal's plane parallel to Π , and separating therefore the variables.

The effect of the crystal potential can then be assimilated to that of a plane structure, and the potential of this structure will be represented by a periodic function $V(z)$ of a variable only.

The spinor wave function of the electron can then be approximately represented (for positive energies) for instance by the following spinor

$$\left\{ \begin{array}{l} \psi_3^I = \psi \\ \psi_2^I = 0 \\ \psi_3^I = -\frac{i}{E-V+m} \frac{\partial \psi}{\partial z} \\ \psi_4^I = \frac{p_x + i p_y}{E-V+m} \psi \end{array} \right. \quad \left\{ \begin{array}{l} \psi_2^I = 0 \\ \psi_2^I = \psi \\ \psi_3^I = \frac{p_x - i p_y}{E-V+m} \psi \\ \psi_4^I = \frac{i}{E-V+m} \frac{\partial \psi}{\partial z} \end{array} \right.$$

where

$$\psi(x, y, z) = \sqrt{E+m} \exp(i(p_x x + p_y y)) \sqrt{\frac{2pE'}{k^2(z)}} \exp(i \int_0^z k(z) dz)$$

$$k(z) = \sqrt{2p(E'-V(z))}; \quad p = \sqrt{p_x^2 + p_y^2}$$

$$E' = E - p_x - \frac{m^2 + p_y^2}{2p_x}$$

and for the Dirac's matrices the customary representation has been adopted

$$\alpha_k = \begin{vmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{vmatrix} \quad \beta = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

Supposing then that the potential energy $V(z)$ varies between zero and V_0 the approximation W.K.B. is good enough if

$$E' \gtrsim 3V_0$$

To calculate the probability of radiation we have to compute the matrix element of the interaction with the radiation field between an initial electron state $\psi^{(a)}$ and a final state $\psi^{(b)}$.

To determine completely $\psi^{(a)}$ and $\psi^{(b)}$ one has to specify the appropriate boundary conditions. This can be made in the simplest way if one thinks that our plane structure, simulating the crystal, is composed of N parallel sheets (we shall then take the limit $N = \infty$).

The "initial state" will then be determined by an "incoming plane wave" impinging on the "lowest layer" and the "final state" by an "outgoing plane wave" emerging from the upper layer of the structure.

This way, given the plane symmetry of the "scatterer", "initial" and "final" states will be represented by waves of the same type exactly like in the Born approximation.

The probability of transition will then be proportional to:

$$|H_{ab}(N)|^2 = |H_{ab}|^2 \frac{\sin^2(N\Delta g/2)}{\sin^2(\Delta g/2)}$$

where

$$\Delta g = g_b(\frac{a}{2}) - g_a(\frac{a}{2}) + (p_{bz} + k_z - p_{az})a$$

and

$$g_a(z) = \sqrt{2} p_a \int_{-\frac{a}{2}}^z [\sqrt{E_a' - V(z')} - p_{az}] dz'$$

$$\vec{p}_a \equiv p_{ax}, p_{ay}, p_{az}$$

a is the pitch of the structure and H_{ab} is the matrix element for the case in which $N=1$, i.e.,

$$V = \begin{cases} 0 & \text{for } z < -\frac{a}{2} \\ V(z) & \text{for } -\frac{a}{2} < z < \frac{a}{2} \\ 0 & \text{for } z > \frac{a}{2} \end{cases}$$

[it is of course supposed that $V(-a/2) = V(a/2) = 0$]. Results of a numerical calculation for the emission of a gamma ray, having an energy of 6.7×10^8 eV polarized in a plane parallel to the plane x,z by an electron or positron of 10 GeV in function of the parameter

$$w_a = \frac{2pV_0}{p_z^2} \quad \left(w_a = \frac{2kaV_0}{p_{az}^2} \right)$$

are given in Table 1. The data have been calculated in the correspondence of the maxima: $\Delta \varphi = -2\pi$ in all cases but for positron and $w_a = 0.50$ and $w_a = 0.55$ where $\Delta \varphi = -4\pi$ ($\Delta \varphi = -2\pi$ was impossible in this case).

In Table 2 we have given the angles of emission of the gamma rays measured in units $m/(25p)$.

The more striking features which can be remarked in Tables 1 and 2 are the following:

- 1) the maxima have apparently about the same value for electrons, positrons, and the Born approximation;
- 2) the position of the maxima in function of w_a (i.e., of the incident angle on the incoming particle) are, however, displaced and in opposite direction with respect to the Born approximation for positrons and electrons;
- 3) the angle of emission is considerably different for positrons and electrons.

From the fundamental point of view the most important result is the first one. It shows that, to save unitarity, it is not sufficient to treat the motion of the electrons better than in the Born approximation; it will be necessary to treat in a better approximation the radiation process and the consequent creation of pairs. So it appears necessary to introduce, for a complete treatment of the passage of the very high energy electrons and positrons in crystals, the notion of a "coherent shower".

This is a notion which, in a certain sense, goes beyond the usual S matrix frame, and deserves probably a further investigation.

R E F E R E N C E S

- 1) B. Ferretti, Nuovo Cimento 7, 118 (1950).
- 2) A good account can be found, for instance, in a paper by U. Timm, DESY 69/14 (1969).
- 3) L.I. Schiff, Phys.Rev. 117, 1394 (1960).
- 4) B. Ferretti, The bremsstrahlung in crystals and the connected phenomena (Lectures given in the Academic Training Programme of CERN, 1968-1969).

Table 1

$$|H_{a,b}(N)|^2/N^2 (k = 6.7 \times 10^8 \text{ eV}, \lim N = \infty)$$

w_a	Born	WKB(electrons)	WKB(positrons)
0.05	21.2	26	16
0.10	7.9	16	2.3 ^{*)}
0.20	11	1	47
0.30	117	57	340
0.40	361	225	1080
0.50	786	540	99 ^{**)}
0.55	108	790	195 ^{**)}

*) $H_{a,b}$ changes of sign between 0.05 and 0.10.

***) $\Delta\phi = -4\pi$ ($\Delta\phi = -2\pi$ impossible in these cases!)

Table 2

$$\theta' [\text{units} \equiv m(25 \text{ p})]$$

w_a	Born	WKB(electrons)	WKB(positrons)
0.05	- 39 + 39	- 38 + 40	- 41 + 38
0.1	- 30 + 30	- 29 + 32	- 32 + 28
0.2	- 22 + 21	- 19 + 24	- 23 + 18
0.3	- 16 + 16	- 13 + 19	- 18 + 11
0.4	- 12 + 12	- 8 + 14	- 13 + 4
0.5	- 8 + 7	- 3 + 10	- 30 + 19
0.55	- 5 + 5	0 + 7	- 29 + 17