Neutrino propagation in AGN environment

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Assuming the violation of equivalence principle (VEP) by ultra high energy AGN neutrinos we study the effect of random magnetic field fluctuation on conversion of electron neutrinos to tau anti-neutrinos.

I. INTRODUCTION

Active Galactic Nuclei (AGN) are the powerful sources for the production of ultra high energy neutrinos in the Universe [1,2] and with the present day detectors these neutrino fluxes can be detected [3,4]. AGNs are believed to be fueled by the gravitational energy of matter accreting to a supermassive black hole ($10^4 M_\odot$ to $10^8 M_\odot$) at the AGN core, where gravitational energy is converted into luminous energy through the acceleration of high energy protons [2,5,6]. These high energy protons lose their energy through $pp$ collision and also through $p\gamma \rightarrow p + e^- + e^+$ and $p\gamma \rightarrow N\pi$ processes in the central region. The pions decay to neutrinos through the $\pi^\pm \rightarrow \mu^\pm \rightarrow e^\pm$ decay chain. From the pion decay we can expect twice $\nu_\mu$ as $\nu_e$ [6]. A negligible number of $\nu_\tau$ are produced in the AGN environment. The search for such high energy neutrinos by the neutrino telescopes DUMAND, AMANDA, NESTOR and BAIKAL are undertaken [3,4].

Mechanism undergoing neutrino oscillation typically assume that neutrinos have non degenerate masses and the weak eigenstates are different from the mass eigenstates, thereby permitting oscillation between the various flavours [7]. An alternative approach was proposed by Gasperini [8] and independently by Halprin and Leung [9]. If Einstein’s equivalence principle is violated, gravity may not universally couple with neutrinos with different flavours. Then if this occurs, the weak eigenstates are distinct from the gravitational one and neutrino oscillations similar to that of vacuum flavour mixing due to neutrino masses will take place. Thus the violation of equivalence principle (VEP) does not require neutrinos to have nonzero masses to oscillate from one flavour to another [10].

Using VEP the solution to the solar neutrino problem [11] and the atmospheric neutrino anomaly [12] is studied. VEP effect on laboratory neutrino oscillation experiment is also analysed [13]. The effect of VEP on AGN neutrino has been studied by Minakata and Smirnov [14]. Magnetic field effect around AGN have important astrophysical consequence. Neutrinos having magnetic moment or transition magnetic moment (for transition between different flavours) can flip their helicity in the presence of a magnetic field which has a component perpendicular to the direction of motion of the neutrino. The spin-flavour oscillation of high energy neutrinos of AGN origin due to VEP and large magnetic field are studied previously [15–17]. Here we are interested to study the spin-flavour transition of $\nu_e \rightarrow \bar{\nu}_e$ due to random fluctuation in the magnetic field in the AGN environment in the presence of VEP. We consider both massive and massless neutrinos here.

II. NEUTRINO PROPAGATION

The evolution equation for propagation of neutrinos in the medium in the presence of a magnetic field is given by

$$\frac{d}{dt} \begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = \begin{pmatrix} V - \delta & \mu B_{\perp}(t) \\ \mu B_{\perp}(t) & 0 \end{pmatrix} \begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix}$$

(1)

where $x = s, b$ (sterile, active) and $b = e, \mu$ and $\tau$. The $\delta$ is given as $\delta = \cos 2\theta \Delta m^2/2E$. $\mu$ corresponds to its diagonal magnetic moment. On the other hand for Majorana neutrino $\mu$ is the transition magnetic moment. $V$ is the difference of neutrino interaction potential. Here we consider neutrino mixing very small, so that $\cos 2\theta = 1$. For considering the fluctuation in the magnetic field, we can write $B(t) = B_0 + \tilde{B}(t)$ where $\bar{B}(t)$ is the fluctuation over the constant background $B_0$.

In a previous paper, one of the present authors [18] has derived the average probability equation for active sterile/active conversion in the presence of a randomly fluctuating magnetic field (fluctuation in both $B_{\perp}$ and $B_{\parallel}$ components). The system consists of magnetic domain structure with a size $L_0$ and the magnetic field is uniform and constant within each domain. Fields in different domains are randomly aligned. For the derivation of the probability equation the simple delta correlation of the magnetic fields in different domains are assumed. For the neutrino conversion length greater than the domain size $l_{\text{conv}} >> L_0$ (where $l_{\text{conv}} \sim 1/\Gamma$, $\Gamma = \mathcal{P}\Gamma_W$ and $\Gamma_W$ is the weak interaction rate), a neutrino will cross many magnetic field domains before it flips its helicity. Thus the neutrino will experience an average field before it flips its helicity. The solution for the probability equation is obtained and found the necessary condition for
the positive definiteness of the neutrino conversion probability \(0 \leq \mathcal{P}(t) \leq 1\). The condition is given by (here we consider only the \(B_{\perp}\))

\[
4(\mu B_{\perp})^2 + (V - \delta)^2 > \frac{4}{3} \Gamma_{\perp}^2
\]  
(2)

where the transverse magnetic field damping parameter \(\Gamma_{\perp}\) is given by

\[
\Gamma_{\perp} = \frac{4}{3} \mu^2 B_{\text{rms}}^2 L_0.
\]  
(3)

\(B_{\text{rms}}^2 = B^2 > 0\) and \(L_0\) is the domain size. The inequality in eq.(2) has to satisfy, irrespective of the form of neutrino potential and the magnetic field. The effect of random magnetic field on neutrino propagation as well as their conversion in the early universe hot plasma and in the supernova core is studied [18,19]. Here we are interested in the propagation of high energy neutrinos in the AGN in the presence of its own gravitational potential. The effective potential experienced by the neutrinos at a distance \(r\) from a gravitational source of mass \(M\) due to VEP is \((V = V_G)\)

\[
V_G = \frac{1}{2} E \Delta f \phi(r),
\]  
(4)

where \(\phi(r) = GM/r\). The VEP is parameterised by a dimensionless parameter \(\Delta f\) and \(E\) is the neutrino energy. The gravitational effect at the neutrino production site is of order \(\phi(r) \sim 5 \times 10^{-3}\) for a \(10^8 M_\odot\) black hole. Then for neutrino energy \(E = 10^{15}\) eV (1 PeV), the gravitational potential for neutrino is \(V_G \sim 2.5 \times 10^{12} \Delta f\) eV. The matter density in the vicinity of AGN is found to be \(\rho \sim 10 - 10^4\) eV. The magnitude of the matter potential for neutrino propagating in the AGN is \(G_{\text{FP}}/m_\nu \sim 10^{-29} - 10^{-33}\) eV, where \(G_F\) is the Fermi coupling constant. Comparison of \(V_G\) with the matter potential shows that for the gravitational effect to be dominant over the matter effect, \(\Delta f > 10^{-45}\). The optimal sensitivity on VEP can be achieved by next generation water Cerenkov detector is \(\Delta f \sim 10^{-18} - 10^{-16}\). Also it has been shown that, for PeV neutrinos satisfying the resonance will give \(\Delta f \sim 10^{-28} \Delta m^2/eV^2\). Considering the vacuum mixing value \(\Delta m^2 \sim 10^{-10}\) eV\(^2\) we obtain \(\Delta f \sim 10^{-38}\), which corresponds to \(V_G \sim 10^{-26}\) eV and this is again order of magnitude larger than the matter effect. So in our analysis we will neglect the matter effect on neutrino propagation and only consider the gravity effect.

In our analysis we will consider both \(\Delta m^2 \neq 0\) and \(\Delta m^2 = 0\) situations. We are interested to consider the process \(\nu_e \rightarrow \bar{\nu}_\tau\). This is because the initial fluxes of these neutrinos in the AGN are maximally asymmetric \((\nu_\tau/\nu_\nu \leq 10^{-3})\). So any enhancement in this ratio can signal the spin flavour conversion of \(\nu_\nu\) to \(\bar{\nu}_\tau\). Let us consider first \(\Delta m^2 \neq 0\). Neutrino propagating in the presence of a constant magnetic field \(B\), the conversion probability is given by

\[
\mathcal{P}(r) = \frac{(2\mu B_{\perp})^2}{(2\mu B_{\perp})^2 + V_G^2} \sin^2 \left(\frac{[(2\mu B_{\perp})^2 + V_G^2]^{1/2} r}{2}\right),
\]  
(5)

where \(V_G = V_G - \delta\). The potential for the massive neutrinos is

\[
V_G - \delta = (2.5 \times 10^{12} \Delta f - 0.5 \times 10^{-13} \frac{\Delta m^2}{eV^2}) eV.
\]  
(6)

The resonance condition is obtained when \(V_G - \delta = 0\) and this gives the value of \(\Delta f \sim 2 \times 10^{-28} \Delta m^2/eV^2\). Let us assume that neutrinos propagating in the AGN medium, will satisfy the same resonance condition in the presence of a random magnetic field fluctuation. Then in that case the inequality in eq.(2) becomes

\[
\mu B_{\perp0} > \frac{1}{\sqrt{3}} \Gamma_{\perp}.
\]  
(7)

Putting

\[
\Gamma_{\perp} = 2.2 \times 10^{-12} \mu_0^2 \left(\frac{B_{\text{rms}}}{G}\right)^2 \left(\frac{L_0}{\text{cm}}\right) eV,
\]  
(8)

we obtain

\[
\frac{B_0}{G} > 0.3 \times 10^{-3} \mu_0 \left(\frac{B_{\text{rms}}}{G}\right)^2 \left(\frac{L_0}{\text{cm}}\right),
\]  
(9)

where \(\mu_0 = \mu/\mu_B\). We take the constant magnetic field \(B_0 \sim 10^4\) G. For considering domain size \(L_0\) of order \(\sim 1\)pc \(\sim 3 \times 10^{18}\text{cm}\) we obtain

\[
\mu_0 < 10^{-12} \left(\frac{B_{\text{rms}}}{G}\right)^{-2}.
\]  
(10)

For considering fluctuation in the magnetic field is of order one Gauss, in different domains of size \(\sim 1\)pc, then we obtain \(\mu \sim 10^{-12}\mu_B\). This shows that one Gauss fluctuation in magnetic field in pc scale might affect the neutrino propagation in the AGN environment. We obtain the average conversion probability \(\mathcal{P} = 0.5\) for the above parameters \(^1\) (magnetic field, domain size and magnetic moment). Thus half of the \(\nu_e\) can be converted into \(\bar{\nu}_\tau\) when propagating in the fluctuating magnetic field of the AGN.

For considering \(4(\mu B_{\perp})^2 >> (V_G - \delta)^2\), in eq.(5) the inequality in eq.(2) will be the same as given in eq.(7).

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\(^1\)We use eq.(25) of ref [18] to compute the conversion probability numerically.
Also we will get the same constraint on the magnetic moment as shown in eq.(10). But in this case constraint on $\Delta f$ will come from the inequality $\Delta f << 2 \mu B_{0,\perp}/\phi(r) E$ (for $\delta \sim 0$). Thus for $B_{0,\perp} \sim 10^4 G$ and $\mu_0 \sim 10^{-12}$ we obtain $\Delta f < 10^{-31}$.

If the AGN neutrinos satisfy the resonance condition $V_G - \delta = 0$ or for massless neutrinos $V_G << 2 \mu B_{1,0}$, we obtain the same constraint on the neutrino magnetic moment $\mu$ and the conversion probability is $\sim 0.5$. But the $\Delta f$ is different in both the cases. In the first case it depends on the $\Delta \mu^2$ value and in the second case on the product $\mu B_{1,0}$.

The inter galactic magnetic field is $\sim 10^{-6} G$ and this is very small to reverse the helicity of the neutrino which is traveling $\sim 100 Mpc$ from the AGN to the earth. So once the neutrino flips its helicity in the AGN fluctuating magnetic field, it will not re-flip in the inter galactic magnetic field. The initial fluxes of high energy neutrinos originating from the AGN are estimated to have the following ratios: $\nu_\tau/\nu_\mu \approx 0.5$ and $\bar{\nu}_\tau/\bar{\nu}_e \leq 10^{-3}$. So if any enhancement of $\bar{\nu}_\tau/\nu_e$ is found correlated to the direction of the AGN source it might be because of the random magnetic field in the AGN and VEP.

For VEP, the weak eigenstates are different from the gravitational one, so neutrino oscillation occurs due to flavour non-diagonal coupling of neutrinos to gravity. Thus even the massless neutrinos can also oscillate. Let us consider the propagation of massless neutrinos in the AGN medium with the constant background magnetic field $B_{1,0}$ very small compared to the random fluctuation $B_{rms}$. In the central region of the AGN, neutrinos are produced because of the accelerating high energy protons colliding with the dense radiation fields ($\gamma \gamma$) and high energy $pp$ collisions. According to the spherical accretion model by Kazanas, Protheroe and Ellison [6], close to the black hole the accretion flow becomes spherical and a shock is formed. The distance from the AGN center to the shock is called as the shock radius and is of order $10^{14} - 10^{20} eV^{-1}$ [15, 16]. In this region we assume that the non linear plasma processes might be the dominating one and magnetic flux lines will be co-moving with the turbulent plasma thus creating the randomness in the magnetic field. We also assume that in this region the magnetic fields will be having domain structures like the solar spots with intense magnetic field of order $\sim 10^5 G$ and the domain size $L_0$ is of order $10^{14}$ to $10^{20} eV^{-1}$, the shock radius. Then the inequality in eq.(2) will give

$$V_G > \frac{2}{\sqrt{3}} \Gamma_\perp (11)$$

For the domain size $L_0 \sim 10^{14} eV^{-1}$ and $\Delta f \sim 10^{-31}$ we obtain $\mu < 10^{-12} \mu_B$ and for $L_0 \sim 10^{20} eV^{-1}$ and the same $\Delta f$ as above we obtain $\mu < 10^{-15} \mu_B$.

If we consider that in the AGN environment there is a magnetic field fluctuation $B_{rms} \sim 1 G$ in the pc scale over the constant background of order $10^4 G$, then we obtain $\mu < 10^{-12} \mu_B$ for normal resonance case ($V_G - \delta = 0$) as well as for $2 \mu B_{1,0} >> V_G$. For considering the magnetic field to be random within the shock radius of order $10^{-14} - 10^{20} eV$ we obtain $\mu < 10^{-12} \mu_B$ to $\mu < 10^{-15} \mu_B$. In this case also we obtain $\mathcal{P} = 0.5$

III. CONCLUSIONS

We have studied the effect of random fluctuation of magnetic field and VEP on the PeV neutrinos originating from AGN. We used the perviously found condition for positive definiteness of the average neutrino conversion probability to constraint the transition magnetic moment for $\nu_e \to \bar{\nu}_\tau$. We found that random fluctuation in the AGN magnetic field with VEP will effect the spin-flavour precession of neutrino. For $\Delta \mu^2 \neq 0$, if we assume that the condition $V_G - \delta = 0$ holds, then for $B_{rms} \sim 1 G$ we obtain $\mu < 10^{-12} \mu_B$. Also for considering $2 \mu B_{1,0} >> V_G$ we obtain the same constraint on $\mu$. For considering $B_{rms} >> B_{1,0}$ within the sock radius we found that for massless neutrinos the positive definiteness condition on the conversion probability gives $\mu < 10^{-15} \mu_B$ for $L_0 \sim 10^{20} eV^{-1}$ and $\mu < 10^{-12} \mu_B$ for $L_0 \sim 10^{14} eV^{-1}$ respectively. Thus the positive definiteness condition on the conversion probability of $\nu_\tau \to \bar{\nu}_\tau$ put constraints on the neutrino magnetic moment. But this constraint on the magnetic moment is not the physical constraint, because it comes from the positive definiteness of the conversion probability not from any physical conditions. But the average conversion probability calculation shows that, maximum half of the original $\nu_e$ can be converted into $\bar{\nu}_\tau$ when propagating in the magnetic field of the AGN. So if any enhancement of $\bar{\nu}_\tau/\nu_e$ flux is found correlated to the direction of the AGN source it might signal the combined effect of VEP and random fluctuation in the magnetic field. Thus the new generation neutrino telescopes DUMAND, AMANDA, NESTOR and BAIKAL might be able to shed more light on this.

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