Two-neutron removal reactions of $^6$He treated as a three-body halo

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Abstract

We formulate a method to compute breakup processes of three-body halo systems reacting with a target. The reaction between one of the particles and the target as well as the corresponding pairwise final state interactions are carefully treated. Both absorption and diffraction processes are included. Detailed differential and total cross sections are calculated for two-neutron (2n) removal from $^6$He by fragmentation on $^{12}$C. Effects due to the core shadowing are taken into account. Good agreement with available experimental data is obtained.

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Introduction. Breakup reactions of three-body systems are difficult to describe in general if the formulation is required to be both practical and fairly accurate. However, due to the steady improvement of radioactive beam facilities an increasing amount of fragmentation data demand analyses where interpretation in terms of few-body reactions appear to be an efficient framework [1–5]. Previous attempts are based on the eikonal approximation and the adiabatic assumption of frozen intrinsic degrees of freedom during the collision [6–13]. In most of these approaches very simple three-body wave functions are used and the final state interaction often needs better treatment.

The crucial ingredients in these processes are the final state interaction [2,8,14] and the interaction between one of the three particles (the participant) and the target. The remaining two particles are essentially spectators and reactions simultaneously involving more than one particle are much less important. The large spatial extension of halo nuclei guarantees that this model is a good first approximation which could be improved in future more elaborate investigations if needed. We shall here fully exploit these simplifying assumptions to keep the approach practical enough to compute many observables. We shall describe the reaction between each particle and the target by the optical model. The final state two-body interactions are those used to compute the bound state wave function of the three-body projectile.

Theory. When a spatially extended three body halo hits a relatively small target at high energy the probability that more than one of the constituents interacts strongly with the target is small. The cross section can then be written as a sum of three terms where each term describes the contribution to the reaction caused by the interaction between the target and the corresponding particle

\[ d\sigma = \sum_{i=1}^{3} d\sigma^{(i)} = \sum_{i=1}^{3} \frac{1}{v} \frac{2\pi}{\hbar} |T^{(i)}|^2 d\nu_f^{(i)}, \]

where \( v \) is the relative projectile-target velocity, \( T^{(i)} \) is the transition matrix element and \( d\nu_f^{(i)} \) is the density of final states. Each of the three terms includes elastic and inelastic scattering of the corresponding particle on the target. Neglecting the Coulomb interaction
and assuming that the target has zero spin we write the transition matrix element for the elastic contribution as (compare with the formulation for a weakly bound projectile [15]):

\[ T^{(i)} = \langle \phi_{p_i}^{(i)(+)} | e^{ipR'} | V_0 | \Psi^{JM} e^{ipR} \rangle = \sum_{\Sigma_i} T^{(0i)}_{\Sigma_i \Sigma_i} M_{\Sigma_i \Sigma_i}^{JM}, \]  
\[ T^{(0i)}_{\Sigma_i \Sigma_i} = \langle \phi_{p_i}^{(i)(+)} | V_0 | e^{ip_0 R_0} \chi_{s_i \Sigma_i} \rangle, \]  
\[ M_{\Sigma_i \Sigma_i}^{JM} = \langle \phi_{p_i}^{(i)(+) \Sigma_i} | e^{ipR_{r_i} \Sigma_i} \chi_{s_i \Sigma_i} | \Psi^{JM} \rangle, \]

where \( V_0 \) is the interaction between particle \( i \) (the participant) and the target, \( \Psi^{JM} \) is the projectile internal wave function, \( \phi_{p_i}^{(i)(+)} \) and \( \phi_{p_{i}}^{j(k)(+) \Sigma_i} \) are the distorted waves in the corresponding two-body subsystems and \( \chi_{s_i \Sigma_i}, \chi_{s_i, \Sigma_i} \) are the spin functions \( (s_{jk} = s_j + s_k) \).

The coordinates \( r_{jk}, R \) and \( R' \) are defined in fig. 1 and the conjugate momenta are denoted by the corresponding \( p \). Primes are used for the final states. The density of final states is

\[ d\nu^{(i)} = \delta(E_{0i} - E_{0i}) \frac{d^3 p_{0i} \, d^3 p'_{jk} \, d^3 p'_i}{(2\pi\hbar)^3 (2\pi\hbar)^3 (2\pi\hbar)^3}, \]  

where \( E_{0i} \) is the relative energy of particle \( i \) and the target and

\[ r_{0i} = R - \frac{m_j + m_k}{m_i + m_j + m_k} r_{i, jk}, \quad p_{i, jk} = p' - \frac{m_j + m_k}{m_i + m_j + m_k} P, \quad p_{0i} = P - \frac{m_0}{m_i + m_0} P'. \]  

The diffraction cross section for non-polarized projectile is then obtained after summing over final and averaging over initial states

\[ \frac{d^3 \sigma_{el}^{(i)}(p', p'_{jk}, p_{0i})}{d^3 p'_{jk} \, d^3 p_{0i}} = \frac{d^3 \sigma_{el}^{(0i)}(p_{0i} \rightarrow p'_{0i})}{d^3 p_{0i}} \frac{1}{2J + 1} \sum_{M_{\Sigma_i \Sigma_i}^{JM}} |M_{\Sigma_i \Sigma_i}^{JM}|^2, \]  
\[ \frac{d^3 \sigma_{el}^{(0i)}(p_{0i} \rightarrow p'_{0i})}{d^3 p_{0i}} = \frac{1}{2 \pi \hbar} \frac{1}{2s_i + 1} \sum_{\Sigma_i, \Sigma_i'} |T^{(0i)}_{\Sigma_i \Sigma_i'}|^2 \delta(E_{0i} - E_{0i}) \frac{d^3 p_{0i}}{(2\pi\hbar)^3}. \]

The absorption cross section where only two projectile particles survive in the final state is simply obtained by replacing the elastic \( \sigma_{el} \) by \( \sigma_{abs} \), i.e.

\[ \frac{d^6 \sigma_{abs}^{(i)}(p', p'_{jk})}{d^3 p'_{jk}} = \sigma_{abs}^{(0i)}(p_{0i}) \frac{1}{2J + 1} \sum_{M_{\Sigma_i \Sigma_i}^{JM}} |M_{\Sigma_i \Sigma_i}^{JM}|^2, \]

The total cross section arising from \( V_0 \) is the sum of eqs. (7) and (9). The function \( M_{\Sigma_i \Sigma_i}^{JM} \) is the transition matrix computed in [11].
Parameters. We now consider the nucleus $^6$He (n+n+α) with the wave function obtained by solving the Faddeev equations in coordinate space [10] by using the potentials from [16]. The resulting three-body wave function has 88% of $p^2$- and 12% of $s^2$-configurations. The binding energy is 0.95 MeV and the root mean square radius is 2.45 fm.

For the neutron-target interactions we use non-relativistic optical potentials obtained from relativistic potentials through a reduction of the Dirac equation into a Schrödinger-like equation [17]. In particular we focus on a carbon target, and for the neutron–$^{12}$C interaction we use the parametrization EDAI-C12 [18] valid for a range of neutron energies from 29 to 1040 MeV. We include 35 partial waves in the calculations.

The detected 2n removal contribution from the interaction between α-particle and target is expected to be relatively small. First the elastic scattering cross section for α-particles on $^{12}$C is about 35% of the total cross section [19], which in turn is of the same order as the neutron $^{12}$C total cross section. Second a substantial fraction of these 35%, where the α-particle survives the reaction with the same energy, are in fact elastically scattered $^6$He-particles and as such not contributing to the 2n removal cross section. Third the very forward angles, containing most of the elastic cross section, are experimentally excluded. We shall therefore neglect this contribution in the present letter.

The final state two-body continuum wave functions ($\phi^{i(k(+)}, \phi^{0i(+)})$ are now calculated with the appropriate boundary conditions and normalization [11]. Then eqs.(7) and (9) are integrated over the unobserved quantities and different momentum distributions or differential cross sections are obtained. However, the so-called core shadowing problem remains [20–22]. Basically this means that the finite sizes of the target and the α-core prohibit some of the neutron-target reactions namely those where the α-particle also interacts with the target. In the eikonal approximation this effect is accounted for by multiplying the halo wave function by a profile function. A simpler approximation is the black disk model where the profile functions are step functions [14], and the interior part of the halo wave function is eliminated. This amounts to eliminate the configuration which is not consistent with the reaction in question.
In particular, in a process where the neutron is removed by the target without destroying the α-core, we must exclude the part of the three-body wave function, where the removed neutron is too close (or even inside) the core. We do this by assuming that $\Psi_i = 0$ when the distance between the interacting neutron and the center-of-mass of the remaining two projectile particles is sufficiently small. This amounts to assuming that the Jacobi coordinate $y = r_{n,\alpha n} \sqrt{(m_\alpha + m_n)/(m_\alpha + 2m_n)}$ is smaller than a certain cut off value $y_c$, which presumably is better left as a parameter, but clearly is related to the sizes of the reacting particles. In fact, the sum of the target and core radii is around 4 fm, and we expect that for $r_{n,\alpha n} < 4$ fm the neutron can not be removed without disturbing (or breaking) the core.

**Observables.** In fig. 2 we compare the calculated and experimental [5] transverse $^5$He momentum distribution after $^6$He fragmentation on $^{12}$C at 240 MeV/u. Without shadowing a too broad momentum distribution is found, while the shadowing with $y_c = 6$ fm gives a too narrow distribution. For the estimated value of $y_c = 4$ fm, the central part of the distribution up to about 60 MeV/c is rather well reproduced. The momentum tails receive contributions from distances where the neutron is inside the core. The model is in other words not applicable for these momenta. Thus $y_c = 4$ fm seems to be an appropriate value. This is consistent with the interpretation in [5,13].

The absolute value of the 2n removal cross section is sensitive to the value of $y_c$ as seen in fig. 3a, where we show results for breakup of $^6$He on $^{12}$C as function of beam energy for different shadowing parameters. The absorption processes amounts to 70-75% of the cross section depending on energy, consistent with the known data of neutron scattering on $^{12}$C [18]. In all cases the shadowing for $y_c = 4$ fm reduces the cross section by a factor around 3.6, but the absolute cross sections should be taken with caution due to the high sensitivity to $y_c$. However, the numbers are encouraging and in agreement with both experimental values [23] and theoretical estimates [6,13]. The predicted minimum at about 200 MeV arises from a corresponding minimum in the n-$^{12}$C cross section [18].

In fig. 3b we show the value of the total 2n removal cross section, with $y_c$ equal to 3, 4 and 5 fm relative to its value at a beam energy of 100 MeV/u ($0.430$ b for $y_c = 3$ fm, $0.250$ b
for \( y_c = 4 \text{ fm and } 0.130 \text{ b for } y_c = 5 \text{ fm} \). We also show the width of the transverse neutron momentum distribution relative to the width at 100 MeV/u (77 MeV/c, 75 MeV/c and 73 MeV/c for \( y_c = 3, 4 \) and 5 fm, respectively). Both these relative quantities are essentially independent of the shadowing. In addition we note that the width of the neutron momentum distribution is almost constant for these high beam energies.

In fig. 4a we compare the calculated and measured transverse neutron momentum distribution after \(^6\text{He}\) fragmentation on carbon. We also show the results of calculations with and without final state interaction based on the sudden approximation as described in [10,11]. These results resemble those without shadowing. This fact supports the validity of the sudden approximation with inclusion of final state interaction as a first approximation to the description of these reactions. However, even if this approximation is rather good, the agreement with the experiment is improved when shadowing with \( y_c = 4 \text{ fm} \) is used. In fig. 4b we compare the calculated longitudinal and transverse neutron momentum distributions. They are very similar, but a small asymmetry, due to the energy dependence of the n-\(^{12}\text{C}\) optical potential, is present in the longitudinal distribution.

**Conclusions.** A practical method treating one particle-target interaction at a time is formulated for spatially extended three-body systems colliding with a light target. Absolute values of differential cross sections including their dependence on beam energy and target structure can then be computed. The longitudinal and transverse momentum distributions in breakup reactions are now distinguishable. Both diffractive and absorptive processes are treated.. The method is subsequently applied to breakup reactions of the halo nucleus \(^6\text{He}\) (n+n+\(\alpha\)) on a \(^{12}\text{C}\) target. The final state interaction and the shadowing are crucial for the neutron momentum distributions and the absolute cross sections respectively. Remarkable agreement is achieved with the observed quantities for absolute as well as for differentials cross sections. If not a correct description we provide at least a very good parametrization.

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REFERENCES


**Figure Captions:**

**Fig.1:** Sketch of the reaction and the coordinates used. The target is labelled by 0 and \(i, j, k\) label the particles within the three-body projectile.

**Fig.2:** Transverse \(^5\text{He}\) momentum distribution after fragmentation of \(^6\text{He}\) on \(^{12}\text{C}\) at 240 MeV/u.

**Fig.3:** (a) Calculated 2n removal cross section after \(^6\text{He}\) fragmentation on \(^{12}\text{C}\) as a function of the beam energy. (b) The same reaction as (a). The circles and squares respectively indicate the ratio between the total 2n removal cross section and the width of the transverse neutron momentum distribution as a function of the beam energy.

**Fig.4:** (a) The transverse neutron momentum distribution after \(^6\text{He}\) fragmentation on \(^{12}\text{C}\) at 240 MeV/u. The results of the calculations described in ref. [11] with (long dashed curve) and without (short dashed curve) inclusion of final state interactions. (b) Computed longitudinal and transverse neutron momentum distribution for the same case as (a).