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REGGE POLES WITH KINEMATIC CUTS IN π N BACKWARD SCATTERING

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A B S T R A C T

We investigate the effects of including kinematic cuts to suppress parity doublets in the Regge pole description of π N backward scattering.

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A basic problem in Regge theory is that MacDowell symmetry requires each fermion trajectory $\alpha(W)$ to connect states of one parity for positive W with those of opposite parity for negative W . Taken together with the empirical observation of trajectories linear in $u \equiv W^2$, this implies the existence of pairs of states of the same mass, spin and signature but of opposite parity, whereas experimentally such parity doublets are not found. Even the F_{15} and D_{15} πN resonances near 1680 MeV should not be taken as parity doublets since they have very different F/D ratios when considered in $SU(3)$.

The traditional approach has been to include zeros in the residue functions to extinguish each of the lowest such unobserved parity partners. This may even be extended to remove all such states ¹⁾. Recently, however, it has been suggested ²⁾ that a fixed square root branch point may occur in the ℓ plane at $\ell = \alpha(0)$ and that the unwanted parity partners all lie on the corresponding unphysical sheet. The cut may be taken to be of kinematical origin since it arises from the fermion mass $m \sim (\ell - \alpha(0))^{\frac{1}{2}}$ being considered rather than $m^2 \sim \ell - \alpha(0)$ as for mesons. This leads essentially to a simple moving pole in the $(\ell - \alpha(0))^{\frac{1}{2}}$ plane instead of in the usual ℓ plane. The kinematic cut should be contrasted with the dynamical Regge cuts arising from absorption and related model calculations which lie in a similar position but which have quite different prescriptions for their discontinuities. Indeed the absorption model cuts are relatively strongest in non-flip amplitudes while the kinematic cut contributes only to the spin-flip amplitude in the present application.

We investigate the effect of the kinematic cuts by analyzing the high energy data for backward πN scattering in terms of u channel N and Δ Regge trajectories. In order to provide a standard for comparison we first performed a conventional Regge pole analysis of the data. For this purpose we used the following parametrizations for the Regge pole contributions to the s channel πN invariant amplitudes

$$A_i + \eta_N B_i = (a_i + b_i u) e^{c_i u} \Gamma(1 \mp \frac{1}{2} - \alpha_i) \left[1 \pm e^{-i\pi(\alpha_i - \frac{1}{2})} \right] \left(\frac{s-t}{2} \right)^{\alpha_i - \frac{1}{2}} \quad (1)$$

$$B_i = \pm \frac{1}{m_i} (a_i + b_i m_i^2) e^{c_i u} \Gamma(1 \mp \frac{1}{2} - \alpha_i) [1 \pm e^{-i\pi(\alpha_i - \frac{1}{2})}] \left(\frac{s-t}{2}\right)^{\alpha_i - \frac{1}{2}} \quad (2)$$

where the upper, lower signs correspond to $i = N, \Delta$. Linear trajectories are assumed $\alpha = \alpha_0 + \alpha' u$, with units in GeV. This parametrization removes the unobserved parity partners at $\frac{1}{2}^-$ on α_N and $\frac{3}{2}^-$ on α_Δ whilst allowing the ratios $(A+m_N B)/B$ to vary with u . It is equivalent to the usual parametrization ³⁾ in terms of f_1 and f_2 . The relations between the residue functions and the particle pole couplings are

$$g_N^2/4\pi = (a_N + b_N m_N^2) e^{c_N m_N^2} / 6\pi m_N \alpha'_N \quad (3)$$

$$\Gamma_\Delta = -(a_\Delta + b_\Delta m_\Delta^2) e^{c_\Delta m_\Delta^2} [(m_\Delta + m_N)^2 - \mu^2] g_\Delta^3 / 6\pi m_\Delta^3 \alpha'_\Delta \quad (4)$$

The Δ residue function includes a zero at $\alpha_\Delta = \frac{1}{2}$ to be in accord with the predictions ⁴⁾ of exchange degeneracy for baryons. This zero reflects the absence of a $\frac{1}{2}^-$ state of mass about 700 MeV on the exchange degenerate ~~N~~ trajectory. Such a zero had already been suggested by Igi et al. ⁵⁾ in order to reconcile the backward $\pi^- p$ data with the value of the residue of the Δ pole. For instance we predict $\Gamma_\Delta \sim 1$ to 2 MeV if we fit the data omitting the $(\frac{1}{2}^- \alpha_\Delta)$ factor whereas $\Gamma_\Delta \sim 200$ MeV when it is included. Moreover the presence of the zero changes the relative sign of the N and Δ Regge pole contributions and so leads to improved fits ⁶⁾ in the near backward direction to the $\pi^- p \rightarrow n \pi^0$ data. However, the dip predicted in the $\pi^- p \rightarrow n \pi^0$ cross-section is moved to $u \sim -0.08$ which is in the wrong direction ⁶⁾ as compared to the data.

The solid curves of Fig. 1 show the fit to the $\pi^- p \rightarrow p \pi^-$ and $\pi^+ p \rightarrow p \pi^+$ data ⁷⁾ obtained with the pole parameters given in the Table. The charge-exchange data ⁸⁾ was predicted (see the Figure) rather than fitted because the 6 GeV data conflict with each other. The $\pi^+ p$ polarization is predicted to be negative for $u > -0.15$, zero at -0.15 and positive for larger $|u|$ values. The trend of the 2.75 and 3.25 GeV data ⁹⁾ is, however, positive around $u \sim -0.15$ and turns negative for $u \lesssim -0.3$.

The defects of the predictions of the two-pole fit are clearly seen to be (i) the dip at $\alpha_\Delta = -\frac{3}{2}$ in $\pi^- p$ backward scattering, (ii) the wrong position of the dip for $\pi^- p \rightarrow n \pi^0$, (iii) the dip in $\pi^+ p$ backward scattering is too deep, particularly at the lower energies fitted, (iv) the polarization changes sign at $u \sim -0.15$ where there is some evidence that it should be at a maximum. These features have been discussed either by including a $N\Delta$ or $N\Delta$ trajectory ^{10),11)} or by including absorptive model cuts ^{6),12)}. However, here we investigate the effect of including the kinematic cuts mentioned above. We find that this way of suppressing parity doublets also leads to excellent fits to the data. The parametrization of the Δ residue is very satisfactory and corresponds to a sensible residue at the Δ pole. However, from the fits we see that the simple parametrization of the N residue has unphysical features.

The simplest way of including the kinematic cut to suppress parity doublets ²⁾ leaves the parametrization of $A + m_N B$ unchanged so that the cut contributes only to the B amplitude. By deforming the original Sommerfeld-Watson contour to the left in the ℓ plane we find Eq. (2) is replaced by

$$B = \pm \frac{1}{\pi} \int_{-\infty}^{\alpha_0} dJ \frac{\Gamma(J - \frac{1}{2}) (a + b\lambda)}{\alpha' \sqrt{-\lambda} (\lambda - u)} e^{c\lambda} \left[1 \pm e^{i\pi(J - \frac{1}{2})} \right] \left(\frac{s - t}{2} \right)^{J - \frac{1}{2}} \quad (5)$$

where again the upper, lower signs correspond to N, Δ ; and where $\lambda \equiv (J - \alpha_0)/\alpha'$. For $u < 0$, Eq. (5) is the complete expression for B if principal value integration is understood, whereas for $u > 0$

there is an additional pole contribution given by $\pm(A+m_N B)/\sqrt{u}$ which removes all the relevant parity doublet states as claimed. It is straightforward to obtain a single expression for B , valid for all u , which is explicitly free of $\sqrt{u}$ singularities and which is suitable for numerical evaluation. In practice the integral along the cut was cut off at $J = -10$. However its numerical value is insensitive to considerable variation of the cut-off and only for very large negative J does the bad asymptotic behaviour of the Γ function begin to dominate. Approximating the Γ function by a constant ²⁾ preserves the convergence of the integral and was found to make little difference for $u \lesssim 0$. Nevertheless we retained expression (5) since it allows a discussion of the particle pole residues.

The results of the fit incorporating the kinematic cuts are shown by the dashed curves in Fig. 1. Although there are no extra parameters the fit is appreciably better than the conventional two-pole fit. Our predictions for $\pi^- p \rightarrow n \pi^0$ are excellent and we agree at 6 GeV with the Boright et al. and Schneider et al. data rather than with the shallow dip from the conflicting Chase et al. data. It is to be noted that absorption model calculations ^{5),12)} have not given satisfactory descriptions of the $\pi^- p \rightarrow n \pi^0$ data. Our predictions for $\pi^+ p$ polarization lie near -70 % over most of the u range. It will be interesting to have data on the reaction $\bar{p} p \rightarrow \pi^+ \pi^-$ for $s > 10 \text{ GeV}^2$ since this may be predicted from both our models and so will provide further tests.

The residues at the N and Δ poles are given in the Table for both of our fits. These are predictions in the sense that they are not imposed on the fits and so the agreement within a factor 2 for the pure pole fit is acceptable. In a Veneziano model the scale of s is $1/\alpha' \approx 1 \text{ GeV}^2$ which would imply that our parameter c should satisfy $|c| \ll 1$. In the pole fit c_Δ violates this condition and confirms the observation ¹¹⁾ that the relation between the Δ residue and the magnitude of the backward $\pi^- p$ data is extremely difficult to accommodate in a Veneziano model. On the other hand the nucleon residue is more reasonable and can be incorporated in a Veneziano approach.

For the fit including kinematic cuts there is a marked improvement in the Δ residue function, which has a small c_Δ and an excellent prediction for the Δ width. The zero in the residue function at $u = -a_\Delta/b_\Delta \approx -0.5$ is not inevitable and tolerable fits can be obtained with $b_\Delta = 0$. However, in this fit the N residue is less reasonable. This can be understood by recalling that the deep dip at $u \approx -0.2$ in the $\pi^+p \rightarrow p\pi^+$ cross-section arises in the pole model from the signature zero in the N residue at $\alpha_N = -\frac{1}{2}$. When the kinematic cut is included, the amplitude B_N is no longer zero here, but is required by the data to be small. Since a_N is determined from the $u = 0$ data, this is arranged by the large value of c_N and the superfluous zero at $u = -a_N/b_N \approx -0.1$. The large prediction for $g_N^2/4\pi$ is a consequence of this artifice. We attempted fits with $b_N = 0$ and with $g_N^2/4\pi$ constrained but they result in dips in $\pi^+p \rightarrow p\pi^+$ which are much too shallow at lower energies. With a reasonable N residue function these fits show that the cut discontinuity is too large near the branch point in Eq. (5). Unitarity requires a non-singular discontinuity at the branch point and a modification ²⁾ of the formalism to accommodate this should yield a much more consistent scheme ^{*)}. Such an approach would lead to results rather similar to the present fit which has an artificially reduced contribution to B_N . Thus the predictions of our model are relevant and we understand the steps necessary to achieve a satisfactory description of the N pole residue.

Our conclusions are that the inclusion of kinematic cuts (i) improves the over-all fit to the high energy backward $\pi^\pm p$ elastic cross-section data and leads to particularly good predictions for the charge-exchange cross-section, (ii) gives a satisfactory Δ residue function with an excellent prediction for the Δ width, whereas the N residue is unreasonable. Modifications of the simple formulation such as including unitarity constraints should improve the description of the N residue.

^{*)} Including an additional $\sqrt{-\lambda}$ in the residue achieves this simply but the cut then occurs in $A + m_N B$ only. Such a cut is completely unable to reproduce the required dip in the $\pi^+p \rightarrow p\pi^+$ data at $u \sim -0.2$. A less drastic modification of the cut in B is needed.

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	Regge poles		Regge poles + cuts	
	N	Δ	N	Δ
a	124.0	- 4.32	132	- 4.6
b	- 67.5	- 5.96	985	- 9.9
c	0.804	1.50	2.65	0.74
α_0	- 0.39	0.22	- 0.40	0.18
α'	0.9	0.9	0.9	0.9
$g_N^2/4\pi; \Gamma_\Delta$	8.2	240 MeV	887	105 MeV

- Table -

Parameters of fits and pole residue predictions

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FIGURE CAPTIONS

Figure 1 Differential cross-sections, in mb/GeV^2 for $\pi^- p \rightarrow p \pi^-$,
 $\pi^+ p \rightarrow p \pi^+$ and $\pi^- p \rightarrow n \pi^0$ versus u in GeV^2 . The
data are taken from Refs. 8) and 9). The solid curve re-
presents the N and Δ Regge pole fit and the dashed
curve corresponds to the fit including the kinematic cuts.
The parameters are given in the Table ; α' was fixed at 0.9.

FIG.1