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CROSSING AND ISOSPIN SYMMETRIC DESCRIPTION OF THE
 $\bar{K}N \pi \pi \bar{\Lambda}$ SYSTEM USING THE GENERALIZED VENEZIANO MODEL

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A B S T R A C T

The Veneziano five-point function model, applied previously to $K^-p \rightarrow \pi^- \pi^+ \Lambda$, is used to describe four other reactions which are related to $K^-p \rightarrow \pi^- \pi^+ \Lambda$ by crossing and isospin symmetry. It is found that with no new parameters a global description of the five different processes can be obtained.

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1. INTRODUCTION

The successful application of the multi-Veneziano model to different reactions related by crossing symmetry ($K^+p \rightarrow K^0 \pi^+ p$, $K^-p \rightarrow \bar{K}^0 \pi^- p$, $\pi^- p \rightarrow K^- K^0 p$)¹⁾ motivates a further study of the crossed reactions of the process $K^-p \rightarrow \pi^- \pi^+ \Lambda$, to which the multi-Veneziano model was first applied²⁾. A step in this direction was already taken in Ref.³⁾ where the reaction $\pi^+ p \rightarrow K^+ Y^{*+}$ was studied.

In this work we complete the study of the reactions of the system $\bar{K} N \pi \pi \bar{\Lambda}$ (considering all particles as incoming) for which data are available. By considering different charge configurations one can distinguish three classes in which the different reactions are related by crossing or charge symmetry:

$$\text{I} \quad K^- p \pi^- \pi^+ \bar{\Lambda}$$

$$\text{II} \quad K^- n \pi^+ \pi^0 \bar{\Lambda}$$

$$\text{III} \quad K^- p \pi^0 \pi^0 \bar{\Lambda}$$

The reactions for which data are available are

$$\text{Class I} \quad K^- p \rightarrow \pi^- \pi^+ \Lambda \quad (1)$$

$$\pi^+ p \rightarrow \pi^+ K^+ \Lambda \quad (2)$$

$$\pi^- n \rightarrow \pi^- K^0 \Lambda \quad (2')$$

$$\pi^- p \rightarrow \pi^- K^+ \Lambda \quad (3)$$

$$\pi^+ n \rightarrow \pi^+ K^0 \Lambda \quad (3')$$

$$\text{Class II} \quad K^- n \rightarrow \pi^- \pi^0 \Lambda \quad (4)$$

$$\pi^- p \rightarrow \pi^0 K^0 \Lambda \quad (5)$$

$$\pi^+ n \rightarrow \pi^0 K^+ \Lambda \quad (5')$$

Due to the isosinglet Λ there are only two isospin amplitudes in all reactions. From the amplitude of (1) one can predict the amplitudes for all the other processes (2) - (5) using crossing symmetry and isospin invariance. This provides ample tests of the crossing and isospin symmetry properties of the original model used in Ref. ²⁾.

2. THE MODEL

a) Class I, $K^- p \pi^+ \pi^- \bar{\Lambda}$

The amplitude used in this paper is the same as in Ref. ³⁾ with no new parameters introduced for the crossed reactions.

The amplitude for the reactions in Class I is

$$\mathcal{A}_I = \beta \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma ((A) + (B) + (F)) \quad (6)$$

where (A), (B) and (F) denote Bardakçi-Ruegg functions ⁴⁾ (hereafter called B_5 functions) corresponding to the graphs A, B and F in Fig. 1. In this figure we also show the other three graphs allowed, i.e., which do not contain the exotic two-body subsystem $(\pi^+ K^+)$.

To limit the number of functions, i.e. to have at most one graph of each type A-F, we choose in each subsystem a "dominating" trajectory. In Fig. 1 these are denoted by their first member. The trajectories are taken to be of the form

$$\alpha(s) = \alpha_0 + \alpha' s + i \theta(s-s_0) \text{Im} \alpha(s)$$

The imaginary part of α serves to remove the poles from the real axis, and gives the resonances a Breit-Wigner shape. The imaginary parts are determined from the experimental total widths of the resonances. They are taken to be increasing functions of s corresponding to total widths which increase with mass. The choice of trajectories, graphs and kinematic factor is extensively motivated in Refs. ²⁾ and ³⁾ (cf. also Ref. ¹⁾). We summarize here the more important points:

- i) For the dominant trajectory in the $\bar{\Lambda} p$ system we choose the K^* trajectory based upon the experimental information from the subprocesses $K^- p \rightarrow \rho^0 \Lambda$ ⁵⁾ and $\pi^- p \rightarrow K^{*0} \Lambda$ ⁶⁾. In both processes an analysis of the decay matrix element gives $1 - \rho_{00} \approx 3/4$ which indicates that vector meson exchange dominates.
- ii) The choice of the baryon trajectories is somewhat ambiguous. For example, the trajectory in the $K^- p$ system is chosen to be the Λ trajectory. This is in accordance with the fact that the Λ recurrences dominate the low energy part of the process $K^- p \rightarrow \pi^- \pi^+ \Lambda$ ³⁾. More generally it has been shown by Schmid ⁷⁾ that this trajectory couples strongest to the $K^- p$ system. However, in Ref. ²⁾ the Y_1^* trajectory was used instead. Nevertheless, the predictions for the reaction $K^- p \rightarrow \pi^- \pi^+ \Lambda$ given there will not be much changed with the Λ trajectory because for the experiments discussed the energy of the $K^- p$ system is above the resonance region.
- iii) The qualitative consequences of our choice of graphs are:
 - a) an exact non-exchange degeneracy of the trajectory in the $\pi \Lambda$ system, i.e., $Y_1^*(1765)$ is supposed not to be produced in these reactions.
 - b) strong exchange degeneracy of the $K^*(890) - K^{*}(1420)$ trajectory in the $\bar{\Lambda} p$ system, leading, e.g., to the

absence of a signature zero at $t \approx -0.3$ in the subprocesses $K^-p \rightarrow \rho^0 \Lambda$ and $\pi^-p \rightarrow K^{*0} \Lambda$. These properties in turn also determine the amplitude. A further consequence is the absence of resonances in the π^+p system. This means that there is no backward peak in $K^-p \rightarrow \pi^- Y^{*+}$. As far as the above predictions can be tested they are in agreement with experimental data ^{3),6),5)}. The consequences of another choice of graphs in accordance with the model of Harari and Rosner will be discussed below.

- iv) The kinematic factor used here neglects the baryon spin. We expect this not to be serious because
- a) the proton and Λ spins are summed over in the distributions considered and
 - b) baryon exchange, where baryon spin might be important, is experimentally small.

To ensure correct Regge behaviour for baryon exchange we shift the intercept of the baryon trajectory below threshold. Without such a shift the asymptotic behaviour of the cross-section for fixed u would be

$$A \approx \epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma f(u) s^{\alpha(u)-J}$$

$$\approx g(u) s^{\alpha(u)-J+1} \neq g(u) s^{\alpha(u)} \text{ if } J \neq 1$$

(J is the spin of the first particle on the exchanged trajectory).

- b) Class II, $K^-n \pi^+ \pi^0 \bar{\Lambda}$

The isospin structure of the $\bar{K}N \pi \pi \bar{\Lambda}$ system is particularly simple because the Λ has isospin zero. There are only two isospin amplitudes, and by permuting the two pions we can obtain both of them, once we know the amplitude for the Class I reactions. Thus the isospin one (zero) component of the reaction $K^-p \rightarrow \pi^- \pi^+ \Lambda$ is given by the amplitude antisymmetrized (symmetrized) with respect to the two pions.

The amplitudes for the Classes II and III are

$$\mathcal{A}_{\text{II}} = \frac{1}{\sqrt{2}} \mathcal{A}(\tau_{\bar{K}N} = 1) = \frac{1}{\sqrt{2}} (\mathcal{A}_{\text{I}} - \mathcal{A}_{\text{I}}(\pi^+ \leftrightarrow \pi^-)) \quad (7)$$

$$\mathcal{A}_{\text{III}} = \frac{1}{\sqrt{6}} \mathcal{A}(\tau_{\bar{K}N} = 0) = -\frac{1}{2} (\mathcal{A}_{\text{I}} + \mathcal{A}_{\text{I}}(\pi^+ \leftrightarrow \pi^-)) \quad (8)$$

where $\mathcal{A}_{\text{I}}(\pi^+ \leftrightarrow \pi^-)$ represents the amplitude obtained from $\mathcal{A}_{\text{I}}$ when the momenta of the pions have been permuted. In the following we shall denote this operation by a prime. Inserting (6) in (7) we get

$$\mathcal{A}_{\text{II}} = \frac{\beta}{\sqrt{2}} \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma \left((A) + (B) + (F) + (A') + (B') + (F') \right) \quad (9)$$

The minus sign in (7) has been cancelled by the antisymmetric kinematic factor. We note

- i) in Group II no exotic states can be formed. The six graphs which were excluded for the Group I reactions are therefore allowed in Group II.
- ii) in our model all the meson resonances have correct isospin. Thus the πK system only resonates when it is in an isospin $\frac{1}{2}$ state, whereas the isospin of the ρ -f trajectory alternates.
- iii) the trajectory in the $\bar{K}N$ system must be a degenerate $T = 0$, $T = 1$ trajectory. In (7) the $\bar{K}N$ system has $T = 1$ while in (8) it has $T = 0$. This is an example of a well-known feature of Veneziano models, namely that one obtains isospin and SU_3 degenerate trajectories ⁸⁾.

If we allow resonances in exotic channels we can avoid the isospin degeneracy of the $\bar{K}N$ trajectory. We can put an isospin 1 trajectory Σ in the $\bar{K}N$ system in (7) and the Λ (now

non-degenerate with the Σ in (8). Then the amplitude for Class I is

$$\begin{aligned} \mathcal{A}_I &= -\frac{1}{2} \mathcal{A}(\tau_{\bar{K}N} = 1) - \frac{1}{\sqrt{6}} \mathcal{A}(\tau_{\bar{K}N} = 0) \\ &= \frac{1}{2} \left[\mathcal{A}_I^\Lambda + \mathcal{A}_I^\Sigma + \mathcal{A}_I^\Lambda (\pi^+ \leftrightarrow \pi^-) - \mathcal{A}_I^\Sigma (\pi^+ \leftrightarrow \pi^-) \right] \end{aligned} \quad (10)$$

where the upper index indicates whether the Λ or the Σ trajectory has been used in the original amplitude (6).

If $\alpha_\Lambda \neq \alpha_\Sigma$ the third and the fourth terms in (10) do not cancel and will give rise to exotic $\pi^+ K^+$ resonances in the diagrams A and B. However, since the slopes are equal the leading terms cancel and the resonances will occur only at the daughter level. Thus no prominent bumps in the πK system of the reaction $\pi^+ p \rightarrow \pi^+ K^+ \Lambda$ are expected.

The main phenomenological consequence of the modified amplitude (10) will be at low energies in $K^- p \rightarrow \pi^- \pi^+ \Lambda$, where both Λ and Σ resonance structures will appear. Of course, in order to reproduce the whole resonance structure one must also include the Y_1^* trajectory.

In calculating the predictions of our model we have used the Monte-Carlo program FOWL to generate events, and evaluate the B_5 function by means of the program written by Hopkinson⁹⁾.

3. COMPARISON WITH EXPERIMENT

a) Total cross-sections

Once the model is determined, all predictions are absolute, except for the normalization constant. This parameter we fix from the value of the total cross-section of $K^- p \rightarrow \pi^- \pi^+ \Lambda$ at

5.5 GeV/c⁵). We get

$$\beta = 1.1 \times 10^3 \text{ GeV}^{-5}$$

In Ref. ²⁾ it is shown that the energy dependence of the total cross-section for this reaction is correct. We now obtain

$$\frac{\sigma_t(K^-n \rightarrow \pi^- \pi^0 \Lambda)}{\sigma_t(K^-p \rightarrow \pi^- \pi^+ \Lambda)} = 1.3 \quad \text{at } 3 \text{ GeV}/c$$

compared to the experimental value 1.5 ± 0.2 ¹⁰⁾. To see the significance of this prediction we compare it with what follows only from isospin requirements. Generally,

$$\sigma_t(K^-n \rightarrow \pi^- \pi^0 \Lambda) \leq 2 \sigma_t(K^-p \rightarrow \pi^+ \pi^- \Lambda)$$

For the dominating resonances

$$\sigma(K^-n \rightarrow \rho^- \Lambda) = 2 \sigma(K^-p \rightarrow \rho^0 \Lambda)$$

and

$$\sigma(K^-n \rightarrow f \Lambda) = 0.$$

With the additional assumption that the $Y^*(1385)$ resonance is produced via isospin $\frac{1}{2}$ exchange in the t channel

$$\sigma(K^-n \rightarrow \pi^- Y^{*0}) = \frac{1}{2} \sigma(K^-p \rightarrow \pi^- Y^{*+})$$

These relations are built into the model. More model-dependent predictions, e.g., the relative strength of the ρ , f and Y^* resonances are, however, necessary for the obtained value.

Comparing the cross-sections for the processes $K^-p \rightarrow \pi^- \pi^+ \Lambda$ and $\pi^- p \rightarrow \pi^- K^+ \Lambda$ we find that our model predicts them to be equal. Experimentally⁶⁾ the cross-section for $\pi^- p \rightarrow \pi^- K^+ \Lambda$ is smaller by a factor 2 to 3. We note that the same factor is present in the crossing predictions of Ref.¹⁾. In Fig. 2 we have "renormalized" our predictions for the processes (3) and (5) to the experimental value of the total cross-section for $\pi^- p \rightarrow \pi^- K^+ \Lambda$ at 4.2 GeV/c. When this is done we find good agreement for the energy dependence of reactions (3) and (5). Furthermore, we obtain a correct prediction of the ratio of the cross-sections for the isospin related reactions (3) and (5).

b) Effective mass distributions

In Figs. 3-5 we give the effective mass distributions for the reactions (3), (4) and (5). It should be noted that the relative strength between the resonances on a given trajectory depends on the detailed structure of the B_5 functions. E.g., they could be changed by adding satellite terms.

The shape of the meson mass spectra is not very sensitive to our specific choice of graphs; the graphs A and C give about the same predictions. The production ratio of competing resonances, however, is sensitive to this choice. Therefore, the correctly predicted ratios of $Y^{*+}(1385)/\rho$ in $K^-p \rightarrow \pi^- \pi^+ \Lambda$ and Y^{*0}/K^* in $\pi^- p \rightarrow \pi^0 K^0 \Lambda$ support our choice.

The $Y^{*-}(1385)$ resonance can be produced in reactions (1) and (3) only by baryon exchange since exotic exchanges have been excluded. The predicted cross-section has the correct energy dependence but is too small in normalization for $\pi^- p \rightarrow K^+ Y^{*-}$. This discrepancy is not surprising since the baryon exchange has been described in only a crude way.

The enhancement in the low energy part of the ΛK^+ mass spectrum in $\pi^- p \rightarrow \pi^- K^+ \Lambda$ (Fig. 4) may be due to the diffraction dissociation $\pi N \rightarrow \pi N(1700)$, which is not included in our model. The effect of Pomeron exchange in the reaction $\pi^+ p \rightarrow \pi^+ K^+ \Lambda$ can also be seen in Fig. 6, where we plot the longitudinal momentum distributions of the π^+ and the K^+ at 8 GeV/c¹¹⁾. The Pomeron should contribute mainly to the fast forward π^+ mesons, with the K^+ being slow or moving backwards. From the discrepancy of our predictions with the data, however, the Pomeron only seems to be a 10-20 % effect even at this high energy.

c) Resonances

In comparing our predictions for the production and decay angular distributions of a resonance with experiment it is useful to remember that near a resonance the B_5 function can be approximated by the pole term. Thus for the first resonance on the trajectory z

$$B_5(z \approx 0) \approx B_4(u, v) \frac{1}{z} \quad (11)$$

where u, v are trajectories non-dual to z . The production mechanism depends on the B_4 function and the kinematic factor, whereas the decay angular distributions only depend on the kinematic factor.

In Fig. 7a we compare the production angular distribution of the K^{*0} in reactions (3) and (5) with experiment. It can be seen that due to the high intercept of the exchanged Y_1^* trajectory our model predicts a backward peak which persists at high energies. This is not in agreement with the data. The inclusion of the $\Sigma(1190)$ trajectory in the $\pi^+ \Lambda$ system would give a lower effective intercept.

The decay angular distributions of the K^{*0} are shown in Figs. 7b,c. The general agreement with the data shows that K^* exchange is dominant. The discrepancies may be due to K exchange.

In Fig. 8¹²⁾ we give the prediction for the total (forward) cross-section and the slope parameter of the $Y_1^{*+}(1385)$ in the reactions $K^-p \rightarrow \pi^- Y^{*+}$ and $\pi^+p \rightarrow K^+ Y^{*+}$. The slope parameter is in good agreement with the data. The cross-sections are, however, predicted to be equal for the two processes. This is due to the exchange degeneracy of the K^* trajectory. Experimentally the π^- induced reaction seems to have a consistently lower cross-section. The dashed line in Fig. 8a shows the prediction when we "renormalize" to the experimental value of $\sigma_{\text{tot}}(\pi^-p \rightarrow \pi^- K^+ \Lambda)$ at 4.2 GeV/c, as in Fig. 2.

4. COMMENTS

- i) We emphasize that although the amplitude (6) does not have exotic states in the reactions which we have studied, the graphs A and B in (6) imply such states through an SU_3 rotation. We have chosen the amplitude (6) because it seems to give better over-all agreement with the data than an amplitude based on the allowed graphs C, D, E and F. In particular, one notices the following disagreements when one uses an amplitude which is allowed by the duality rules.
- a) Experimentally there is no evidence for direct channel Δ resonances in $\pi^+p \rightarrow K^+ Y^{*+}(1385)$. This observation is based on the smooth energy dependence of the cross-section and on the structureless distribution in the scattering angle (cf. Ref. ³⁾). In any model based on the Harari-Rosner graphs one must include Δ poles, and requiring that the Δ trajectory is non-exchange degenerate one gets strongly oscillating distributions in the scattering angle. This prediction follows from

the signature of the Δ trajectory and is independent of the exact form of the amplitude and the Δ intercept. The amplitude (6) contains no Δ resonances and agrees well with the data.

- b) Regardless of any specific model, the Harari-Rosner rules predict that the $Y_1^*(1385)$ is exchange degenerate in our reactions (since permuting the π and the Λ leads to a forbidden graph). Yet the $Y_1^*(1765)$ is not seen in any of our processes, the bump around 1.7 GeV in the $M(\pi^+\Lambda)$ distribution of $K^-p \rightarrow \pi^-\pi^+\Lambda$ generally being interpreted ¹³⁾ as the lower spin resonance $\Sigma(1670)$.
- c) Experimentally one sees strong direct channel resonances (Λ , Σ , Y_1^*) in reaction (1) at low energies. This is what one would expect from the amplitude (6) where all the graphs have resonances in the K^-p system. Since the diagrams C and D do not have such resonances there would be a considerable smooth background in a model based on the graphs C to F. Before quantitative comparisons with experiment can be made in the low energy region our model must, however, be refined.
- d) At least in the framework of our assumptions the general quality of the fits to the data decreases when we use only the graphs allowed by the Harari-Rosner rules. We have compared the predictions of our amplitude (6) with the amplitude

$$\begin{aligned} \mathcal{A}_I^{H-\Lambda} = & \beta' \epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma \times \\ & \times ((C) + (D) + (E) + (F)) \end{aligned} \quad (12)$$

Due to the high intercept of the Δ trajectory the contribution from Δ exchange is overestimated by the

Veneziano model. This gives rise to too large cross-sections and backward peaks which are not experimentally seen. We have for the alternative model (12) used a makeshift method to suppress Δ exchange by lowering the intercept of the Δ trajectory by $\frac{1}{2}$.

As an example of the results we compare the predictions of (6) (solid line) and (12) (dashed line) for the $\pi^+\Lambda$ and $\pi^+\pi^-$ effective mass distributions of the reaction $K^-p \rightarrow \pi^-\pi^+\Lambda$ at 5.5 GeV/c in Fig. 9. The situation for the other processes is similar in that the predictions of (12) definitely are not better than those of (6). By varying the weights of the different graphs in (12) one can obtain a slight improvement in reaction (1), but the agreement becomes worse in the crossed channels.

Finally we note that although there are many arguments against the Harari-Rosner rules for our set of reactions, most of our tests are really model-dependent. Thus one cannot exclude the possibility that the duality rules are correct and the discrepancies arise from the model.

- ii) When crossing a pion with a kaon to get from reaction (1) to (3) we observe that the π induced reaction is predicted to be too high. This is also the case for the subprocesses $K^-p \rightarrow \pi^-Y^{*+}$ and $\pi^+p \rightarrow K^+Y^{*+}$, which are related by line reversal (Fig. 7). In two-body reactions it is known¹⁴⁾ that models which have an exchange degenerate K^*-K^{*+} or $\rho-\rho$ trajectory in the t channel cannot (without absorption corrections) account for the experimentally unequal cross-sections of line-reversed processes. This same feature is present also in three-body reactions.
- iii) The model as discussed here and in Refs. 2), 3) gives in principle a definite form for the amplitude and leaves fairly little freedom. It should be stressed, however, that several assumptions have been made which actually limit the applicability of the model to the domain discussed above. We note in

particular the following limitations:

- a) the approximate way in which we have treated the baryon spin and the baryon trajectories;
- b) the neglect of pseudo-scalar exchange;
- c) the "dominant trajectory" hypothesis - we use only one trajectory in each channel;
- d) the unwanted isospin degeneracy of the Λ and the Σ trajectories.

If one wants to apply the model at lower energies and for backward scattering modifications may be needed. Some of the difficulties can easily be avoided at the expense of the simplicity of the model - e.g., the $\Lambda - \Sigma$ degeneracy can be broken by introducing small contributions from exotic states (see Section 2). Particularly for reaction (2) the modifications seem to be less important and our model gives a good qualitative description almost down to threshold³⁾. This is due to the fact that the process is dominated by $Y^{*+}(1385)$ production, where O^- exchange is forbidden. Also there are no direct channel resonances and the isospin degeneracy is not important since it occurs in reaction (2) only for a low-lying exchanged baryon trajectory.

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FIGURE CAPTIONS

Figure 1: Diagrams for the B_5 functions allowed for the reactions of Class I. All particles are taken as incoming. The label between each pair of adjacent lines represents the trajectory assumed to be dominant in that channel.

Figure 2: The total cross-sections for reactions (3) and (5), $\pi^- p \rightarrow K^+ \pi^- \Lambda$ and $\pi^- p \rightarrow K^0 \pi^0 \Lambda$ ⁶⁾. The predictions are normalized to the $\pi^- p \rightarrow K^+ \pi^- \Lambda$ experiment at 4.2 GeV/c.

Figure 3: The effective mass distributions in percentages for reaction (4) $K^- n \rightarrow \pi^- \pi^0 \Lambda$ at 3 GeV/c ¹⁰⁾. Note that the $(\pi^- \Lambda)$ and $(\pi^0 \Lambda)$ distributions should be equal by isospin symmetry.

Figure 4: The effective mass distributions in percentages for reaction (3) at 4 and 6 GeV/c ⁶⁾.

Figure 5: The effective mass distributions in percentages for reaction (5) at 4 and 6 GeV/c ⁶⁾.

Figure 6: The c.m. longitudinal momentum distributions in percentages for reaction (2) at 8 GeV/c ¹¹⁾.

Figure 7: The production and decay angular distributions in percentages for the $K^*(890)$ resonances in reaction (3) at 4 GeV/c ⁶⁾:

- a) the production angle;
- b) the Jackson angle;

c) the Treiman-Yang angle.

The Jackson angle θ and the Treiman-Yang angle ϕ are the polar co-ordinates of the outgoing K^+ in the $(K\pi)$ rest frame, where the z axis is the direction of the incoming π^- in this frame, and the y axis is the normal to the production plane.

- Figure 8:
- a) The total cross-sections for the reactions $K^-p \rightarrow \pi^- Y^* (\phi)$ and $\pi^+p \rightarrow K^+ Y^{*+} (\phi)^{12)}$. The solid line is the prediction normalized to the total cross-section of $K^-p \rightarrow \pi^- \pi^+ \Lambda$ at 5.5 GeV/c ⁵⁾. The dashed line shows the same prediction, but normalized to the total cross-section of $\pi^-p \rightarrow K^+ \pi^- \Lambda$ at 4.2 GeV/c ⁶⁾.
 - b) The slope parameter B , resulting from a fit of the form $(d\sigma/dt) = Ae^{Bt}$. The solid line is the prediction for $K^-p \rightarrow \pi^- Y^{*+}$ and the dashed line for $\pi^+p \rightarrow K^+ Y^{*+}$ ¹²⁾.

- Figure 9: Mass distributions for reaction (1) at 5.5 GeV/c ⁵⁾. The solid line shows the predictions of the model described in Section 2 of this paper; the dashed line shows the predictions resulting from a use of the amplitude given in formula (12).

FIG.2

$K^- n \rightarrow \pi^- \pi^0 \Lambda$ 3 GeV/c (1951 events)

FIG.3

FIG. 4

$\pi^- p \rightarrow \pi^0 K^0 \Lambda$

4 GeV/c (72 events)

6 GeV/c (99 events)

FIG.5

FIG. 6

$\pi^- p \rightarrow K^{*0} \Lambda$ 4 GeV/c

a)

b)

FIG.8

FIG.9