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MESON SCATTERING AS A PROBE FOR DEFORMED NUCLEI

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A B S T R A C T

Meson absorption and charge exchange scattering on oriented deformed nuclei should provide most valuable information on proton and neutron densities. Effects of the order of 10% or more could be obtained at presently accessible temperatures.

1. INTRODUCTION

High energy mesons are interesting probes for the study of the shape and outer structure of large nuclei. Combining elastic and charge exchange scattering, one may further analyze how protons and neutrons may separately contribute to them. The Glauber theory ¹⁾ is sufficiently accurate for small angle high energy scattering, that the observed cross-sections can be readily interpreted in terms of proton and neutron densities inside the probed nucleus. The purpose of this note is to study the very interesting case of deformed nuclei which may be at present relatively easily oriented. They then offer different cross-sections to an incoming beam when the polarization or alignment axis is changed with respect to the direction of the beam. One may thus answer most naïve but yet unsettled questions such as :

- i) if a large quadrupole moment, or, more generally, electron scattering data and at present μ mesic atom data give evidence for a deformed charge distribution, does the neutron distribution follow the proton distribution or not ?
- ii) does the observed anisotropy in the charge distribution correspond mainly to a deformation of an outer shell around a spherically symmetric core, or does the deformation affect all the protons ? In both cases again what are the main features of the neutron distribution ?

We show how such questions could be answered studying forward elastic and charge exchange scattering of pions off oriented deformed nuclei. Our analysis is general but, in order to give specific figures, we emphasize the particular case of $^{165}_{67}\text{Ho}$ which, with a quadrupole moment of 2.82 barn, a spin 7/2 and an internal magnetic field at the nucleus of 7×10^6 oersted, is a priori a very good candidate for such a study. Shape effects soon disappear when the temperature increases above 1° . Nevertheless, for a target temperature of 0.1° , which is by now not very difficult to reach, effects of the order of 15% could be observed. Such polarization effects have already been used in neutron scattering at low energy ^{2),3)}. It has been long recognized that a deformed nucleus should cast different shadows when probed by a beam of high energy hadrons for

which it is practically opaque ⁴⁾. We here present the result of a slightly more involved calculation where the Glauber theory is used in order to compute cross-sections resulting from different extreme distributions. Three cases are considered. In case 1, the neutrons and the protons are equally deformed. In case 2, the protons are deformed as in case 1, thus giving the same electron scattering parameters, but the neutrons keep a spherical distribution. In case 3, an outer shell grouping half of the nucleons suffers a stronger deformation in order to give still the same value for the quadrupole moment. The other nucleons remain in a spherical core. In all three cases, we give the expected cross-sections when the polarization is along the incident beam or perpendicular to it.

2. FORWARD SCATTERING OFF A DEFORMED NUCLEUS

We limit ourselves to the forward scattering of high energy mesons. Even though the nucleus rotates the collision time is much shorter than the rotation time. As a result an incident pion or kaon will interact as if the nucleus had a fixed direction in space. The orientation of the axis of symmetry of the nucleus with respect to the incident beam is specified by two angles : θ and ϕ (Fig.1). The shape of the nucleus is approximated by an ellipsoidal Saxon-Woods distribution. The ratio of the longitudinal and transversal axis defines a deformation parameter d with

$$\left(\frac{R_l}{R_t}\right)^2 = 1 + d \quad (1)$$

The nucleon density is then given by

$$\rho(\vec{r}) = n \left(1 + \exp(R-c)/a \right)^{-1} \quad (2)$$

with

$$R = \left(x^2 + y^2 + (1+d) z^2 \right)^{1/2}$$

We take n such that

$$\int \rho(\vec{r}) d^3\vec{r} = 1 \quad (3)$$

the nucleon density is $A\rho$, with A the mass number of the nucleus. The parameters c and a characterize the size and surface thickness of the nucleus.

Relation (2) describes a nucleus which is deformed as a whole. It is readily extended to a sum of terms in order to describe different neutron and proton distributions, or the superposition of two structure such as a spherical core and an outer ellipsoid shell.

The cross-sections of interest, namely the total inelastic cross-section, giving the forward beam intensity observed behind the sample, and the charge exchange cross-sections can be obtained in terms of the effective nucleon numbers ⁵⁾ introduced in the Glauber theory ¹⁾. They are defined as

$$N(\theta) = \int \rho_0(x, y, z) e^{-\sigma \int \rho dz'} dx dy dz \quad (4)$$

where ρ_0 stand for the nucleon, proton and neutron density. This effective number, which can be calculated from (2) and (4) is a function of the distribution parameters already introduced, the total cross-section σ and the orientation of the nucleus with respect to the incident beam. In effect, it is enough to specify the angle θ between the axis of symmetry and the beam direction (Fig.1).

Relation (4) assumes that a unique meson-nucleon cross-section is considered even for charge exchange scattering. It also assumes that the different nucleons in the nucleus scatter incoherently. We have in mind forward scattering of mesons of a few GeV and these assumptions are highly justified ¹⁾. The final meson should be detected with a lower bound on the transfer, $t \gg 5 \times 10^{-2} \text{ GeV}^2/c$ say, to avoid any coherent effects and the resolution in energy should be good enough (100 MeV, say) so that the two-body character of the reaction could be ascertained. It should not be too good, though, so that the closure property could be used as implied

in (4). Needless to say, the study of coherent effects would be highly interesting. They are yet outside the scope of the first exploration here proposed.

The transmission of the sample is sensitive to the nucleon density. The $\pi^+ \pi^0$ charge exchange reaction is sensitive to the neutron density, while the $\pi^- \pi^0$ charge exchange reaction is sensitive to the proton density. K meson scattering could also be used ⁵⁾.

Various models can be tried for the nucleon distribution but they are constrained to be in agreement with the electron scattering data and the value of the quadrupole moment which give information on the proton density.

The moments of the charge distribution which are determined by the electron scattering data can be expressed in terms of integrals involving only c/a as defined in (2), Namely :

$$\begin{aligned} \langle r^2 \rangle &= 4\pi n \frac{3+d}{3(1+d)^{3/2}} a^5 \int_0^\infty \left(1 + \exp\left(x - \frac{c}{a}\right)\right)^{-1} x^4 dx \\ \langle r^4 \rangle &= 4\pi n \frac{8d^2 + 20d + 15}{15(1+d)^{5/2}} a^7 \int_0^\infty \left(1 + \exp\left(x - \frac{c}{a}\right)\right)^{-1} x^6 dx \end{aligned} \quad (5)$$

with

$$n^{-1} = \frac{4\pi}{(1+d)^{1/2}} a^3 \int_0^\infty \left(1 + \exp\left(x - \frac{c}{a}\right)\right)^{-1} x^2 dx$$

This yields a value for the nuclear radius and surface thickness which one would obtain for a spherical object. In the particular case of $^{165}_{67}\text{Ho}$, one thus obtains $c = 1.08 A^{1/3} = 5.9$ fermi and $a = 0.55$ fermi ⁶⁾. The spectroscopic value of the quadrupole moment $Q = 2.82$ barn leads to an intrinsic quadrupole moment

$$Q_0 = \frac{(I+1)(2I+3)Q}{I(2I-1)} = 6.04 \text{ barn}$$

using the spin value $I = 7/2$. This gives an average over the proton density

$$\langle 3z^2 - r^2 \rangle = \frac{Q_0}{2} = 9.03 \cdot 10^{-26} \text{ cm}^2$$

Our next step is to determine values for a and c as a function of d such that for any particular choice of d the same values would be obtained for $\langle r^2 \rangle$ and $\langle r^4 \rangle$. If all protons participate in the deformation, the quadrupole moment is given by

$$\frac{Q_0}{2} = 4\pi n \frac{3-2d}{3(1+d)^{3/2}} a^5 \int_0^\infty \left(1 + \exp\left(x - \frac{c}{a}\right)\right) x^4 dx \quad (6)$$

The deformation parameter d being essentially determined by the value of the quadrupole moment, Eq. (5) can be inverted numerically to give the required values for n , a and c . These values are then used in (2) in order to calculate the effective proton number. The effective neutron number is obtained in the same way, assuming that the neutron distribution follows the proton distribution or remains spherical, limiting our analysis to these two rather extreme cases. We further consider the case where the proton and neutron distributions are the same but that only half of the proton build up the quadrupole moment while the rest retains a spherical shape.

In order to illustrate the type of results obtained at this stage, we give in Fig. 2 the effective nucleon number as a function of the orientation angle θ , as it is obtained when the protons and the neutrons have the same distribution and when the neutrons are spherically distributed.

3. CROSS-SECTIONS FOR TRANSVERSE AND LONGITUDINAL POLARIZATIONS

If the nucleus may be assumed to remain fixed in orientation during the meson impact, its wobbling has to be introduced before the effective nucleon number so far determined can be translated in terms of observed cross-sections. If the nucleus is oriented in the direction of the beam, the diagonal elements of the density matrix will be related to one another as :

$$\rho_m = \rho_{m+1} \exp\left(-\frac{\omega_L}{kT}\right) \quad (7)$$

where ω_L stands for the Larmor frequency.

The observed cross-section will then be given by :

$$\sigma_{\parallel} = 4\pi A \sum_m \rho_m \int N(\theta) \left(d_{7/2, m}^{7/2}(\theta)\right)^2 d\cos\theta \quad (8)$$

On the other hand, if the nucleus is oriented in a direction perpendicular to the beam, the observed cross-section will be :

$$\sigma_{\perp} = \frac{4\pi A}{2\pi} \sum_m \rho_m \int N(\theta) \left(d_{7/2, m}^{7/2}(\theta')\right)^2 d\cos\theta d\phi \quad (9)$$

with

$$\cos\theta' = \sin\theta \cos\phi \quad (10)$$

The $d_{m', m}^J$ functions needed for the calculation are given in Table I. The nucleus being in its ground state, the spin component along the axis of symmetry is taken equal to 7/2. Using the relation

$$d_{m', m}^J(\theta) = (-1)^{J+m'} d_{m'-m}^J(\theta)$$

these cross-sections are readily seen to depend only upon the alignment of the nucleus, as they obviously should.

The nucleus $^{165}_{67}\text{Ho}$ is a particularly interesting candidate. The magnetic field at the nucleus is 7×10^6 oested. This corresponds to a Larmor frequency of $10 \pi 10^9$ herz. As a result, the observed effect is almost maximum when the temperature is equal to 0.1° . The results obtained in the three cases considered are displayed in Table II. They correspond to a temperature of 0.05° . The variation of the

observed effects with temperature is illustrated in Fig.3. Temperatures as low as 0.1° being presently accessible for targets, the estimated effects are large enough to provide proper answers to the questions corresponding respectively to the three cases singled out here. The effect looked for further eliminates partly possible systematic errors. Our calculation assumes a complete magnetic orientation of the sample. This might be hard to achieve in practice but large enough orientations have already been achieved ^{2),3)} and variation of the observed effect as a function of the applied magnetic field could clear up this problem.

In the particular case of holmium, one could also use the helicoidal pattern of the internal magnetic field selecting special direction in the crystal ²⁾. Using a mono-crystal, one could avoid an external magnetic field and obtain alignment or full disorder by cooling the crystal or warming it up above 1° . The observed effect would of course be at most only about half of what is given in Table II, only the perpendicular alignment being accessible.

4. CONCLUSIONS

Recent progress in cryogenics makes such an analysis much easier than it might have seemed a few years ago. We have stressed that simple reactions such as pion total cross-sections and, as a next step, pion charge exchange in the few GeV range, would already provide extremely valuable information. Refinements along these lines would of course lead to many more detailed studies of nuclear shapes and of long range correlations among nucleons.

Besides pion and kaon charge exchange, one could also study K_S regeneration from a K_L beam, as a function of the nucleus orientation. This was proposed by Abragam ⁷⁾ as a first step towards an ingenious experiment where the precession of the deformed nuclei, tuned to a radio-frequency field, would induce a resonance transition between the two states.

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TABLE I - Nucleus wobbling wave functions for the different spin components.

$d_{7/2 \ 7/2}^{7/2}(\theta) = \frac{1}{8} (1+\cos\theta)^3 \cos\frac{\theta}{2}$	$d_{7/2-7/2}^{7/2}(\theta) = -\frac{1}{8} (1-\cos\theta)^3 \sin\frac{\theta}{2}$
$d_{7/2 \ 5/2}^{7/2}(\theta) = -\frac{\sqrt{7}}{8} (1+\cos\theta)^3 \sin\frac{\theta}{2}$	$d_{7/2-5/2}^{7/2}(\theta) = \frac{\sqrt{7}}{8} (1-\cos\theta)^3 \cos\frac{\theta}{2}$
$d_{7/2 \ 3/2}^{7/2}(\theta) = \frac{\sqrt{21}}{4} \sin^2\theta \cos^3\frac{\theta}{2}$	$d_{7/2-3/2}^{7/2}(\theta) = -\frac{\sqrt{21}}{4} \sin^2\theta \sin^3\frac{\theta}{2}$
$d_{7/2 \ 1/2}^{7/2}(\theta) = -\frac{\sqrt{35}}{8} \sin^3\theta \cos\frac{\theta}{2}$	$d_{7/2-1/2}^{7/2}(\theta) = \frac{\sqrt{35}}{8} \sin^3\theta \sin\frac{\theta}{2}$

TABLE II - Expected effect at $T=0.05^{\circ}$. The three cases considered are :

- 1) neutrons and protons equally deformed, $d=0.394$;
- 2) protons as in case 1). Neutrons spherical;
- 3) half of the nucleon in a strongly deformed structure, $d=0.60$ and the other in a spherical core.

The first column gives $\sigma_{\perp} / \sigma_{\parallel}$ for the total absorption, the second column gives $\sigma_{\perp} / \sigma_{\parallel}$ for the neutron sensitive reaction, i.e., $\pi^{+} \pi^{0}$ charge exchange. Two figures are given, the first one corresponds to a nucleon total cross-section of 39 mb, the second one to a nucleon total cross-section of 17 mb.

case	$(\sigma_{\perp} / \sigma_{\parallel})_{\text{total}}$	$(\sigma_{\perp} / \sigma_{\parallel})_{\text{neutron sensitive}}$
1	1.15 1.17	1.15 1.17
2	1.06 1.07	0.90 0.98
3	1.06 1.16	1.06 1.17

R E F E R E N C E S

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FIGURE CAPTIONS

Figure 1 : Orientation of a deformed nucleus with respect to an incident beam.

Figure 2 : Effective nucleon number as a function of the polar angle θ . The calculation is made with $\sigma = 17$ mb. Curve a) corresponds to identical proton and neutron deformation. Curve b) corresponds to a spherical neutron configuration.

Figure 3 : Variation of the observed effect with target temperature.

Nucleus Axis

FIG.1

FIG. 2

FIG.3