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ELECTRON-POSITRON ANNIHILATION AT HIGH ENERGY

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A B S T R A C T

The connection between the structure functions νW_2 and $\nu \bar{W}_2$ of deep inelastic electron scattering and electron-positron annihilation is discussed. A Veneziano-like model that gives a good fit with experiment for νW_2 suggests that it may be $\bar{W}_2$, rather than $\nu \bar{W}_2$, that obeys a scaling law.

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In a recent paper ¹⁾, a Veneziano-like model was proposed for the amplitude that describes the scattering of a virtual photon on a proton. Although the model is somewhat crude (it omits the proton spin, the Δ trajectory and the Pomeron), it results in a surprisingly good fit with experiment ²⁾ for the structure function ³⁾ νW_2 of deep inelastic electron-proton scattering. This is shown in Fig. 1. In the model, the photon amplitude is closely related to the amplitude that describes the scattering of the charged isovector current. The latter amplitude possesses the Fubini - Gell-Mann pole at $J = 1$, and it is this that indirectly results in the Bjorken scaling property ⁴⁾ for νW_2 . The success of the model suggests the possibility that, when it is refined, it will be able fully to explain the data for νW_2 without bringing in the Pomeron. That is, although the Pomeron contribution is presumably important when the photon has zero mass, it may be negligible when the photon mass is far from zero. This suggestion is to be contrasted with the work of authors ⁵⁾ who make essential use of the Pomeron to explain the scaling law. A feature of the model is that, in the deep inelastic region, it does not allow saturation by a finite number of resonances.

The purpose of this note is to discuss what can be learnt from the model about the corresponding function $\nu \bar{W}_2$ of electron-positron annihilation ³⁾. It has been suggested ⁶⁾ that, under quite general conditions, the function $\nu \bar{W}_2$ should be connected with νW_2 by a simple analytic continuation. This, however, turns out not to be true in the model : although νW_2 obeys the Bjorken scaling law, and so has a finite limit as $\nu \rightarrow \infty$, the function $\nu \bar{W}_2$ diverges as $\nu \rightarrow \infty$. If this prediction is a realistic one, the cross-section for

$$e^- + e^+ \rightarrow (\text{proton or antiproton}) + (\text{many hadrons}) \quad (1)$$

at high energy will be even larger than had previously been hoped, and much larger than the cross-section for a two-hadron final state.

The amplitude involved both in the deep inelastic scattering and in the annihilation process (1) is drawn in Fig. 2, where the wavy line is the virtual photon that couples to the leptons. In the inelastic scattering the particle labelled p is the initial-state proton ; in the annihilation process it is the distinguished nucleon. For both processes its momentum p will be drawn inwards, and variables ν and ω will be defined by

$$\nu = \frac{p \cdot q}{M} \quad , \quad \omega = - \frac{2M\nu}{q^2} \quad (2)$$

where M is the nucleon mass. For the inelastic scattering

$$q^2 < 0 \quad , \quad \nu > 0 \quad , \quad \omega > 1 \quad (3)$$

and for the annihilation

$$q^2 > 0 \quad , \quad \nu < 0 \quad , \quad 0 < \omega < 1 \quad . \quad (4)$$

For either reaction, one takes the square of the modulus of the amplitude in Fig. 2, integrates over the phase space for the many hadrons, and then sums over all possible numbers of those hadrons. In the region (3) of the variables this results, through unitarity, in the imaginary part of the forward amplitude for the two-body elastic scattering of the virtual photon on the proton. As has been stressed elsewhere ^{6),7)}, this is not the case in the region (4), where unitarity contains extra, unwanted terms.

Given that, for large ν , the function νW_2 defined in region (3) is a function of ω alone, the simplest situation is that one can just continue it directly from $\omega > 1$ to $\omega < 1$ and so arrive at $\nu \bar{W}_2$, defined in region (4). However, to pass from region (3) to

region (4) involves changing q^2 from negative to positive values, and so takes q^2 onto the cut in that variable (Fig. 3). The calculation of the square of the modulus of the amplitude in Fig. 2 involves the product of the physical amplitude with its complex conjugate, and as we increase q^2 we must take it round the upper side of any branch point we encounter in the amplitude, but round the lower side of the same branch point in the complex-conjugate amplitude. That is, unless these branch points are unimportant for some dynamical reason, the square of the modulus is not an analytic function of q^2 or, alternatively, of ω . In the parton model, it turns out ⁷⁾ that the Feynman graphs that dominate for large ν do not have branch points in q^2 , but there is no reason why this simple situation should be found in a model that has more structure.

Consider the following model for the virtual photon/proton elastic double-flip amplitude (Fig. 4) :

$$A(s, t, q_1^2, q_2^2) + A(u, t, q_1^2, q_2^2) \quad (5)$$

where ⁸⁾

$$\begin{aligned} A(s, t, q_1^2, q_2^2) = & \alpha' N^2 \int_0^\infty \frac{du_1 du_2 dz}{u_1 u_2 z} \left[1 + \frac{z(u_1 + u_2 + u_1 u_2)}{1 + z + u_1 u_2} \right]^{-m} \\ & \left[1 + \frac{1}{z} \right]^{\alpha(t) - 2} \left[1 + \frac{z}{1 + \frac{u_1 u_2}{1 + z + u_1 + u_2}} \right]^{\beta(s)} \\ & \left[1 + \frac{1}{u_1} \right]^{\alpha(q_1^2) - 1} \left[1 + \frac{1}{u_2} \right]^{\alpha(q_2^2) - 1} \left[1 + \frac{N(u_1 + u_2)}{1 + z} \right]^{-1}. \end{aligned} \quad (6)$$

Here α is the exchange-degenerate isospin-degenerate vector meson trajectory (ρ, ω, A_2, f):

$$\alpha(q^2) = \alpha_0 + q^2 \alpha'$$

and $\beta(s) + \frac{1}{2}$ is the nucleon trajectory, taken to have the same slope α' . The constant m is determined from the asymptotic behaviour of the nucleon elastic form factor that is derived by factorizing the residue of the nucleon pole at $\beta(s) = 0$:

$$F(q^2) \sim (q^2)^{-m} \quad \text{as} \quad q^2 \rightarrow \infty. \quad (7)$$

The normalizing constant N is fixed by the requirement that $F(0) = 1$:

$$N^{-1} = B(m, 1 - \alpha_0) \quad (8)$$

where B is the beta function.

Since passing from νW_2 to $\overline{\nu W}_2$ involves a continuation along different paths in q_1^2 and q_2^2 , consider the asymptotic limit

$$\begin{aligned} s, q_1^2, q_2^2 &\rightarrow \infty \\ q_1^2 &= c_1 s, \quad q_2^2 = c_2 s \end{aligned} \quad (9)$$

and then afterwards make $c_1^{-1} = c_2^{-1} = 1 - \omega$. Consider first the inelastic scattering, for which one can take the imaginary part of the forward elastic amplitude. First suppose that s, q_1^2, q_2^2 all $\rightarrow -\omega$, and take the limit under the integral in (6) after making the changes of variable $u_1 = -\beta(s) v_1$, $u_2 = -\beta(s) v_2$. Then make s become positive by continuing to $c_1, c_2 < 0$ and take the imaginary part ⁹⁾, which at $t = 0$ works out to be

$$\frac{N\pi}{s(c_1 - c_2)} \int_{-c_1}^{-c_2} dz (1+z)^{-m} \left(1 + \frac{1}{z}\right)^{\alpha_0 - 1} \quad (10)$$

To calculate νW_2 , let $c_1 \rightarrow c_2$ and multiply by $2M\nu/\pi$, with the result ¹⁰⁾

$$[B(m, 1 - \alpha_0)]^{-1} \omega^{-m + \alpha_0} (\omega - 1)^{m - 1} \quad (11)$$

Since there is no reason why m should be an integer [see Eq. (7)], it is clear that in general one cannot obtain $\nu \bar{W}_2$ by a simple continuation of (11) to $\omega < 1$; this would produce a non-real result. Instead, continue (10) into the region $c_1, c_2 > 1$ by giving c_1 a small positive imaginary part and c_2 a small negative imaginary part (or vice versa, it makes no difference). This continuation is equivalent to a continuation in q_1^2 and q_2^2 at fixed s , and takes q_1^2 and q_2^2 round opposite sides of any branch point that may be encountered. It results in c_1 and c_2 being on opposite sides of the branch cut that is associated with the factor $(1+z)^{-m}$ when m is not an integer, so that when now $c_1 \rightarrow c_2$ the integral no longer vanishes, and (10) diverges because of the factor multiplying the integral.

Since the original integral (6) does not have a pole at $q_1^2 = q_2^2$, this must be interpreted as an indication that the asymptotic behaviour is now greater than s^{-1} . An example of a function with this property is

$$\frac{N\pi}{(c_1 - c_2)s/I + \psi(s)} \quad (12)$$

where I is the integral in (10). If, for large s , $s^{-1}\psi(s) \rightarrow 0$, the leading asymptotic behaviour of (12) for $c_1 \neq c_2$ is just (10), and ψ contributes only to the non-leading behaviour. But when $c_1 \rightarrow c_2$, the leading behaviour is larger, $N\pi [\psi(s)]^{-1}$.

It is not surprising that a term that previously contributed only to the non-leading behaviour should now be important. This is because the effects under study are supposed to arise from the cut ⁹⁾ in the q^2 variable, and the q^2 singularities in (6) arise from the region of integration where u_1 and u_2 are small, while the leading asymptotic behaviour that gives νW_2 arises from the region where u_1 and u_2 are large.

It would be a difficult mathematical task to extract the precise form of $\psi(s)$ from the model ; and it would not be very useful to do so, since the model was not designed with any particular non-leading behaviour in mind. If one guesses that $\psi(s)$ goes to a constant for large s , it is $\bar{W}_2$ rather than $\nu \bar{W}_2$ that scales, and $\bar{W}_2$ then contributes a finite contribution to the cross-section at high energy.

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- 3) For the definitions of the structure functions see, for example, F.J. Gilman - Fourth International Symposium on Electron and Photon Interactions at High Energy, Daresbury 1969, (Preprint SLAC-PUB-674).
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- 7) S.D. Drell, D.J. Levy and T-M. Yan - Preprint SLAC-PUB-685.
- 8) This form of A differs in detail from that proposed in Ref. 1), which was a little more complicated and had some unwanted structure in asymptotic behaviour on unphysical sheets ⁹⁾. The function $A(s, t, q_1^2, q_2^2)$ is supposed to be the amplitude that describes the scattering of the negatively-charged isovector current on the proton, and it has the Fubini - Gell-Mann fixed pole at $J = 1$; this pole cancels between the terms of (5), but is indirectly responsible for the scaling property.
- 9) As explained in Ref. 1), the cut in the asymptotic form of the amplitude is synthesized by the infinite numbers of poles along the positive real s and q^2 axes, in the same way as the Regge asymptotic form of the ordinary Veneziano beta function has a cut.
- 10) The curve in Fig. 1 is plotted for $m = 3.5$ and $\alpha_0 = 0.5$, but it is not very sensitive to the values of these parameters.

FIGURE CAPTIONS

- Figure_1 The solid curve represents the function νW_2 as given by the model of Ref. 1). The experimental data is from Ref. 2), as plotted by Gilman [Ref. 3)].
- Figure_2 The amplitude involved in the two processes. The line labelled q is the virtual photon.
- Figure_3 The lowest threshold in the variable q^2 for the amplitude of Fig. 2.
- Figure_4 The two-body photon/proton amplitude.

FIG.1

FIG.2

FIG.3

FIG.4