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SCALING LAWS IN HIGH-ENERGY NEUTRINO REACTIONS

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A B S T R A C T

Bjorken has proposed that the structure functions F_1 , F_2 , F_3 of deep inelastic neutrino scattering are functions of $\omega = -2M\nu/q^2$ only. These scaling laws are studied in Veneziano-like models and are found to emerge naturally for the scattering of neutrinos on a neutron or of antineutrinos on a proton, together with the Callan-Gross sum rule for $\int_1^\infty d\omega F_2/\omega^2$, the Gross-Llewellyn Smith sum rule for $\int_1^\infty d\omega F_3/\omega^2$ and the relation $F_1 = \frac{1}{2}\omega F_2$. (It is assumed that certain equal-time commutators of currents contain no operator Schwinger terms.) The models suggest the possibility that the structure functions for the scattering of neutrinos on a proton, or of antineutrinos on a neutron, are small in the deep inelastic limit.

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1. - INTRODUCTION

The reactions

$$\begin{aligned}\bar{\nu} + p &\rightarrow \mu^+ + \text{hadrons} \\ \nu + p &\rightarrow \mu^- + \text{hadrons}\end{aligned}\tag{1.1}$$

are described by the matrix elements ^{1),2)}

$$\begin{aligned}& \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \langle p | [J_{\pm}^{\mu}(x), J_{\mp}^{\nu}(0)] | p \rangle_{Av} \\ &= \left(\frac{q^{\mu} q^{\nu}}{q^2} - g^{\mu\nu} \right) M W_1^{(\bar{\nu}, \nu)} \\ &+ \left(p^{\mu} - \frac{p \cdot q}{q^2} q^{\mu} \right) \left(p^{\nu} - \frac{p \cdot q}{q^2} q^{\nu} \right) W_2^{(\bar{\nu}, \nu)} / M \\ &- i \epsilon^{\mu\nu\alpha\beta} p_{\alpha} q_{\beta} W_3^{(\bar{\nu}, \nu)} / 2M + \dots\end{aligned}\tag{1.2}$$

Here J is the Gell-Mann-Cabibbo current, M is the nucleon mass and "Av" denotes an average over proton polarizations. It is understood that only the connected part of the matrix element is included. The terms not written explicitly are proportional to $(p^{\mu} q^{\nu} + q^{\mu} p^{\nu})$ and $q^{\mu} q^{\nu}$. By charge symmetry of the strong interactions, the same matrix elements are also respectively associated with the reactions

$$\begin{aligned}\nu + n &\rightarrow \mu^- + \text{hadrons} \\ \bar{\nu} + n &\rightarrow \mu^+ + \text{hadrons}.\end{aligned}\tag{1.3}$$

Bjorken ²⁾ has proposed that at high-energy the structure functions W_i obey the scaling laws

$$\begin{aligned} M W_1^{(\bar{\nu}, \nu)} &\rightarrow F_1^{(\bar{\nu}, \nu)}(\omega) \\ \nu W_2^{(\bar{\nu}, \nu)} &\rightarrow F_2^{(\bar{\nu}, \nu)}(\omega) \\ \nu W_3^{(\bar{\nu}, \nu)} &\rightarrow F_3^{(\bar{\nu}, \nu)}(\omega) \end{aligned} \quad (1.4)$$

where

$$\nu = p \cdot q / M, \quad \omega = -2M\nu/q^2. \quad (1.5)$$

The amplitude of which W_2 is the imaginary part contains ³⁾ the Fubini-Gell-Mann fixed pole at angular momentum $j = 1$. In a previous paper ⁴⁾ it was shown that, when this information is combined with the analyticity properties present in a Veneziano-like model for the amplitude, it leads naturally to the scaling property for the function $\nu W_2^{(\bar{\nu})}$. By crossing and isospin manipulations, the part of $F_2^{(\bar{\nu})}(\omega)$ that corresponds to the vector current is proportional to the contribution from isovector-photon exchange in the deep inelastic scattering of an electron on a proton. If it is assumed that isoscalar-photon exchange contributes an equal amount in the deep inelastic limit, the resulting form of $F_2(\omega)$ agrees ^{4),5)} surprisingly well with the experimental data ⁶⁾, both in shape and in magnitude. (If it is really true that the isoscalar and isovector parts of the photon contribute equally, the structure function F_2 for the deep inelastic scattering of an electron on a neutron will be zero within the present explanation of the scaling laws.)

Since the residue of the Fubini-Gell-Mann pole is the nucleon elastic form factor, the scaling properties of $\nu W_2^{(\bar{\nu})}$ depend indirectly on the equal-time commutation relations of the time component of the vector (axial-vector) current with the space components of the vector (axial-vector) current. In this paper it will be shown

that the scaling law for $\nu W_3^{(\bar{\nu})}$ has its origin indirectly in the equal-time commutation relations of the space components of the vector current V with the space components of the axial current A .

It is found that, at least so far as the contribution from the vector current V is concerned, the scaling property of $W_1^{(\bar{\nu})}$ is closely linked to that of $\nu W_2^{(\bar{\nu})}$, and that

$$F_1^{(\bar{\nu})}(\omega) = \frac{1}{2}\omega F_2^{(\bar{\nu})}(\omega). \quad (1.6)$$

This relation follows independently of the detailed structure of the equal-time $[V_+^0, V_-^j]$ commutation relations, provided only that they have no operator Schwinger terms ⁷⁾. For deep inelastic electron scattering, it implies that there is no contribution from the exchange of longitudinal photons, which seems to agree with experiment ⁶⁾.

The sum rule of Callan and Gross ⁸⁾ is obtained in the form

$$\int_1^\infty \frac{d\omega}{\omega^2} F_2^{(\bar{\nu})}(\omega) = b \quad (1.7)$$

where b is the coefficient of $p_i p_j$ in

$$i \int d^3x \langle p | [\dot{J}_+^i(0, x), J_-^j(0)] | p \rangle_{Av}. \quad (1.8)$$

For F_3 is obtained the sum rule of Gross and Llewellyn Smith ⁹⁾

$$\frac{1}{2} \int_1^\infty \frac{d\omega}{\omega^2} (F_3^{(\bar{\nu})} + F_3^{(\nu)}) = -f \quad (1.9)$$

where

$$\int d^3x \langle p | [A_+^i(0, x), V_-^j(0)] | p \rangle_{Av} = -i \epsilon^{ijk} f p_k. \quad (1.10)$$

It is found that, at least to the extent that the coupling of the Δ trajectory may be small compared with that of the nucleon, all the structure functions $F_1^{(\nu)}$, $F_2^{(\nu)}$, $F_3^{(\nu)}$ for the scattering of a neutrino on a proton are small compared with those for the scattering of an antineutrino. It is argued that there is reason to suppose a stronger result : that even if the coupling of the Δ trajectory is not weak, the (νp) structure functions $F_i^{(\nu)}$ are much less than the $(\bar{\nu} p)$ functions $F_i^{(\bar{\nu})}$ in the deep inelastic limit.

In this, and other, respects the present explanation of the scaling laws differs markedly from a mechanism that relies entirely on diffractive effects ¹⁰⁾. Indeed, the Pomeron does not ⁹⁾ couple to the amplitude $W_3^{(\bar{\nu})}$. It presumably has to be included in $W_1^{(\bar{\nu})}$ and $W_2^{(\bar{\nu})}$, since these are related by crossing and isospin manipulations to the Compton amplitude, but the present approach to the scaling laws does not require the Pomeron term to participate in the deep inelastic limit ¹¹⁾. It may also be noted that the Veneziano-like models do not allow, in the deep inelastic region, saturation by a finite number of resonances ; an approximation of this type, either in the q^2 or s channels, leads to a very different large q^2 behaviour. The models allow the structure functions to have rather more structure than is predicted in parton models ¹²⁾. In particular, it was shown in a previous paper ⁵⁾ that they can lead to a larger e^+e^- annihilation cross-section than might have been expected (it might even go to a constant at high energy).

Section 2 of this paper is concerned with what can be learnt about the amplitudes concerned from current algebra ; in Section 3 this information is used to construct Veneziano-like models for them, and their behaviour in the deep inelastic limit is then discussed in Section 4.

2. - CURRENT ALGEBRA

For simplicity, it will be supposed that the small value of the Cabibbo angle θ allows the pretence that it is actually zero. (It would be straightforward to take account of a non-zero θ .) It will be assumed throughout that the strong interactions are P and T invariant.

Consider first the amplitude

$$\begin{aligned} T^{\mu\nu} &= \frac{1}{\pi} \int d^4x e^{iq \cdot x} \theta(x) \langle p | [A_+^\mu(x), V_-^\nu(0)] | p \rangle_{Av} \\ &= \epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta T(s, q^2), \end{aligned} \quad (2.1)$$

where $s = (p+q)^2$. The imaginary part of T is $W_3^{(\overline{\nu})}/4M$. In the limit

$$\begin{aligned} q^0 &\rightarrow \infty \\ \underline{q}, p &\text{ fixed} \end{aligned} \quad (2.2)$$

Bjorken's asymptotic theorem ¹³⁾ gives

$$q^0 T^{\mu\nu} \rightarrow \frac{i}{\pi} \int d^3x e^{-iq \cdot x} \langle p | [A_+^\mu(0, x), V_-^\nu(0)] | p \rangle_{Av}. \quad (2.3)$$

If, as has been implicitly assumed in (2.1), there are no operator Schwinger terms in the equal-time commutators of the currents, the $(\mu 0)$ and (0ν) components of the integral (2.3) vanish, while the (ij) component can be written ¹⁴⁾ as in (1.10). Then in the limit (2.2)

$$T(s, q^2) \sim -\frac{1}{\pi} f/q_0^2. \quad (2.4)$$

It is this information that, when incorporated into a Veneziano-like model for T , will result in the Bjorken scaling property (1.4) for $W_3^{(\bar{\nu})}$.

If one considers only the forward amplitude, as in (2.1), one cannot incorporate known properties of analyticity in the momentum transfer variable t . In order to make use of such information, define $T^{\mu\nu}$ for the case where the momenta of the two protons are not necessarily equal :

$$T^{\mu\nu} = \frac{1}{\pi} \int d^4x \ e^{iq_2 \cdot x} \theta(x) \langle p_2 | [A_+^\mu(x), V_-^\nu(0)] | p_1 \rangle_{Av}. \quad (2.5)$$

For $p_1 \neq p_2$ the notion of an average over proton spin polarizations in an amplitude is not well defined ¹⁵⁾. The definition that will be adopted here is that if ¹⁶⁾

$$\langle p_2, s_2 | M | p_1, s_1 \rangle = \bar{u}_{s_2}(\underline{p}_2) \gamma n u_{s_1}(\underline{p}_1),$$

then

$$\langle p_2 | M | p_1 \rangle_{Av} = \frac{1}{2} \text{Tr} \left\{ \frac{\gamma \cdot p + m}{2m} \gamma n \right\} \quad (2.6)$$

where $p = \frac{1}{2}(p_1 + p_2)$.

With no operator Schwinger terms in the equal-time commutators of the currents, the current conservation condition $\partial V = 0$ results in $T^{\mu\nu} q_{1\nu} = 0$, and $T^{\mu\nu}$ has the expansion

$$T^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} \{ p_\alpha T + q_{2\alpha} T' \} q_{1\beta} . \quad (2.7)$$

Then in the limit

$$q_1^0 \sim q_2^0 \rightarrow \infty$$

$$\underline{q}_1, \underline{q}_2, p_1, p_2 \text{ fixed}, \quad (2.8)$$

where q_1 and q_2 are the momenta carried by the currents, the amplitude T has the asymptotic behaviour

$$T(s, t, q_1^2, q_2^2) \sim -\frac{1}{\pi} f(t) / q_{10}^2 \quad (2.9a)$$

where

$$\int d^3x e^{-iq \cdot x} \langle p_2 | [A_+^i(0, x), V_-^j(0)] | p_1 \rangle_{Av}$$

$$= -i \epsilon^{ijk} f(t) p_k . \quad (2.9b)$$

Consider now the amplitude

$$M^{\mu\nu} = \frac{i}{\pi} \int d^4x e^{iq \cdot x} \theta(x) \langle p | [V_+^\mu(x), V_-^\nu(0)] | p \rangle_{Av}, \quad (2.10)$$

it being understood that only the connected part of the amplitude is included. The current conservation conditions $q_\mu M^{\mu\nu} = -p^\nu/\pi$ $M^{\mu\nu} q_\nu = -p^\mu/\pi$ lead to the decomposition

$$M^{\mu\nu} = \left[p^\mu p^\nu - (p^\mu q^\nu + q^\mu p^\nu) \frac{p \cdot q}{q^2} + \frac{(p \cdot q)^2}{q^2} g^{\mu\nu} \right] A \\ + \left[q^\mu q^\nu - q^2 g^{\mu\nu} \right] B \\ - \frac{p^\mu q^\nu + q^\mu p^\nu}{\pi q^2} + \frac{p \cdot q}{\pi q^2} g^{\mu\nu}, \quad (2.11)$$

which has been written so that the functions A and B are both free of kinematical singularity at $q^2 = 0$. The imaginary part of A is one part of $W_2^{(\bar{\nu})}/M$; the other part is a corresponding contribution from the matrix element of two axial currents. Similarly, $\text{Im } B'$ contributes to $M W_1^{(\bar{\nu})}$, where

$$B' = q^2 B - \frac{(p \cdot q)^2}{q^2} A. \quad (2.12)$$

[At $q^2 = 0$, $A = -(\pi p \cdot q)^{-1}$, so B' has a kinematical singularity and is less suitable than B for the construction of a model.]

In the limit (2.2) we have ¹³⁾, still assuming that there are no operator Schwinger terms,

$$q^\circ M^{\circ\nu} \rightarrow -p^\nu/\pi, \quad (2.13)$$

from which we find that $q_0^2 B \rightarrow 0$. From the work of the next Section, it will be found that a consequence of this is that in the Bjorken scaling limit $q^2 B \rightarrow 0$. Hence we find from (2.12) that, because $\nu W_2^{(\bar{\nu})}$ obeys the Bjorken scaling law (1.4) so also does $W_1^{(\bar{\nu})}$, with the

relation (1.6) connecting the functions $F_1^{(\bar{\nu})}$ and $F_2^{(\bar{\nu})}$. Strictly, this argument applies only to the vector-current contributions, since it makes use of current conservation. But it is possible that the axial current behaves effectively as a conserved current in the deep inelastic limit.

A further application of the Bjorken asymptotic theorem in the limit (2.2) gives

$$q_0^2 M^{ij} \rightarrow \frac{-i}{\pi} \int d^3x e^{-iq \cdot x} \langle p | [\dot{V}_+^i(0, x), V_-^j(0)] | p \rangle_{Av} \quad (2.14)$$

with a similar relation for the amplitude corresponding to two axial currents. It is these relations that will result in the Callan-Gross sum rule (1.7) ; they are needed only for this.

As with $T^{\mu\nu}$, we extend the definition of $M^{\mu\nu}$ to non-zero momentum transfers, $t \neq 0$. The construction ⁴⁾ of the amplitude A takes account of the Fubini - Gell-Mann limit

$$\begin{aligned} s &= (p_1 + q_1)^2 \rightarrow \infty \\ q_1^2, q_2^2, t &= (q_1 - q_2)^2 \text{ fixed} \end{aligned} \quad (2.15)$$

in which ³⁾

$$A \sim - \frac{2 F(t)}{\pi s} \quad (2.16)$$

Here $F(t)$ is the Dirac isovector form factor of the proton, normalized such that $F(0) = 1$. It is important for what follows that, although the result (2.16) is independent of q_1^2 and q_2^2 , it is only supposed to be valid when q_1^2 and q_2^2 are kept fixed while $s \rightarrow \infty$.

3. - CONSTRUCTION OF THE AMPLITUDES

A Veneziano-like model for the amplitude A was constructed in Ref. 4). A review of this will be given here, stressing only those features that are important for the Bjorken scaling limit. The construction of the amplitudes T and B will then also be discussed.

For simplicity, the only Regge trajectories that are included are the exchange-degenerate vector-meson trajectory (ρ, ω, A_2, f), denoted by α_V , the exchange-degenerate (π, A_1) trajectory, denoted by α_A , and the exchange-degenerate nucleon trajectory, $\beta + \frac{1}{2}$. All these trajectories are supposed to be linear, with common slope α' . Because of the neglect of the Δ trajectory, the u channel is effectively exotic. The effect of including the Δ or other trajectories and breaking the exchange degeneracy is discussed in the next Section. As is usual in the context of Veneziano theory, no attempt is made to include the Pomeron; as discussed in Section 1, the only definite thing that can be said about the Pomeron is that it is possible to achieve scaling without it.

Consider the function

$$\int_0^{\infty} \frac{du_1 du_2 dz}{u_1 u_2 z} \left(1 + \frac{1}{u_1}\right)^{\alpha_V(q_1^2) - 1} \left(1 + \frac{1}{u_2}\right)^{\alpha_V(q_2^2) - 1} \\ \left(1 + \frac{1}{z}\right)^{\alpha_V(t) - 2} \left(1 + z X(u_1, u_2, z)\right)^{\beta(s)} \\ Y(u_1, u_2, z). \quad (3.1)$$

We require this integral to converge for negative values of the exponents, so when u_1 or u_2 is large the function X is bounded and Y tends to zero. If also X and Y are neither zero nor infinite when the integration variables tend to zero or when $z \rightarrow \infty$, the integral has the same pole structure in the variables q_1^2, q_2^2, s and t as is desired for the amplitude A . The s and t channels are dual to each other, their poles being associated with different ends of the same integration z , and their resonances produce Regge pole terms in the appropriate asymptotic limits. For example, when ¹⁷⁾
 $\beta(s) \rightarrow -\infty$ with t, q_1^2, q_2^2 fixed, the dominant contribution to the integral arises from the region of integration where zX is small. The Regge pole term arises from small z ; to extract it make the change of variable $z = -z'/\beta(s)$, take the limit under the integral, so that for example

$$\left(1 + z X(u_1, u_2, z)\right)^{\beta(s)} \rightarrow e^{-z' X(u_1, u_2, 0)}, \quad (3.2)$$

and perform the z' integration :

$$\Gamma(2-\alpha_v(t)) (-\beta(s))^{\alpha_v(t)-2} \int_0^\infty \frac{du_1 du_2}{u_1 u_2} \left(1 + \frac{1}{u_1}\right)^{\alpha_v(q_1^2)-1} \quad (3.3)$$

$$\left(1 + \frac{1}{u_2}\right)^{\alpha_v(q_2^2)-1} \left(X(u_1, u_2, 0)\right)^{2-\alpha_v(t)} Y(u_1, u_2, 0).$$

The functions X and Y must be chosen in such a way that the integral (3.1) has also the term (2.16) in its s channel asymptotic behaviour. As the coefficient of s^{-1} in this term depends on t but not on q_1^2 or q_2^2 , the term must arise from the part

$$\begin{aligned} u_1, u_2 &\rightarrow \infty \\ z &\text{ finite} \end{aligned} \quad (3.4)$$

of the integration region. That is, the function X must tend to zero in this region. We give here a choice of the function X that differs slightly from that in the previous papers ^{4),5)}

$$X(u_1, u_2, z) = \left[1 + \frac{u_1 u_2}{e^z + u_1 + u_2} \right]^{-1}. \quad (3.5)$$

[There will not be an attempt here to establish the essential uniqueness of this choice of X , as ultimately the choice depends on what is desired from the spectrum of daughter particles ; the analysis of this is altogether beyond the scope of the present work. However, it will be remarked that the choice of X is very severely constrained by several other requirements, in addition to those already mentioned on its behaviour at the ends of the ranges of integration. Among these is a requirement that there be no further structure in the s or t channel asymptotic behaviour ¹⁷⁾ (so that, for example, X does not tend to zero when u_1 alone is large) and that the function goes to zero exponentially ¹⁸⁾ when $s \rightarrow \infty$ at fixed $u \equiv (p_1 - q_2)^2$. Another natural requirement is that $X(0,0,z) = Y(0,0,z) = 1$, so that the amplitude for the purely hadronic process

$$e^- + p \rightarrow e^- + p$$

extracted from the residue of the twofold pole at $\alpha_V(q_1^2) = \alpha_V(q_2^2) = 1$ be an ordinary Veneziano beta function. In order that the residues of the s channel poles have the appropriate factorization properties, it is also required that $X(u_1, u_2, \infty)$ and $Y(u_1, u_2, \infty)$ reduce to suitably factorized forms in u_1 and u_2 . The details of this are unimportant here, but it will be remarked that the form factors obtained by factorizing the s channel pole residues can be made to vanish as rapidly as one pleases for large current "mass". An explicit example was given in Ref. ⁴⁾.]

We still have to make the correct function $2F(t)/\pi$ appear as the coefficient of $(-s)^{-1}$ in (2.16). Make use of the known analyticity properties of $F(t)$ to write it as the transform :

$$F(t) = \int_0^{\infty} \frac{dz}{z} \left(1 + \frac{1}{z}\right)^{\alpha_V(t)-1} \tilde{F}(z) . \quad (3.6)$$

(By making the change of variable $1 + \frac{1}{z} = e^x$ one can see that this is essentially a Laplace transform.) Choose the function Y such that in the region (3.4)

$$Y \sim \frac{2\alpha'}{\pi} \frac{1+z}{u_1+u_2} \tilde{F}(z) . \quad (3.7)$$

Then on making the changes of variable

$$\begin{aligned} u_1 &= -\beta(s) u'_1 \\ u_2 &= -\beta(s) u'_2 , \end{aligned} \quad (3.8)$$

taking the limit under the integral and performing the u'_1, u'_2 integrations, we find the result (2.16). Note that (2.16) is the leading term in the asymptotic behaviour only when $\alpha_V(t) < 1$. At $\alpha_V(t) = 1$ there is a pole in (2.16) which is not present in the complete amplitude A . This pole is cancelled ¹⁹⁾ by a similar one in the Regge term (3.3), which dominates the asymptotic behaviour for $\alpha_V(t) > 1$.

Consider now, instead of the asymptotic limit (2.15), the Bjorken scaling limit ²⁾

$$\begin{aligned} s \rightarrow \infty , \quad s/q_1^2 \sim s/q_2^2 \sim 1-\omega \\ t, \omega \text{ fixed} . \end{aligned} \quad (3.9)$$

In this limit the factors in (3.1) that involve q_1^2 and q_2^2 reduce to finite exponentials when one makes the changes of variable (3.8) and takes the limit under the integral ¹⁷⁾; that is the dominant contribution again arises from the region of integration (3.4). Using (3.7), we can perform the u_1' and u_2' integrations and find the asymptotic form

$$- \phi_2(t, \omega) / s \quad (3.10)$$

where

$$\phi_2(t, \omega) = \frac{2}{\pi} \int_0^\infty dz \frac{(1 + \frac{1}{z})^{\alpha_V(t) - 1} \tilde{F}(z)}{z + \frac{1}{1 - \omega}} \quad (3.11)$$

Note that for $\alpha_V(t) < 1$, $\phi_2(t, \omega) = 2F(t)/\pi$; this is not surprising since the limits (2.15), and (3.9) with $\omega = \omega$, overlap. Notice also that the s^{-1} behaviour of A in the limit (3.9) holds even for $t \neq 0$ and applies to the complete amplitude rather than just to its imaginary part; these features are not found with the diffractive explanation of the scaling laws ¹⁰⁾.

The amplitude corresponding to two axial currents is treated similarly, and is given the representation

$$\int_0^\infty \frac{du_1 du_2 dz}{u_1 u_2 z} \left(1 + \frac{1}{u_1}\right)^{\alpha_A(q_1^2)} \left(1 + \frac{1}{u_2}\right)^{\alpha_A(q_2^2)} \left(1 + \frac{1}{z}\right)^{\alpha_V(t) - 2} \left[1 + \frac{z}{1 + \frac{u_1 u_2}{e^z + u_1 + u_2}}\right]^{\beta(s)} Y'(u_1, u_2, z) \quad (3.12)$$

(We have again used the function X of (3.5), though in principle it seems possible to use a function that differs from X in detail.) As this amplitude has the same Fubini-Gell-Mann pole (2.16), Y' is given the same asymptotic form (3.7) as Y in the region (3.4), and we obtain the same contribution (3.10) in the Bjorken scaling limit.

For the amplitude T , as we have no information concerning the Fubini-Gell-Mann limit (2.15), we instead incorporate the term (2.9a) that arises in the $|q_0| \rightarrow \infty$ limit (2.8). In terms of invariants, this limit is

$$\begin{aligned} |s| &\rightarrow \infty \\ s/q_1^2 \sim s/q_2^2 &\rightarrow 1 \\ t &\text{ fixed.} \end{aligned} \quad (3.13)$$

If we introduce integration variables u_1, u_2 and z in the same way as before, this asymptotic term must again arise from the region (3.4) of the integration. Write the transform

$$f(t) = \int_0^\infty \frac{dz}{z} \left(1 + \frac{1}{z}\right)^{\alpha_V(t)-1} \tilde{f}(z). \quad (3.14)$$

[The function $f(t)$ is expected to have poles at $\alpha_V(t) = 1, 2, \dots$, and so $\tilde{f}(0)$ will be finite.] Then T has the representation, using again the function X of (3.5),

$$\begin{aligned} T = & \int_0^\infty \frac{du_1 du_2 dz}{u_1 u_2 z} \left(1 + \frac{1}{u_1}\right)^{\alpha_V(q_1^2)-1} \left(1 + \frac{1}{u_2}\right)^{\alpha_A(q_2^2)} \\ & \left(1 + \frac{1}{z}\right)^{\alpha_V(t)-1} \left[1 + \frac{z}{1 + \frac{u_1 u_2}{e^z + u_1 + u_2}}\right]^{\beta(s)} \\ & Y''(u_1, u_2, z) \end{aligned} \quad (3.15)$$

where, in the limit (3.4),

$$Y'' \sim \frac{\alpha'}{\pi} \frac{1+z}{u_1 + u_2} \tilde{f}(z). \quad (3.16)$$

In the Bjorken scaling limit (3.9) we then obtain ¹⁷⁾

$$T \sim -\phi_3(t, \omega) / s \quad (3.17)$$

where

$$\phi_3(t, \omega) = \frac{1}{\pi} \int_0^\infty dz \frac{(1+\frac{1}{2})^{\alpha_v(t)} \tilde{f}(z)}{z + \frac{1}{1-\omega}}. \quad (3.18)$$

In the case of the amplitude B , we found in Section 2 that it goes to zero faster than s^{-1} in the limit (3.13). Hence in a similar construction of B the corresponding function Y''' must go to zero more rapidly than u_1^{-1} or u_2^{-1} in the limit (3.4), and in the scaling limit $q^2_B \rightarrow 0$.

4. - THE SCALING LAWS

As the form factors $F(t)$ and $f(t)$ are real for $\alpha_V(t) < 1$, their transforms $\tilde{F}(z)$ and $\tilde{f}(z)$ are real. Hence the functions ϕ_2 and ϕ_3 , defined in (3.11) and (3.18), are real at $t = 0$ when $\omega < 1$. If they are now continued ¹⁷⁾ to $\omega > 1$ they acquire imaginary parts due to the vanishing of the denominator in each integrand :

$$\text{Im } \phi_2(0, \omega) = -2 \theta(\omega-1) \omega^{\alpha_v(0)-1} \tilde{F}\left(\frac{1}{\omega-1}\right) \quad (4.1)$$

$$\text{Im } \phi_3(0, \omega) = -\theta(\omega-1) \omega^{\alpha_v(0)} \tilde{f}\left(\frac{1}{\omega-1}\right). \quad (4.2)$$

The existence of these imaginary parts implies that ϕ_2 and ϕ_3 have branch points at $\omega = 1$, even though A and T have been constructed to have only poles, and no branch points. The explanation of this is familiar : the apparent cuts ²⁰⁾ are synthesized by the infinite numbers of poles in the variables s and q^2 . The presence of these poles in fact prevents the asymptotic behaviour (3.10) and (3.17) being strictly valid in a wedge containing the positive ω real axis. But it is supposed that it would be valid on the real axis in a unitarized theory, where the trajectory functions have cuts and the poles are displaced from the real axis.

From (4.1) and (4.2) we calculate the structure functions defined in (1.4) :

$$F_2^{(\bar{\nu})}(\omega) = 2 \omega^{\alpha_v(0)} \tilde{F}\left(\frac{1}{\omega-1}\right) / (\omega-1) \quad (4.3)$$

$$F_3^{(\bar{\nu})}(\omega) = 2 \omega^{\alpha_v(0)+1} \tilde{f}\left(\frac{1}{\omega-1}\right) / (\omega-1). \quad (4.4)$$

Since we require the first singularities of both $F(t)$ and $f(t)$ to be poles at $\alpha_v(t) = 1$, we see from (3.6) and (3.14) that $\tilde{F}(0)$ and $\tilde{f}(0)$ are finite numbers. Hence ¹¹⁾ for large ω

$$\begin{aligned} F_2(\bar{\omega}) &\propto \omega^{\alpha_v(0)-1} \approx \omega^{-\frac{1}{2}} \\ F_3(\bar{\omega}) &\propto \omega^{\alpha_v(0)} \approx \omega^{\frac{1}{2}}. \end{aligned} \quad (4.5)$$

Suppose that for large t , $F(t)$ and $f(t)$ are proportional respectively to t^{-m} and t^{-n} . Then for large z , $\tilde{F}(z)$ is proportional to z^{-m} and $\tilde{f}(z)$ is proportional to z^{-n} . So ²¹⁾ near $\omega = 1$

$$\begin{aligned} F_2(\bar{\omega}) &\propto (\omega-1)^{m-1} \\ F_3(\bar{\omega}) &\propto (\omega-1)^{n-1}. \end{aligned} \quad (4.6)$$

Consider the function

$$\psi_2(\omega) = \frac{1}{\omega-1} \phi_2(0, \omega). \quad (4.7)$$

This vanishes at infinity and satisfies an unsubtracted dispersion relation in ω . According to the work of Section 2 [See Eq. (2.14)], the value of the function at $\omega = 0$ is b_V/π , where b_V is the coefficient of $p_i p_j$ in

$$i \int d^3 \underline{x} \ e^{-i \underline{q} \cdot \underline{x}} \langle p | [\dot{V}_+^i(0, \underline{x}), V_-^j(0)] | p \rangle_{Av}. \quad (4.8)$$

On evaluating the dispersion relation at $\omega = 0$, and adding on a similar contribution from the axial currents, we obtain the Callan-Gross sum rule (1.7). [One can also derive the sum rule

$$\int_1^{\infty} \frac{d\omega}{\omega} F_2^{(\nu)}(\omega) = 2 F(0) \equiv 2 .]$$

The same argument applied to the function

$$\psi_3(\omega) = \frac{1}{\omega-1} \phi_3(0, \omega) \quad (4.9)$$

gives the sum rule

$$\frac{1}{2} \int_1^{\infty} \frac{d\omega}{\omega^2} F_3^{(\nu)}(\omega) = -f(0) \equiv -f. \quad (4.10)$$

The apparent difference between this relation and the sum rule (1.9) of Gross and Llewellyn Smith arises because in the present model all three (νp) structure functions vanish :

$$F_1^{(\nu)}(\omega) = F_2^{(\nu)}(\omega) = F_3^{(\nu)}(\omega) = 0 . \quad (4.11)$$

This comes about in the following way. The amplitudes $\tilde{T}(s, t)$ and $\tilde{A}(s, t)$ for the scattering of a neutrino on a proton are obtained from T and A by interchanging the charges on the currents. By crossing,

$$\tilde{T}(s, t) = T(u, t) \quad , \quad \tilde{A}(s, t) = A(u, t) . \quad (4.12)$$

Hence in the Bjorken scaling limit

$$\begin{aligned}\tilde{A}(s,t) &\sim -\phi_2(t,-\omega)/u \\ \tilde{T}(s,t) &\sim -\phi_3(t,-\omega)/u .\end{aligned}\tag{4.13}$$

Because of the θ functions in (4.1) and (4.2), the imaginary parts of (4.13) are zero²²⁾ in the relevant region, $\omega > 1$. The vanishing of $F_1^{(\nu)}(\omega)$ follows because the relation (1.6) applies equally to the (νp) functions, at least for the vector-current contribution, or alternatively from the inequalities⁹⁾

$$\omega F_2^{(\bar{\nu},\nu)}(\omega) \geq 2 F_1^{(\bar{\nu},\nu)}(\omega) \geq |F_3^{(\bar{\nu},\nu)}(\omega)| .\tag{4.14}$$

The result (4.11) has been derived in a model where the Δ trajectory has been omitted, so that there are no resonances in the (νp) channel. It will now be argued that the absence of a left-hand cut in $\phi_2(0,\omega)$ is independent of this, so that the vector-current contribution to $F_2^{(\nu)}(\omega)$ vanishes even if the Δ is included. The argument makes use of current conservation. If it is assumed that the effect of the non-conservation of the axial current is small in the scaling limit, $F_2^{(\nu)}(\omega)$ will be small. The inequalities (4.14) will then constrain $F_1^{(\nu)}$ and $F_3^{(\nu)}$ to be small also, at least for not too large values of ω .

Suppose then that

$$A(s,t) = V_1^{BM}(s,t) + V_2^{BM}(u,t) + V_3^{BB}(s,u) .\tag{4.15}$$

Here $V_1^{\text{BM}}(s, t)$ denotes a sum of Veneziano-like terms, each with some baryon trajectory in the s channel and some meson trajectory in the t channel, together with any appropriate mesons in the current channels. The functions V_2 and V_3 are similarly defined. The details of precisely which trajectories are included, whether or not they are exchange-degenerate, and the relative strengths of their couplings will not matter. It is now argued that only the term V_1 contains the Fubini-Gell-Mann contribution (2.16) in the appropriate asymptotic limit (2.15). Certainly V_3 is not expected to contribute anything of this form, but at first sight V_2 might. To see that it does not, note that we are concerned with a contribution that is independent of q_1^2 and q_2^2 (according to the work of Section 3 it is only such a contribution that leads to a non-zero term in the scaling limit), so we can investigate it at a special set of values of q_1^2 and q_2^2 . At $q_1^2 = 0$, $q_2^2 = t$, current conservation demands ¹⁹⁾ that the amplitude A is exactly equal to the nucleon-pole term

$$A \equiv \frac{2 F(t)}{M^2 - s} . \quad (4.16)$$

(We did not attempt to incorporate this condition in the construction of A in Section 3.) Because the u channel has isospin $\frac{3}{2}$, the function V_2 does not contain a pole at the nucleon mass, so going now to the Fubini-Gell-Mann limit (2.15), we see that the (q_1^2, q_2^2) independent result (2.16) comes entirely from those terms in V_1 that contain the nucleon pole. It does not matter how the contribution is shared amongst these terms ; according to the work of Section 3 the function $\phi_2(0, \omega)$ arising from their sum will have a cut only for $\omega > 1$.

Finally, return to the inequalities (4.14). It has already been argued that the first of them should be saturated, at least for the vector-current contribution. Little is known ¹⁴⁾ about the form factor $f(t)$, except that it, like $F(t)$, should have poles corresponding to the spin-one daughters of the particles on the trajectories $\alpha_V(t)$. If, for want of any better information, it is assumed that

$f(t)$ is the same shape as $F(t)$, and that its relative normalization is as is given by the trivial model where the Cabibbo current is constructed from bare nucleon fields,

$$J_+^\mu = \bar{p} \gamma^\mu (1 - \gamma^5) n, \quad (4.17)$$

one obtains

$$F_3^{(\nu)}(\omega) = -\omega F_2^{(\bar{\nu})}(\omega). \quad (4.18)$$

This relation, taken together with $F_3^{(\nu)} = 0$, would make the cross-section for the scattering of energetic neutrinos on a heavy liquid (containing roughly equal numbers of neutrons and protons) rather bigger than that for antineutrinos.

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FOOTNOTES AND REFERENCES

- 1) The normalization of the states is $\langle p' | p \rangle = (2\pi)^3 2p^0 \delta^{(3)}(\underline{p}-\underline{p}')$
and the metric is $(+---)$.
- 2) J.D. Bjorken - Phys.Rev. 179, 1547 (1969).
- 3) S. Fubini - Nuovo Cimento 43, 475 (1966).
- 4) P.V. Landshoff and J.C. Polkinghorne - Nuclear Phys. B19, 432 (1970).
- 5) P.V. Landshoff - CERN Preprint TH 1154 (to be published in Physics Letters) (1970).
- 6) M. Briedenbach et al. - Phys.Rev.Letters 23, 935 (1969). See also reports by F.J. Gilman and R.E. Taylor, Fourth International Symposium on Electron and Photon Interactions at High Energy (Daresbury, 1969). Some preliminary data for the neutrino processes are given by I. Budagov et al. - Phys.Letters 30B, 364 (1969).
- 7) A connection between the absence of operator Schwinger terms and the relation (1.6) follows also from recent work of R. Jackiw, R. Van Royen and G.B. West - M.I.T. Preprint C.T.P. 118. (1970).
- 8) C.G. Callan and D.J. Gross - Phys.Rev.Letters 22, 156 (1969).
- 9) D.J. Gross and C.H. Llewellyn Smith - Nuclear Phys. B14, 337 (1969).
- 10) H. Harari - Phys.Rev.Letters 22, 1078 (1969).
- 11) If the Pomeron contribution does disappear at large q^2 , there is a strong possibility that the existing experimental data ⁶⁾ for $F_2(\omega)$ is "contaminated" by a residual Pomeron contribution at large values of ω , so that F_2 falls off more rapidly for large ω than the present data suggest.

- 12) S.D. Drell, D.J. Levy and T-M. Yan - Phys.Rev.Letters 22, 744 (1969) ; Phys.Rev. 187, 2159 (1969), and D1, 1035 (1970) ; C.H. Llewellyn Smith - Nuclear Phys. B17, 277 (1970).

- 13) J.D. Bjorken - Phys.Rev. 148, 1467 (1966). For the purpose of the present work it will be assumed that the troubles with the limit (2.2) in perturbation theory [reviewed, for example, by R. Jackiw at the Trieste Conference on Renormalization Theory, 1969 - CERN Preprint TH. 1065, (1969)] are to be interpreted as a defect in our knowledge of how to handle perturbation theory. It has been shown by J.C. Polkinghorne [Cambridge Preprint DAMTP 70/8] that they are intimately connected with questions of renormalization.

- 14) It is supposed that the constant $f \neq 0$. The algebra of fields [T.D. Lee, S. Weinberg and B. Zumino - Phys.Rev.Letters 18, 1029 (1967)] would make $f = 0$; however, it would also result ⁸⁾ in a vanishing contribution from transverse photon exchange in deep inelastic electron scattering.

- 15) I am grateful to Dr. P. Auvil for discussions on this point.

- 16) The Dirac spinor $u(p)$ satisfies $(\gamma.p - m)u = 0$.

- 17) Throughout this work, asymptotic limits are taken in directions in the complex plane for which the integrals converge. It is implicitly assumed that no extra undesirable terms appear when asymptotic limits of the analytic continuations of the integrals are taken.

- 18) This requirement was not incorporated in the versions of X given in Refs. 4), 5). The forms of X given here and in Ref. 5) are simpler than that of Ref. 4) and incorporate another requirement : that the amplitude be free of undesirable branch points in its asymptotic behaviour on unphysical sheets in the scaling limit.

- 19) J.B. Bronzan, I.S. Gerstein, B.W. Lee and F.E. Low - Phys.Rev. 157, 1448 (1967).
- 20) A precisely analogous cut is that found in the two-Reggeon/
particle coupling function extracted from the B_5 Veneziano
amplitude [I.T. Drummond, P.V. Landshoff and W.J. Zakrzewski -
Nuclear Phys. B11, 383 (1969)]. There the cut is in the
variable $\cos \omega$, where ω is the Toller angle, which is
kept finite as energies tend to infinity.
- 21) S.D. Drell and T-M. Yan - Phys.Rev.Letters 24, 181 (1970) also find
a relation between the large t behaviour of the form factor
and the small ω behaviour of the structure function. How-
ever, their relation is different. Here m and n need not
be integers, which was shown in Ref. 5) possibly to have im-
portant consequences for high-energy e^+e^- annihilation.
- 22) If this had not been the case, the sum rule (1.9) would have been
derived by applying the dispersion-relation argument to
 $[\psi_3(\omega) + \psi_3(-\omega)]$ instead of to $\psi_3(\omega)$.