CM-P00058234

Ref.TH.1117-CERN

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LOCAL CURRENT ALGEBRA MODEL WITH LINEAR TRAJECTORIES

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A B S T R A C T

We construct a model containing an infinite number of particles with arbitrarily high mass and spin corresponding to a family of linear Regge trajectories. The model does not contain states with spacelike momentum and gives rise to local currents obeying the standard equal time commutation relations. The basic structure is reminiscent of the oscillator representation ¹⁾ of the Veneziano model ; in particular ghost states with negative norm occur ²⁾.

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The model may be formulated in terms of an infinite component field $\psi(x)$ satisfying the wave equation

$$\{c_1 \square + i c_2 \Gamma^\mu \partial_\mu + M^2\} \psi(x) = 0 \quad (1)$$

where c_1 and c_2 are two real parameters whereas Γ^μ and M^2 are matrices operating on the components of $\psi(x)$. In order to specify these matrices we introduce a set of four harmonic oscillator modes described by creation and annihilation operators $a_\mu^\dagger, a_\mu$, ($\mu = 0, 1, 2, 3$):

$$\begin{aligned} [a_\mu, a_\nu^\dagger] &= -g_{\mu\nu} 1 \\ [a_\mu, a_\nu] &= [a_\mu^\dagger, a_\nu^\dagger] = 0, \end{aligned} \quad (2)$$

$g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ is the Lorentz metric. The representation of these operators is specified by the requirement that there exists a ground state with the property $a_\mu |0\rangle = 0$. A complete set of basis vectors is generated by vectors of the type

$$\varphi_N = a_{\nu_1}^\dagger a_{\nu_2}^\dagger \dots a_{\nu_n}^\dagger |0\rangle \quad (3)$$

where N stands for the collection $(\nu_1, \nu_2, \dots, \nu_n)$. Clearly, the metric in this vector space is indefinite, e.g., $(0|a_\mu a_\nu^\dagger|0) = -g_{\mu\nu}$. Accordingly the adjoint $a_\mu^\dagger$ is defined by $\eta^{-1} a_\mu^{*\dagger} \eta$, where the operator η stands for the indefinite metric: $\eta a_k = a_k \eta$, $\eta a_0 = -a_0 \eta$, $\eta|0\rangle = |0\rangle$.

The components of our basic field $\psi(x)$ are in one-to-one correspondence with the above basis vectors and may be labelled $\psi_N(x) = \psi_{\nu_1 \dots \nu_n}(x)$. The matrices a_μ and $a_\mu^\dagger$ may be applied to this field in an obvious fashion, e.g., $a_\mu^\dagger \psi_N = \psi_{\mu \nu_1 \dots \nu_n}$.

In terms of a_μ and $a_\mu^\dagger$ the matrices Γ^μ and M^2 are defined by

$$\begin{aligned}\Gamma_\mu &= i(a_\mu^\dagger - a_\mu) \\ M^2 &= \alpha - \beta a_\mu^\dagger a^\mu\end{aligned}\tag{4}$$

where α and β are two real parameters. This completes the specification of the wave equation and we now proceed to its solution. Clearly, if $c_2 = 0$ the plane wave solutions are eigenstates of M^2 and may be identified with the complete set of basis vectors φ_N :

$$M^2 \varphi_N = (\alpha + \beta n) \varphi_N \tag{5}$$

In the general case $c_2 \neq 0$ a complete set of plane wave solutions is obtained by tilting these basis vectors :

$$\psi(x) = e^{-ipx} e^{-i\gamma p A} \varphi_N \tag{6}$$

where $A_\mu = a_\mu + a_\mu^\dagger$, and γ is chosen such that $c_2 = \beta\gamma$. Note that the four parameters c_1, c_2, α, β occurring in the wave equation may be multiplied by a common factor without affecting the model. We find it convenient to make use of this freedom to normalize the coefficient c_1 by $c_1 = 1 + \beta\gamma^2$. With this convention we are left with α, β and γ as intrinsic parameters of the model and one easily verifies that the mass of the solution (6) is given by $p^\mu p_\mu = \alpha + \beta n$. A complete set of positive energy solutions results if φ_N runs through the complete set of basis vectors introduced above. Of course, there exists a complementary set of negative energy solutions - there are, however, no spacelike solutions $p^\mu p_\mu \leq 0$.

The wave equation is explicitly Lorentz invariant provided the four matrices a_μ constitute a Lorentz vector. This transformation rule specifies the spin content of the model : the set of positive energy solutions belonging to a given mass value $m_n^2 = \alpha + \beta n$ consists of a degenerate family of particles with spin $j = n, n-1, \dots, 0$. This spectrum corresponds to a family of linear Regge trajectories ; the leading trajectory and its first daughter are simple, the second and third daughters are occupied twice, etc. The leading trajectory and all even daughters have positive norm, whereas all odd daughters have negative norm.

We now turn to the construction of local currents and observe that the model admits of a conserved current

$$j^\mu = \frac{i}{2} \bar{\psi} \{ c_1 \overleftrightarrow{\partial}^\mu + i c_2 \Gamma^\mu \} \psi \quad (7)$$

where $\bar{\psi} = \psi^{*\top} \eta$ denotes the adjoint of ψ . In order that the model gives rise to a set of local currents it has to be extended in two respects : the particles must be equipped with internal degrees of freedom and the field $\psi(x)$ must be quantized. Concerning the introduction of internal degrees of freedom we observe that it is possible to treat the case of a broken symmetry by allowing the mass operator M^2 to split the internal degeneracy. We shall, however, restrict ourselves to a discussion of the simplest case exhibiting full symmetry. Concerning the need for quantization we mention that the extension to a local many-particle theory can be avoided if one restricts oneself to current algebra relations at infinite momentum which are satisfied by one particle states. From this point of view the introduction of quantized local fields is merely a convenient formal device.

We denote the quantized infinite component field by $\psi_a(x)$, the index a labelling the internal degrees of freedom. Under an infinitesimal transformation of the internal group this field transforms according to $\psi_a \rightarrow \psi_a + i \varepsilon_k \psi_b (Q_k)_{ba}$; the matrices

Q_k are the generators of the relevant representation of the internal group and satisfy the corresponding bracket relations. The currents are now defined as ³⁾

$$j_k^\mu(x) = \frac{i}{2} : \bar{\psi}_a(x) Q_{kab} \{c_1 \overleftrightarrow{\partial}^\mu + i c_2 \Gamma^\mu\} \psi_b(x) : \quad (8)$$

One easily verifies that these currents are conserved and that their time components satisfy the standard commutation relations provided the field and its conjugate momentum

$$\pi_a = \bar{\psi}_a \{c_1 \overleftrightarrow{\partial}_0 - \frac{i}{2} c_2 \Gamma_0\} \quad (9)$$

are quantized according to the canonical equal time commutation rules

$$[\psi_a(x), \pi_b(y)] = [\bar{\psi}_a(x), \bar{\pi}_b(y)] = i \delta_{ab} \delta(x-y) \quad (10)$$

(all other commutators between ψ_a , $\bar{\psi}_a$, π_a and $\bar{\pi}_a$ vanish at equal times).

Finally we indicate the one particle matrix elements of the above currents :

$$\begin{aligned} \langle p, N, a | j_k^\mu(0) | q, M, b \rangle &= \frac{1}{2} (2\pi)^{-3} Q_{kab} \cdot \\ &\cdot \bar{\varphi}_N e^{i\gamma_A p} \{c_1 (p^\mu + q^\mu) - c_2 \Gamma^\mu\} e^{-i\gamma_A q} \varphi_M \quad (11) \end{aligned}$$

In particular, if both N and M denote the ground state we find

$$\langle p, 0, a | j_{\mu}^{\nu}(0) | q, 0, b \rangle = \frac{1}{2} (2\pi)^{-3} Q_{\mu ab} (p^{\nu} + q^{\nu}) e^{\frac{1}{2} \gamma^2 t} \quad (12)$$

where $t = (p-q)^2$ is the invariant momentum transfer. This result shows that the model corresponds to form factors that decrease exponentially in the spacelike region. We have checked that the one-particle matrix elements (11) saturate the current algebra relations at infinite momentum.

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- 3) Note that the metric in the many particle space is also indefinite ; accordingly $\bar{\Psi}_a$ denotes the adjoint with respect to this indefinite metric.