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CURRENT ALGEBRA SUM RULES SUGGESTED BY THE PARTON MODEL

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A B S T R A C T

Those results which depend only on the weak and electromagnetic currents being carried by quarks are extracted from the parton model. We show that the most directly tested consequences of these results can be derived without the parton model if certain quark Hamiltonians are assumed and certain formal manipulations of operators are valid. Bounds and estimates are given for high-energy neutrino and anti-neutrino cross-sections.

1. INTRODUCTION

The recent accumulation of data for e^-p cross-sections in the deep inelastic region at SLAC ¹⁾ has been accompanied by much theoretical speculation (for a review see Ref. ²⁾). It is natural to apply the same speculative ideas to neutrino reactions ^{3),4)}. Despite the experimental difficulties, there is already some evidence in ν reactions ⁵⁾ for the scale invariance ⁶⁾ observed in e^-p experiments ¹⁾ and certain models $\sqrt{\text{vector dominance}}$ ⁷⁾ if simply formulated - see, however, Ref. ⁸⁾ for more complicated formulations - and also the algebra of fields ^{9),3)} if the Bjorken limit ¹⁰⁾ is valid $\sqrt{\text{are probably excluded by both the } e^- \text{ and } \nu \text{ experiments.}}$

"Parton" models ^{11),12),13),14),3),4)} of point-like structure inside the nucleon have been proposed to explain the scale invariance. Particular parton models are easily tested; for example, the e^-p cross-sections are too small to be explained in one particular quark parton model ¹³⁾ and the vector-axial vector interference term in ν cross-sections is very model-dependent ^{3),4)}. However, the parton model is flexible; for instance the e^-p cross-sections predicted by the quark parton model of Ref. ¹³⁾ are easily reduced by adding a background of neutral partons $\sqrt{\text{which could be responsible for binding the quarks (cf. Section 5)}}$. It is therefore interesting to extract those predictions which are independent of the details of the model.

In Section 3 we shall discuss parton models in which the electromagnetic and weak currents are carried by quarks ^{13),3)}, since only models in which the majority of the partons participating in the currents have spin one half are compatible with experiment ^{1),2)} (we assume the spin $\frac{1}{2}$ objects are quarks in order to reproduce the results of current algebra). Neutral partons, which could bind the quarks but have zero electromagnetic and weak charges, may also be present. Those predictions which are independent of the details of the model are given.

Only these predictions have a chance of being true more generally. In Section 4 we show that some of the implications of these predictions may be derived without assuming the parton model. The

derivations rely on the Bjorken limit $q_0 \rightarrow i\infty$ ¹⁰⁾ (see Section 2 for the notation). The validity of this limit has recently been put in doubt by the fact that it does not hold in perturbation theory^{15),16)}. However, there is no reason to believe that perturbation theory in the strong interactions is relevant to high energy behaviour. In perturbation theory the structure functions F_i diverge logarithmically as $q^2 \rightarrow -\infty$ (ν/q^2 fixed), so that if perturbation theory were relevant the parton model and all speculations about scale invariance would be wrong and the observed scale invariance¹⁾ would be a chimera.

In Section 5 we use the results of this paper and previously derived sum rules to give bounds and estimates for high energy neutrino and anti-neutrino scattering in terms of the electron scattering data.

2. NOTATION

The inelastic neutrino cross-section for scattering of a neutrino (anti-neutrino) from an unpolarized nucleon may be written¹⁷⁾

$$\frac{\pi}{EE'} \frac{d^2\sigma^{\nu,\bar{\nu}}}{d\Omega dE'} = M \frac{d^2\sigma^{\nu,\bar{\nu}}}{d|q^2|d\nu}$$

$$= \frac{E'}{E} \frac{G^2}{2\pi} \left(\cos^2 \frac{\theta}{2} W_2^{\nu,\bar{\nu}} + 2 \sin^2 \frac{\theta}{2} W_1^{\nu,\bar{\nu}} + \frac{E+E'}{M} \sin^2 \frac{\theta}{2} W_3^{\nu,\bar{\nu}} \right)^{(1)}$$

with $E(E')$ being the energy of the incident (scattered) neutrino (muon or electron) in the lab. frame, θ the scattering angle of the lepton, $q^2 = -4EE' \sin^2 \theta/2$ the momentum transfer squared to the nucleon, $\nu = q \cdot P = (E-E')M$ the energy transfer to the nucleon and $P(q)$ the momentum carried by the target nucleon (the weak current). Dropping terms which do not contribute to the cross-section if the lepton mass is neglected (as it is here), the structure

functions W_i are given by

$$\begin{aligned}
 W_{\mu\nu} &= \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \langle \vec{p} | [J_\mu^\mp(x), J_\nu^\pm(0)] | \vec{p} \rangle \\
 &= -g_{\mu\nu} W_1^{\nu\bar{\nu}}(q^2, \nu) + \frac{P_\mu P_\nu}{M^2} W_2^{\nu\bar{\nu}}(q^2, \nu) - i \frac{\epsilon_{\mu\nu\alpha\beta} P_\alpha q_\beta}{2M^2} W_3^{\nu\bar{\nu}}(q^2, \nu)^{(2)} \\
 &\quad + \dots
 \end{aligned}$$

Here, and throughout this paper, an average over nucleon spins is implied. Our normalization is such that in terms of the quark fields p , n and λ and the Cabibbo angle θ_c

$$J_\mu^+ = (J_\mu^-)^\dagger = \bar{p} \gamma_\mu (1 - \gamma_5) (n \cos \theta_c + \lambda \sin \theta_c), \quad (3)$$

$$\langle \vec{p} | \vec{p}' \rangle = (2\pi)^3 \frac{E}{M} \delta^3(\vec{p} - \vec{p}').$$

In the approximation $\sin^2 \theta_c = 0.05 \cong 0$, which we shall use in this paper (strictly our results multiplied by $\cos^2 \theta_c$ apply to the cross-sections for non-strange final states), it follows from isospin invariance for the final state that

$$\begin{aligned}
 W_i^{\nu p} &= W_i^{\bar{\nu} n} \\
 W_i^{\bar{\nu} p} &= W_i^{\nu n}.
 \end{aligned} \quad (4)$$

It is convenient to define the variables

$$\begin{aligned}
 x &= \frac{\nu}{ME} \\
 \omega &= -\frac{q^2}{\nu}.
 \end{aligned} \quad (5)$$

We shall assume that Bjorken's conjecture ⁶⁾ is correct and that in the limit $\nu \rightarrow \infty$ with ω fixed

$$\begin{aligned}
 M W_1 &\longrightarrow F_1(\omega) \\
 \frac{\nu W_2}{M} &\longrightarrow F_2(\omega) \\
 \frac{\nu W_3}{M} &\longrightarrow F_3(\omega)
 \end{aligned}
 \tag{6}$$

where the F_i are finite. The F_i satisfy the inequalities

$$0 \leq \frac{1}{2} |F_3^{\nu, \bar{\nu}}(\omega)| \leq F_1^{\nu, \bar{\nu}}(\omega) \leq \frac{1}{\omega} F_2^{\nu, \bar{\nu}}(\omega). \tag{7}$$

For electron scattering the same equations hold when the following substitutions are made

$$\begin{aligned}
 \frac{G^2}{2\pi} &\longrightarrow \frac{4\pi\alpha^2}{q^4} \\
 W_3 &\equiv 0
 \end{aligned}
 \tag{8}$$

$$J_\mu^+ \rightarrow J_\mu^\gamma = \frac{2}{3} \bar{p} \gamma_\mu p - \frac{1}{3} \bar{n} \gamma_\mu n - \frac{1}{3} \bar{\lambda} \gamma_\mu \lambda .$$

3. PARTON MODELS

The quark-parton model previously considered ^{13), 3)} is easily generalized to allow any number of neutral partons to be present (which may serve to bind the quarks), arbitrary distributions of

momenta and all possible coupling schemes for the quarks. In this case we find

$$F_1^{\gamma p}(\omega) = \frac{1}{2} \sum_N P_N \sum_q f_q^N(\frac{\omega}{2}) n_q^N Q_q^2, \text{ etc.} \quad (9)$$

where

P_N = probability of finding a configuration of N partons

$f_q^N(y)$ = probability of a parton of type q having a fraction y of the proton's longitudinal momentum in the N quark configuration

$$(\int f_q^N(y) dy = 1)$$

n_q^N = number of quarks of type q in an N parton configuration of the proton

Q_q = charge of parton of type q .

Using isospin invariance to equate the distribution of p quarks in the proton to the distribution of n quarks in the neutron etc., and noting that the λ and $\bar{\lambda}$ quark contributions always occur in the combination, $f_{\lambda\lambda}^N + f_{\bar{\lambda}\bar{\lambda}}^N$, we see that the F 's are expressed in terms of five arbitrary functions. In models with spin $\frac{1}{2}$ partons necessarily 9),13),3

$$\begin{aligned} \omega F_1^{\gamma} &= F_2^{\gamma} \\ \omega F_1^{\nu, \bar{\nu}} &= F_2^{\nu, \bar{\nu}} \end{aligned} \quad (10)$$

so that there are six independent F 's ($F_1^{\gamma p}, F_1^{\gamma n}, F_1^{\nu p}, F_1^{\nu n}, F_3^{\nu p}, F_3^{\nu n}$). We can therefore derive one equality which is

$$12(F_1^{\gamma p} - F_1^{\gamma n}) = (F_3^{\nu p} - F_3^{\nu n}). \quad (11)$$

Because of the conservation of strangeness $n_{\lambda}^N = n_{\bar{\lambda}}^N$, so that

$$\text{Baryon number of proton} = 1 = \frac{1}{3} (n_p^N + n_n^N - n_{\bar{p}}^N - n_{\bar{n}}^N) \quad (12)$$

$$\text{Charge of proton} = 1 = \frac{2}{3} (n_p^N - n_{\bar{p}}^N) - \frac{1}{3} (n_n^N - n_{\bar{n}}^N).$$

Integrating the F 's over ω and using Eq. (12) and the normalization condition for the f_q^N we find two further relations

$$\int d\omega (F_1^{\nu n} - F_1^{\nu p}) = 2 \quad (13)$$

$$-\frac{1}{2} \int d\omega (F_3^{\nu p} + F_3^{\nu n}) = 6. \quad (14)$$

Equation (13) is the Bjorken backward sum rule ¹⁸⁾ expressed as an integral over ω [using $F_2 = (1/\omega)F_1$ it becomes the infinite q^2 limit of one of Adler's sum rules ¹⁷⁾ - his other two sum rules are invalid for free quarks and hence invalid in the parton model]. Equation (14) was derived from current algebra in Ref. ³⁾. We can also derive the following inequalities by using the fact that every contribution to F_1 is positive

$$F_1^{\nu p} + F_1^{\nu n} \geq \frac{5}{18} (F_1^{\nu p} + F_1^{\nu n}). \quad (15)$$

Further inequalities can be derived by deducing from Eq. (12) that $n_p^N \geq 2$, $n_n^N \geq 1$. Hence

$$\int F_1^{\gamma p} d\omega \gg 1 \quad , \quad \int F_1^{\gamma n} d\omega \gg \frac{2}{3}, \quad (16)$$

$$\int F_1^{\gamma p} d\omega \gg 2 \quad , \quad \int F_1^{\gamma n} d\omega \gg 4.$$

The left-hand sides of these inequalities diverge logarithmically if Regge behaviour is assumed (cf. Section 5) and the first is already almost satisfied by the data.

Apart from the inequalities of Eq. (16) (which we shall not consider further), we have been able to deduce five results which are true in all quark parton models [Eqs. (10), (11), (13), (14), (15)]. Three of these results have been derived previously without using the parton model. In the next section we shall see that some consequences of the remaining two results [Eqs. (11) and (15)] can also be derived without using the parton model.

4. DERIVATIONS

In Ref. 8) and 3) the following equations were derived using the quark model with forces carried by a neutral vector meson (the 'gluon model')

$$\begin{aligned} & \left(\delta_{ij} - \frac{p_i p_j}{\vec{p}^2} \right) \int \omega d\omega F_1^{\gamma p, n} = C_{ij} \\ & = - \lim_{|\vec{p}| \rightarrow \infty} \frac{M}{|\vec{p}|^2} \int d^4x \delta(x_0) \langle p, n | [\dot{J}_i^{\gamma}(x), J_j^{\gamma}(0)] | p, n \rangle \end{aligned} \quad (17)$$

$$\left(\delta_{ij} - \frac{p_i p_j}{\vec{p}^2}\right) \int \omega d\omega (F_1^{\nu p} + F_1^{\nu n}) = D_{ij}$$

$$= - \lim_{|\vec{p}| \rightarrow \infty} \frac{M}{|\vec{p}|^2} \int d^4x \delta(x_0) \langle p | [\dot{J}_i^+(x), J_j^-(0)] + [\dot{J}_i^-(x), J_j^+(0)] | p \rangle \quad (18)$$

These equations are just the $n = 1$ moment sum rules written down by Bjorken ⁶⁾ [Eq. (17) is the $q^2 \rightarrow -\infty$, ω fixed limit of the sum rule of Cornwall and Norton ¹⁹⁾], except that we have used the fact that $\omega F_1 = F_2$ which holds in the gluon model ^{9), 3)}. Actually this latter result holds not only in models with a neutral vector meson field ($B(x)$) but also in models with a neutral pseudo-scalar ($\phi(x)$) or scalar ($\xi(x)$) field. We assume isospin to be a good symmetry so that the quark mass matrix M_q and the fields B , ϕ and ξ are $SU(2)$ scalars. Then, for the currents we are interested in (the isospin raising/lowering current and the electromagnetic current), the Dirac equation gives

$$\dot{J}_i^a(x) = \bar{\psi}(x) \left[-\gamma \cdot \vec{\delta} \gamma_0 \Gamma_i - \Gamma_i \gamma_0 \gamma \cdot \vec{\delta} + i(M_q + g_3 \xi(x))(\gamma_0 \Gamma_i - \Gamma_i \gamma_0) \right. \\ \left. + i g_1 (\gamma \cdot B \gamma_0 \Gamma_i - \Gamma_i \gamma_0 \gamma \cdot B) + g_2 \phi (\gamma_5 \gamma_0 \Gamma_i - \Gamma_i \gamma_0 \gamma_5) \right] \lambda^a \psi(x) \quad (19)$$

where

$$J_i^a(x) = \bar{\psi}(x) \Gamma_i \lambda^a \psi(x). \quad (20)$$

Hence (assuming that Schwinger terms are absent)

$$\int d^4x \delta(x_0) \langle a | [\dot{J}_i^x(x), J_j^x(0)] | a \rangle \\ = - \langle a | \bar{\psi}(0) \left[\gamma_i \vec{\delta}_j + \gamma_j \vec{\delta}_i - 2\delta_{ij} \gamma \cdot \vec{\delta} + 2i g_1 (\gamma_i B_j + \gamma_j B_i - 2\delta_{ij} \gamma \cdot B) \right. \\ \left. - 4i(M_q + g_3 \xi(0))\delta_{ij} - 4g_2 \delta_{ij} \gamma_5 \phi(0) \right] Q^2 \psi(0) | a \rangle \quad (21)$$

where Q is the quark charge matrix, and

$$\begin{aligned}
 & \int d^4x \delta(x_0) \langle P | [\dot{J}_i^+(x), J_j^-(0)] + [\dot{J}_i^-(x), J_j^+(0)] | P \rangle \\
 &= -2 \langle P | \bar{\psi}(0) [\gamma_i \overleftrightarrow{\delta}_j + \gamma_j \overleftrightarrow{\delta}_i - 2 \delta_{ij} \gamma \cdot \overleftrightarrow{\delta} + 2ig_i (\gamma_i B_j + \gamma_j B_i - 2\delta_{ij} \gamma \cdot B) \\
 & \quad - 2i(M_q + g_3 \xi(0)) \delta_{ij} - 2g_2 \delta_{ij} \gamma_5 \phi(0)] (2B+Y) \psi(0) | P \rangle
 \end{aligned} \tag{22}$$

where B and Y are the quark baryon number and hypercharge matrices. Note that it follows from the Lorentz tensor structure of the operators that the terms involving $M_q \xi$ and ϕ do not contribute when C_{ij} and D_{ij} are constructed so that

$$C_{ij}^{pn} = - \lim_{|\vec{p}| \rightarrow \infty} \frac{M}{|\vec{p}|^2} \langle p, n | \bar{\psi}(0) O_{ij} Q^2 \psi(0) | p, n \rangle \tag{23}$$

$$D_{ij} = - \lim_{|\vec{p}| \rightarrow \infty} \frac{2M}{|\vec{p}|^2} \langle P | \bar{\psi}(0) O_{ij} (2B+Y) \psi(0) | P \rangle$$

with the same operator O_{ij} for C_{ij} and D_{ij} .

It is convenient to write Q^2 in the form

$$Q^2 = \frac{1}{9} (B-Y) + \frac{5}{18} (2B+Y) + \frac{1}{3} I_3.$$

A positive quantity F_1^{B-Y} can be defined in terms of the Hermitean current $J_\mu^{B-Y} = \bar{\psi} \gamma_\mu (B-Y) \psi$ just as F_1^Y is defined in terms of J_μ^Y . F_1^{B-Y} satisfies Eqs. (17) and (23) when the substitutions $F_1^Y \rightarrow F_1^{B-Y}$, $J_\mu^Y \rightarrow J_\mu^{B-Y}$, $Q^2 \rightarrow (B-Y)^2 = B-Y$ are made. Hence, summing over neutron and proton states to remove the I_3 term, Eqs. (17) and (23) give

$$\begin{aligned}
 & \left(\delta_{ij} - \frac{p_i p_j}{\vec{p}^2} \right) \int \omega d\omega (F_1^{\gamma p} + F_1^{\gamma n} - \frac{2}{9} F_1^{B-\gamma}) \\
 & = - \lim_{|\vec{p}| \rightarrow \infty} \frac{M}{|\vec{p}|^2} \langle p | \bar{\psi}(0) O_{ij} \frac{5}{9} (2B + \gamma) \psi(0) | p \rangle \\
 & \quad F_1^{B-\gamma} \gg 0
 \end{aligned} \tag{24}$$

and

$$\int \omega d\omega (F_1^{\gamma p} + F_1^{\gamma n}) \gg \frac{5}{18} \int \omega d\omega (F_1^{\nu p} + F_1^{\nu n}). \tag{25}$$

We have therefore derived one consequence of the parton model result, Eq. (15), without using the parton model.

Following the method of Ref. 6), 9) and 3), it is easy to derive the result

$$\begin{aligned}
 & \frac{i \epsilon_{ijk} p_k}{|\vec{p}|} \int \omega d\omega (F_3^{\nu p} - F_3^{\nu n}) \\
 & = \lim_{|\vec{p}| \rightarrow \infty} \frac{2M}{|\vec{p}|^2} \int d^4x \delta(x_0) \langle p | [J_i^+(x), J_j^-(0)] - [J_i^-(x), J_j^+(0)] | p \rangle.
 \end{aligned} \tag{26}$$

By explicit calculation the right-hand side of this equation is

$$\lim_{|\vec{p}| \rightarrow \infty} - \frac{16M i \epsilon_{ijk}}{|\vec{p}|^2} \langle p | \bar{\psi}(0) [\gamma_0 \vec{\partial}_k + 2ig_i \epsilon_{ijk} \gamma_0 B_k] I_3 \psi(0) | p \rangle \tag{27}$$

in models in which $\dot{J}$ is given by Eq. (19).

$$\frac{d^2 \sigma^{\nu, \bar{\nu}}}{d\omega dx} = \frac{G^2 M E}{2\pi} \left(\omega F_1 \left(\frac{x^2}{2} + 1 - x \right) \mp \frac{x}{2} \left(1 - \frac{x}{2} \right) \omega F_3 \right) \quad (E \gg M) \quad (32)$$

where $0 \leq x \leq 1$, $0 \leq \omega \leq 2$ and we have put $\omega F_1 = F_2$ which is true in the models considered here. Eq. (32) is not valid down to exactly $x = 0 (\nu=0)$ or $\omega = 0 (q^2=0)$ as the description in terms of $F_i(\omega)$ is wrong there. In talking of high energies we are indulging the hope that there is a region $E \gg M$ below the energy at which the effective weak interaction fails ($E_{\text{lab}}^{\nu} \sim 10^5 \text{ GeV?}$). From Eq. (32) we obtain

$$\frac{1}{2} \frac{d(\sigma^{\nu(\bar{\nu})p} + \sigma^{\nu(\bar{\nu})n})}{dx} = \frac{3G^2 M E}{\pi} \left(J \left(\frac{x^2}{2} + 1 - x \right) \pm K x \left(1 - \frac{x}{2} \right) \right)$$

$$\frac{\sigma^{\nu p} + \sigma^{\nu n} + \sigma^{\bar{\nu} p} + \sigma^{\bar{\nu} n}}{4} = \frac{J G^2 M E}{\pi} \quad (33)$$

$$\frac{\sigma^{\nu p} + \sigma^{\nu n} - \sigma^{\bar{\nu} p} - \sigma^{\bar{\nu} n}}{4} = \frac{K G^2 M E}{\pi}$$

$$J = \frac{1}{6} \int \omega d\omega (F_1^{\nu p} + F_1^{\nu n})$$

$$K = -\frac{1}{12} \int \omega d\omega (F_3^{\nu p} + F_3^{\nu n}) = -\frac{1}{12} \int_{\frac{1}{2}}^{\infty} \frac{d\rho}{\rho^2} (F_3^{\nu p} + F_3^{\nu n})$$

$$\rho = \frac{1}{\omega} = \frac{\nu}{-q^2}$$

Writing

$$\begin{aligned}\langle P | \bar{\psi}(0) \gamma_\mu \overleftrightarrow{\partial}_\nu \psi(0) | P \rangle &= A P_\mu P_\nu + A' g_{\mu\nu} \\ \langle P | \bar{\psi}(0) \gamma_\mu B_\nu \psi(0) | P \rangle &= B P_\mu P_\nu + B' g_{\mu\nu}\end{aligned}\tag{28}$$

we find

$$\int \omega d\omega (F_3^{\nu P} - F_3^{\nu n}) = -16M(A + 2ig, B).\tag{29}$$

But it follows from the equations above that

$$\int \omega d\omega (F_1^{\gamma P} - F_1^{\gamma n}) = -\frac{4M}{3}(A + 2ig, B)\tag{30}$$

so that

$$12 \int \omega d\omega (F_1^{\gamma P} - F_1^{\gamma n}) = \int \omega d\omega (F_3^{\nu P} - F_3^{\nu n})\tag{31}$$

which is a consequence of the parton result Eq. (11).

5. CONSEQUENCES

We shall now explore the consequences of the sum rules derived in Section 4 for the high-energy ν ($\bar{\nu}$) cross-section which may be written ⁶⁾

Using Eq. (25) we have

$$J \ll \frac{3}{5} \int \omega d\omega (F_1^{\gamma p} + F_1^{\gamma n}).$$

The plausible assumption that $\int \omega d\omega F_1^{\gamma p} \gg \int \omega d\omega F_1^{\gamma n}$ (which in parton language requires that in the proton $n_p^N + n_{\bar{p}}^N \gg n_n^N + n_{\bar{n}}^N$) and the assumption that $F_1^{\gamma p}$ does not behave wildly beyond the measured region, give

$$\begin{aligned} \int \omega d\omega F_1^{\gamma p} &\approx 0.32 \\ J &\ll 0.38. \end{aligned} \tag{35}$$

The inequality $|F_3| \ll 2F_1$ then gives $|K| \ll J \ll 0.38$ (this is simply the requirement of positive cross-sections).

The actual value of J is model-dependent as can be seen by assuming $SU(3)$ is a good symmetry and expressing Eqs. (17), (18) and (23) in terms of octet and singlet couplings.

Denoting the ratio of the F to the D octet couplings by α we find

$$J = \frac{\int \omega d\omega ((1+3\alpha)F_1^{\gamma p} + 2F_1^{\gamma n})}{3+3\alpha}. \tag{36}$$

In parton models in which the nucleon consists of three 'valence' quarks and an $SU(3)$ scalar sea of quarks and anti-quarks (Ref. 13); $F_1^{\gamma p} + F_1^{\gamma n} = 6F_1^{\gamma p}$ [in agreement with the data for small ω (Ref. 1), 5)] $\alpha = \infty$ and $J \approx 0.32$ (such models are called model A below; we do not necessarily assume here that the momentum distribution for the 'valence' quarks is the same as for the others). There is nothing sacrosanct about these models; in models with a quark-anti-quark 'valence' pair and an $SU(3)$ scalar core of quarks

and anti-quarks with baryon number one (model B below) $F_1^{\nu p} + F_1^{\nu n} = 2F_1^{\gamma p} + 4F_1^{\gamma n}$, $\alpha = 0$ and $J = \int \omega d\omega (F_1^{\gamma p} + 2F_1^{\gamma n}) < 0.32$.

The quantity K is also model-dependent. In model A

$$\begin{aligned} 36(F_1^{\gamma n} - F_1^{\gamma p}) &= F_3^{\nu p} + F_3^{\nu n} \\ K &= 3 \int \omega d\omega (F_1^{\gamma p} - F_1^{\gamma n}) \end{aligned} \quad (37)$$

which should be testable. Model B makes no prediction unless it is assumed that the momentum distribution f_q^N is the same for all the quarks, in which case Eq. (37) is obtained.

K can be bounded numerically in a modified version of the model of Bjorken and Paschos¹³⁾ in which

$$\begin{aligned} \int x f^N(x) dx &= \frac{1}{N} \\ \int F_2^{\gamma p} \frac{d\omega}{2} &= \sum_N P_N \frac{\langle \sum_i Q_i^2 \rangle_N^{\text{proton}}}{N} = \text{Mean square charge per parton} \end{aligned} \quad (38)$$

where we have used Eqs. (9) and (10). Previously, no neutral partons were put in and the average value of the sum of the squares of the parton charges in the N parton configuration was

$$\langle \sum_i Q_i^2 \rangle_N^{\text{proton}} = 1 + \frac{2(N-3)}{9} \quad (39)$$

in contradiction to the experimental result in Eq. (38) (assuming, again, that $F_2^{\gamma p}$ is well-behaved beyond the measured region). We can effectively introduce neutral partons by multiplying the second term in Eq. (39) by ϵ , which represents the average fraction of the background which is composed of charged partons (quarks) and will be assumed independent of N . Then

$$0.16 = \frac{2\epsilon}{9} + \left(1 - \frac{2\epsilon}{3}\right) \left\langle \frac{1}{N} \right\rangle$$

$$\left\langle \frac{1}{N} \right\rangle < 0.16 \quad (40)$$

$$\epsilon < 0.72.$$

The small value of $\langle 1/N \rangle$ indicates that many partons are needed in this model. The bound for $\langle 1/N \rangle$ and the parton model values for F_3 given in Ref. ³⁾ give

$$K = 2 \left\langle \frac{1}{N} \right\rangle < 0.32. \quad (41)$$

Putting this bound into Eq. (37) gives a bound for the difference of the e^-p and e^-n cross-section in this model.

K can also be bounded in an empirical fashion, which is of some interest, if we assume Regge behaviour

$$\nu W_3 \xrightarrow[q^2 \text{ fixed}]{\nu \rightarrow \infty} \nu^{\alpha_{\omega, \phi(0)}} \quad (42)$$

$$\alpha_{\omega, \phi(0)} \simeq 1/2.$$

If νW_3 is dominated by a term $c(q^2) \nu^\alpha$ for $\nu \gg -q^2$ even when the limit $-q^2 \rightarrow \infty$, $\omega = -q^2/\nu$ fixed ($\ll 1$) is taken ^{20), 21)} we find

$$F_3 \xrightarrow{\omega \rightarrow 0} \omega^{-\alpha} = \omega^{-1/2} = \rho^{1/2}. \quad (43)$$

[Brandt has partially justified the analogous result for F_1 and F_2 ²²⁾].

To put a limit on K we now make an ansatz for $-(F_3^{\nu p} + F_3^{\nu n})$ which is as large as possible for small ρ , since the integral in the definition of K [Eq. (33)] is weighted in favour of small ρ . We shall assume that the F_3 are everywhere negative (which is true in simple parton models) and saturate the inequality $|F_3| \leq 2F_1$ for small ρ . We put

$$F_3^+ = F_3^{\nu p} + F_3^{\nu n} = -2(F_1^{\nu p} + F_1^{\nu n}), \quad \rho < \rho_1 \quad (44)$$

$$F_3^+ = F_3^+(\rho_1) \left(2\sqrt{\frac{\rho}{\rho_1}} - 1\right), \quad \rho > \rho_1.$$

We now put $F_1^{\nu p} + F_1^{\nu n} = RF_1^{\chi p}$ where R is a constant to be determined shortly. Assuming that the possibly observed decrease in $F_2^{\chi p}$ for large ρ (1) is not a real effect, the Regge limit $F_2^{\chi p} = \text{const.} \approx 0.35$ seems to be reached at $\rho \approx 2$. Provided $\rho_1 \gg 2$ our ansatz and its first derivative are therefore continuous at $\rho = \rho_1$. Putting $\rho_1 = 2$ and using the F_3^+ sum rule ³⁾ [Eq. (14)] and the e^-p data we find $R \approx 5$. Note that in all parton models [cf. Eq. (15)]

$$7.2 F_1^{\chi p} > \frac{18}{5} (F_1^{\chi p} + F_1^{\chi n}) > F_1^{\nu p} + F_1^{\nu n} \quad (45)$$

$$R \leq 7.2$$

if $F_1^{\chi p} \gg F_1^{\chi n}$. Our ansatz for F_3^+ is therefore almost as large as is reasonably possible for small ρ (R could be increased to the limit by decreasing ρ_1 or letting F_3^+ approach Regge behaviour much more rapidly). We conclude that

$$-\int F_3^+ \omega d\omega \lesssim -\int F_3^+(\text{ansatz}) \omega d\omega \approx 3$$

(46)

$$0 < K \lesssim 0.25.$$

The interest of this analysis is that it indicates that if Regge theory is valid then F_3^+ must be substantial for small ρ in order to fulfil the F_3^+ sum rule unless Regge behaviour is approached very slowly.

6. CONCLUSIONS

The main results of this paper are the sum rule of Eq. (31) and the inequality of Eq. (25), which depend on the assumption of certain quark model Hamiltonians and the validity of the Bjorken limit and of certain formal manipulations of field operators. These results are just the consequences of Eqs. (11) and (15) which are most directly amenable to experimental tests, since the integrals involved enter directly into the expressions for high energy ν and $\bar{\nu}$ cross-sections. Since Eqs. (11) and (15) are the only relations which hold in all quarks parton models and have not also been derived otherwise [except for the presumably trivial Eq. (16)] we conclude that the model is hard to substantiate.

Our results for ν and $\bar{\nu}$ cross-sections are summarized in Eqs. (33), (35) and (46). We expect that probably $J \approx 0.3$. The CERN experiment ⁵⁾ gives $J+K = 0.6 \pm 0.15$ when a linear fit is made (if we assume that neutron and proton cross-sections are not very different, so that the fact that the propane target is not exactly an equal p-n mixture is unimportant), suggesting that K is nearer the upper rather than the lower limit in Eq. (46). The CERN data give an x distribution which is compatible with the whole range $0 \ll K \ll J$ in models with $\omega F_1 = F_2$.

When e^-n data are available it will be possible to construct more specific parton models which, combined with 'solutions' of the sum rules (such as the 'solution' of the F_3^+ sum rule in Section 5), should give a reasonable indication of the ν and $\bar{\nu}$ cross-sections if the parton model has any validity. However, until tests of some of the parton/quark model sum rules are made, it must be remembered that all this work could well be unconnected with the real world.

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